

# Quantitative Methods – I

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## Practice 8

Lorenzo Cavallo

For any clarification:

[lorenzo.cavallo.480084@uniroma2.eu](mailto:lorenzo.cavallo.480084@uniroma2.eu)

# THEME #1



## Continuous Random Variables

# Continuous Random Variables

- ✓ **Normal** distribution  $X \sim N(\mu, \sigma^2)$
- ✓ **Standard normal** distribution  $Z \sim N(0,1)$
- ✓ **Chi-square** distribution  $X \sim \chi_n^2$
- ✓ **t-Student** distribution  $X \sim t_n$
- ✓ **F** distribution  $X \sim F_{n,m}$

# Probability distributions of Continuous Variables

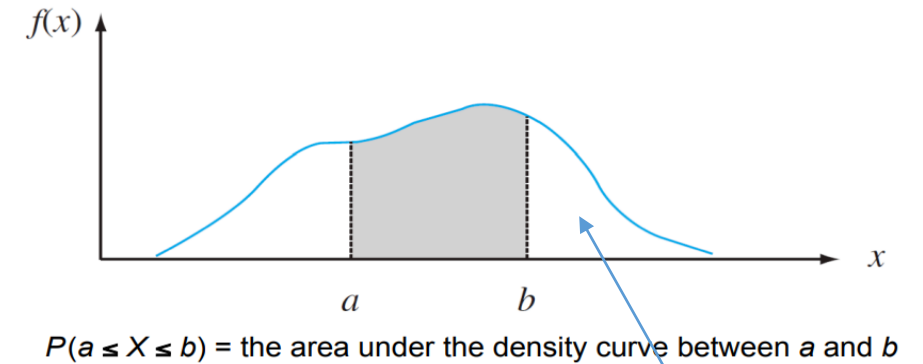
A continuous random variable  $X$  can assume any value over an interval.

2 characteristics (similar to discrete variables):

1. For any interval  $[a, b]$ ,  $0 \leq P(a < X < b) \leq 1$

2. The total probability of all the intervals within which  $X$  can assume a value is 1

$$\sum P(-\infty < X < +\infty) = 1$$



area under the entire graph of  $f(x) = 1$

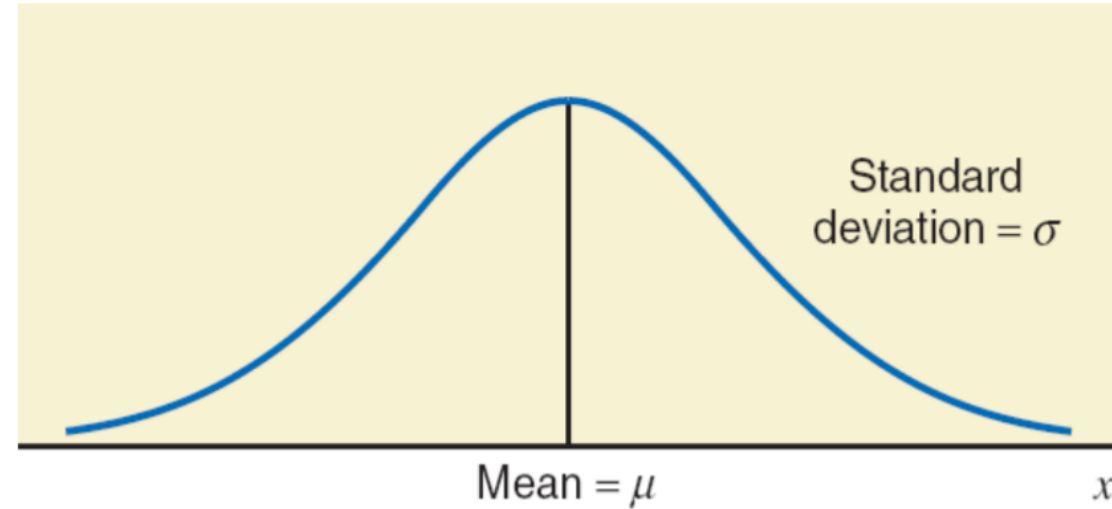
## Normal distribution function, $N(\mu, \sigma^2)$

Most widely used continuous probability distribution

$$x \sim N(\mu, \sigma^2)$$

$$\text{Density function: } f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

parameters:  $\mu$  (mean), and  $\sigma$  (the spread)

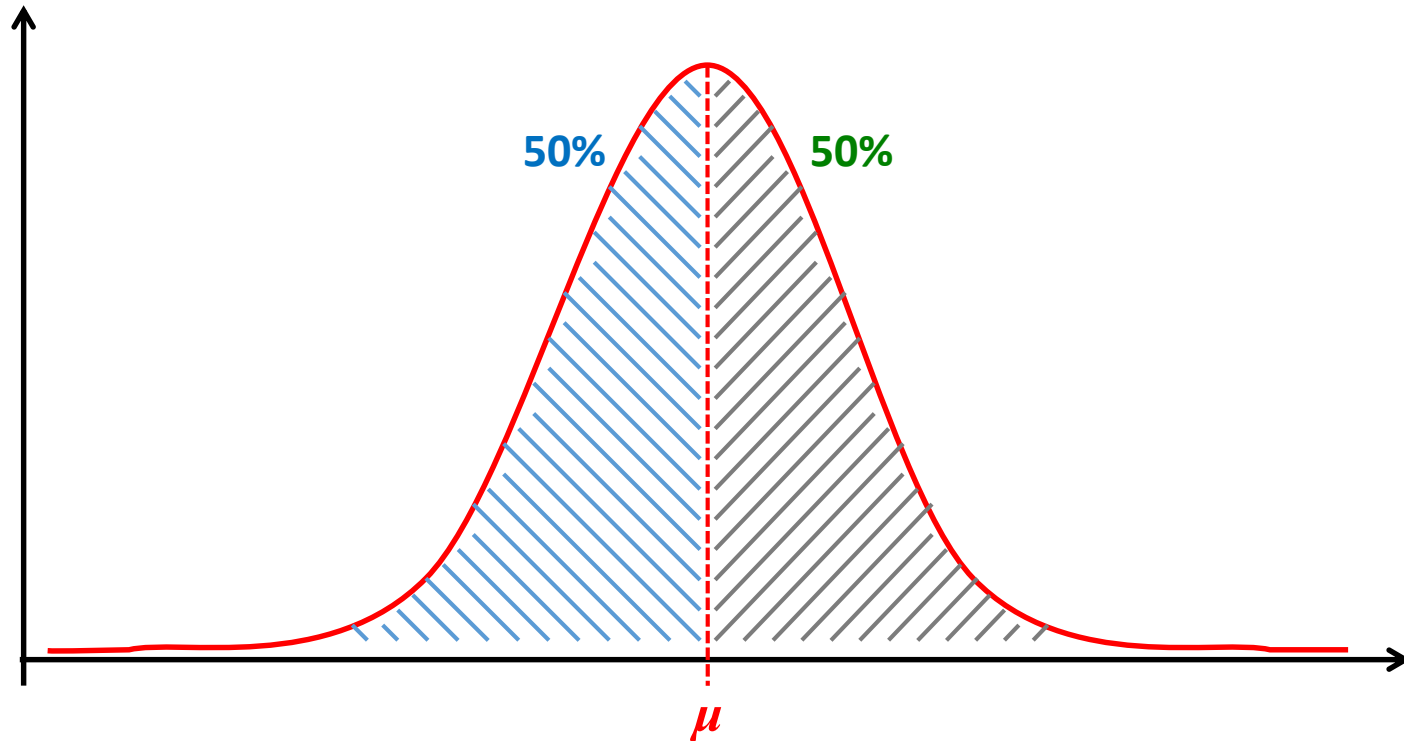


### Some examples of things that follow a Normal Distribution:

- Heights of people
- Size of items produced by a machine
- Errors in measurements
- Blood Pressure
- Test Scores
- ...

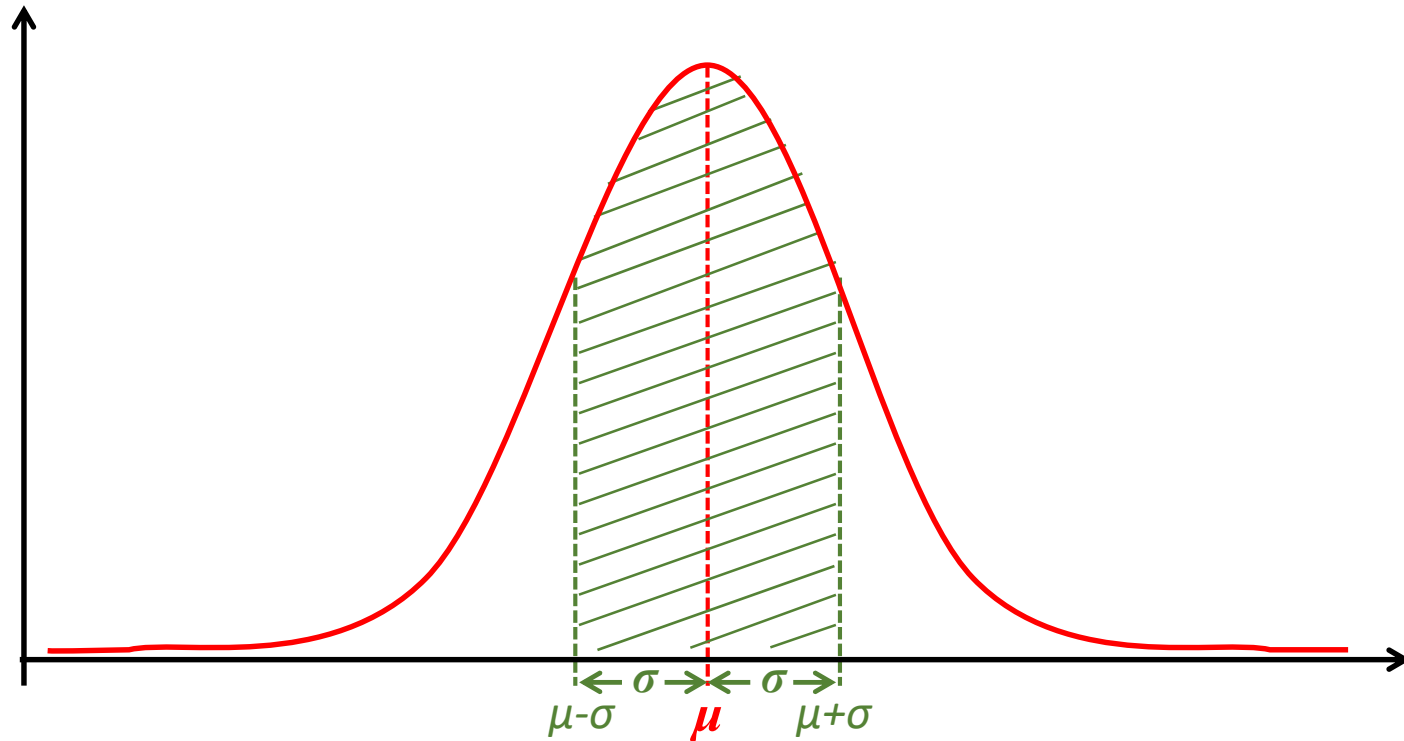
## Normal distribution, $N$ :

- It is a **bell-shaped** curve
- Symmetry about the mean  $\mu$  (mean = mode = median)
- The total area under the curve is equal to 1 (or 100%)
- 50% of the area is to the left of the mean, and 50% to the right



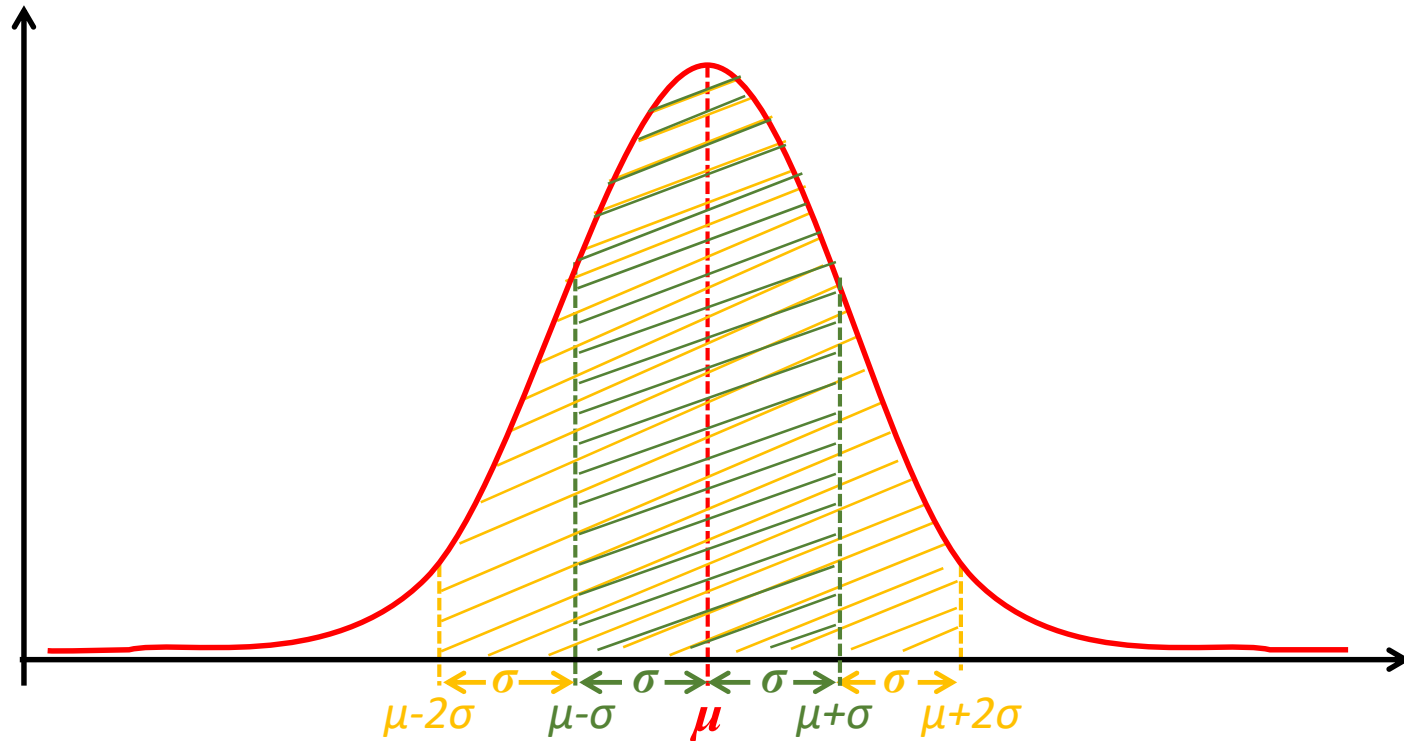
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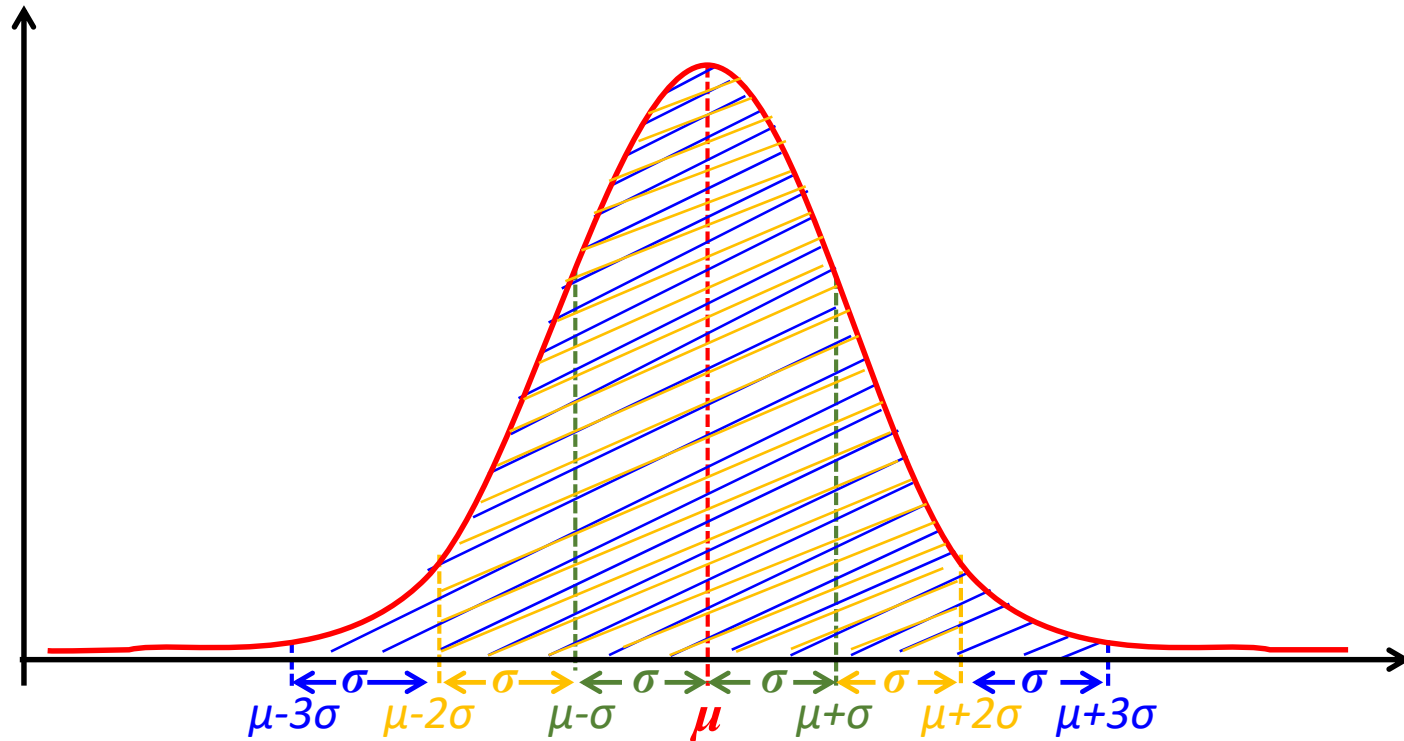
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- Approximately **95.4%** of the area is between  $[\mu - 2\sigma; \mu + 2\sigma]$





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- Approximately **68.3%** of the area is between  $[\mu - \sigma; \mu + \sigma]$
- Approximately **95.4%** of the area is between  $[\mu - 2\sigma; \mu + 2\sigma]$
- Approximately **99.7%** of the area is between  $[\mu - 3\sigma; \mu + 3\sigma]$



**The Standard Normal Distribution,  $Z$ ,** is a Normal Distribution with mean equal to 0 and standard deviation equal to 1,  $Z \sim N(0,1)$

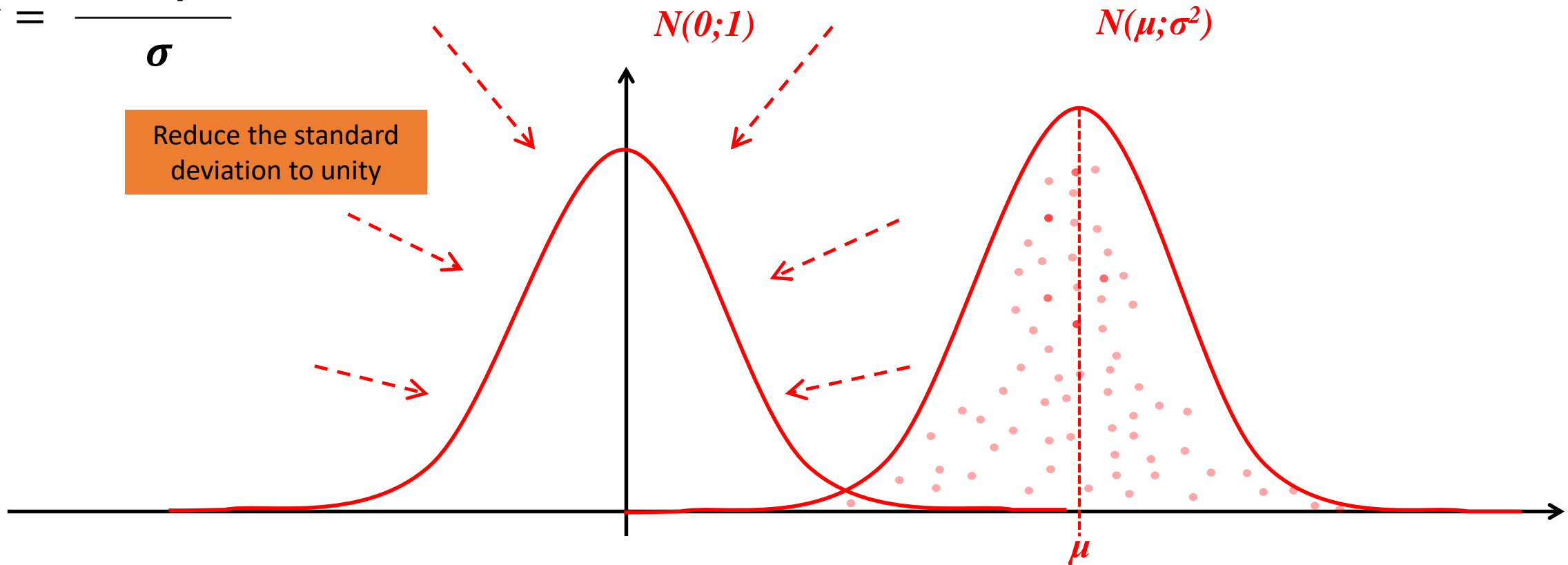
ANY Normal distribution,  $X \sim N(\mu, \sigma^2)$ , can be converted into a Standard Normal,  $Z \sim N(0,1)$ , by standardization:

$$Z = \frac{x - \mu}{\sigma}$$

$$Z = \frac{x - \mu}{\sigma}$$

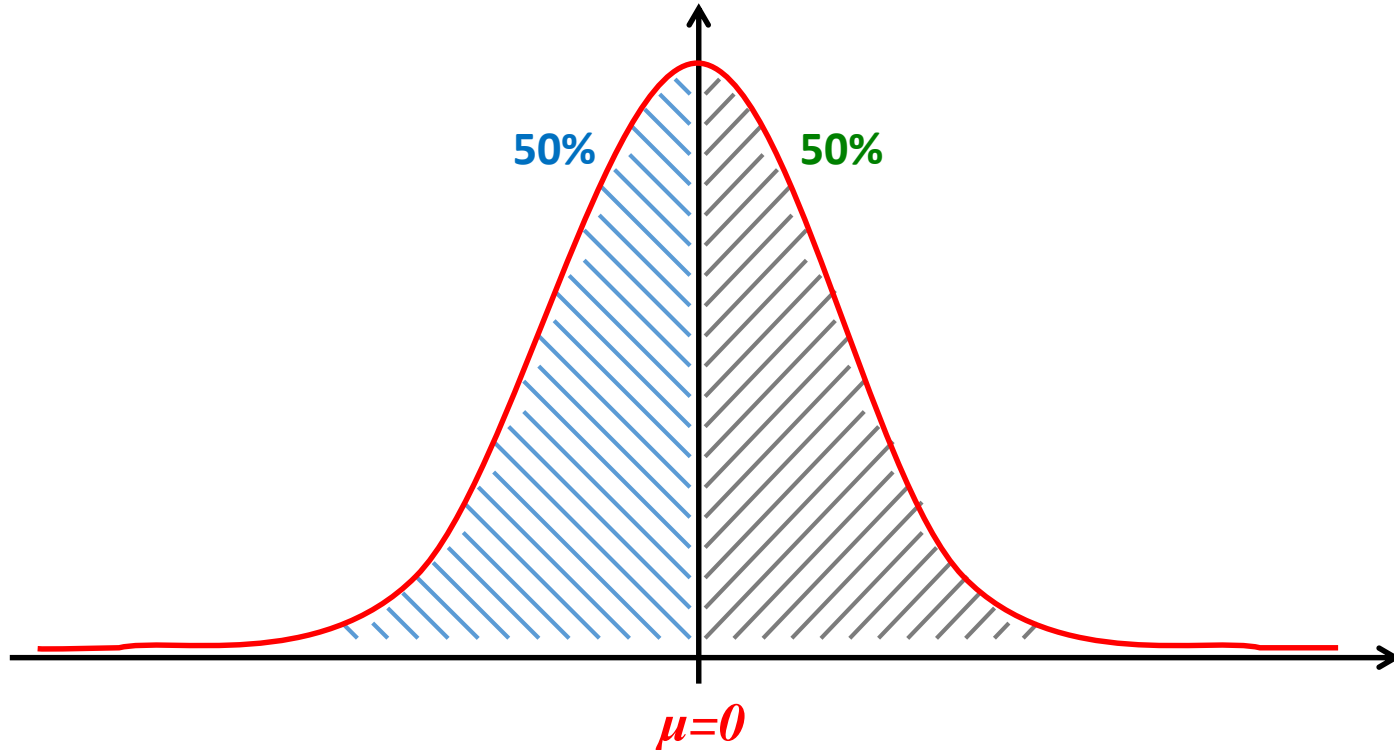
Move the mean to 0

Reduce the standard deviation to unity



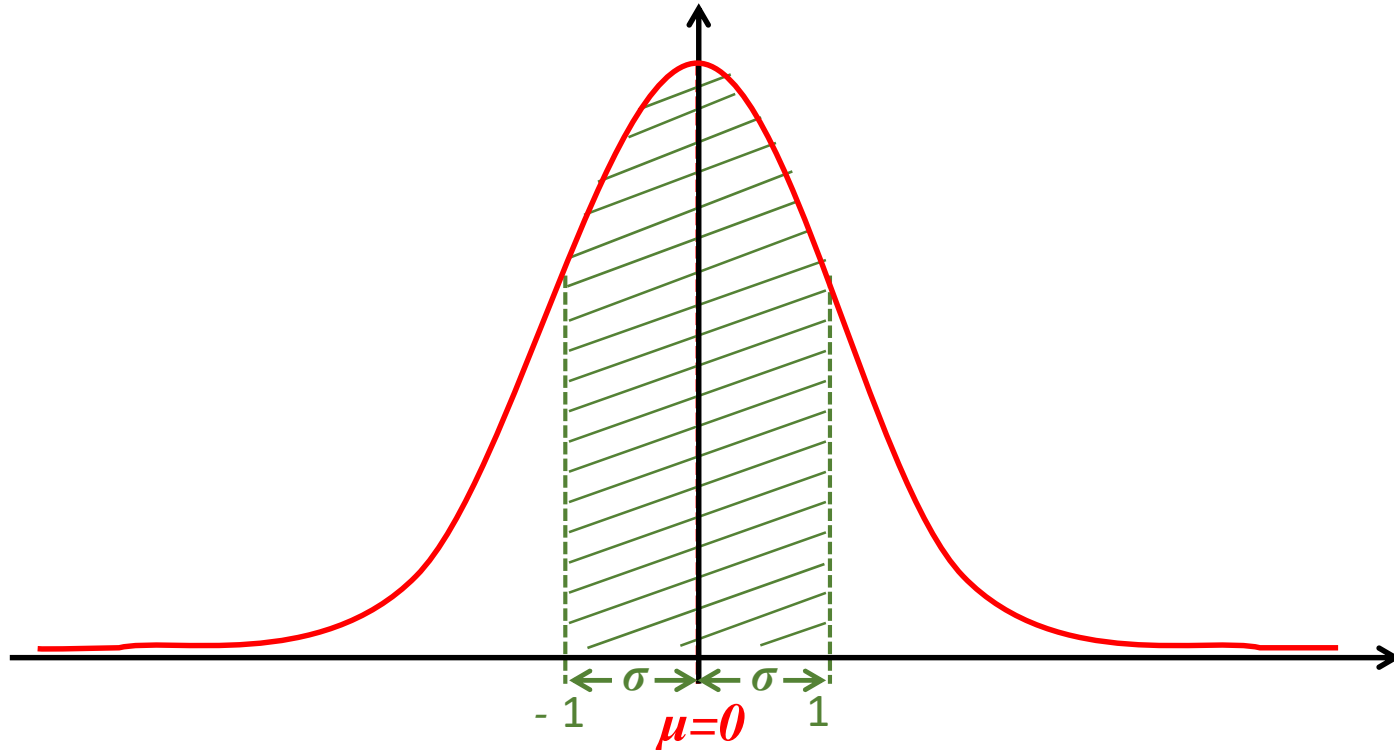
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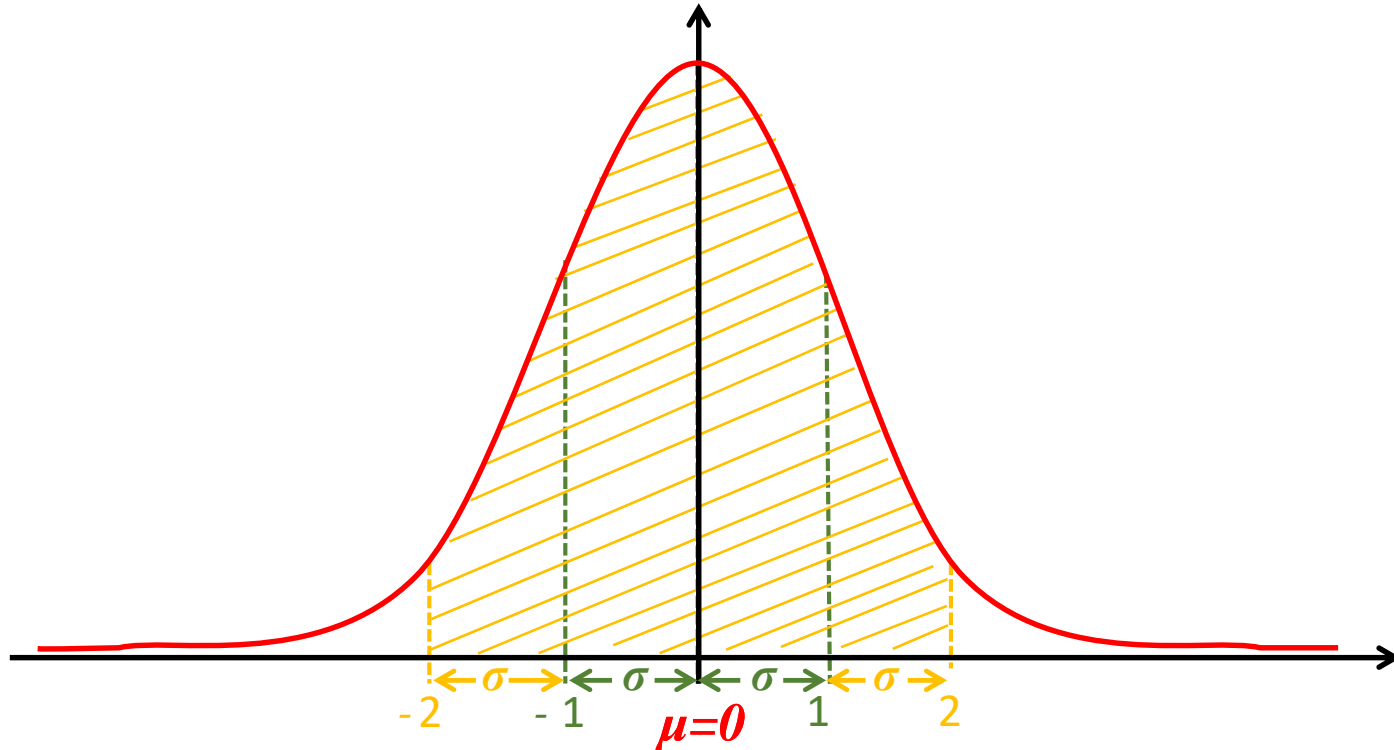
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- The total area under the curve is 1 (or 100%)
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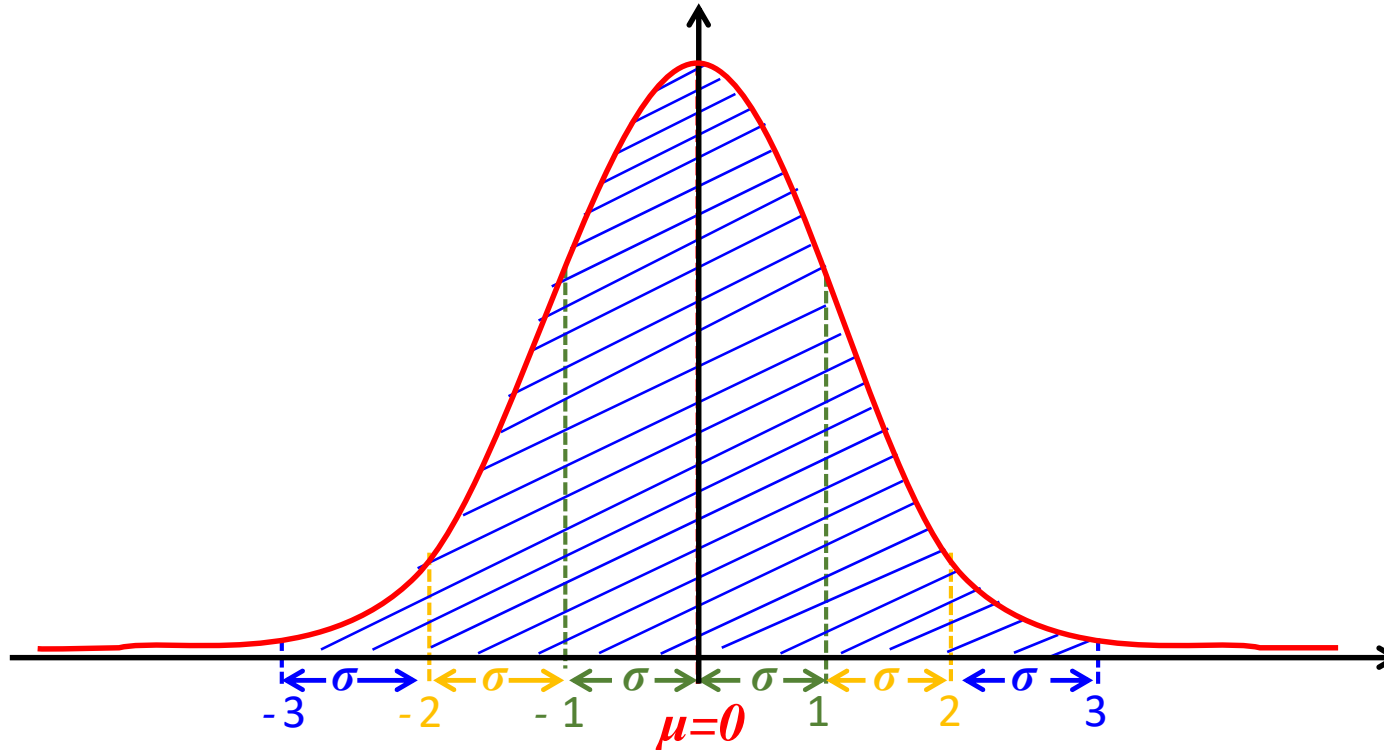
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- Approximately **95.4%** of the area is between  $[\mu - 2\sigma = -2; \mu + 2\sigma = 2]$

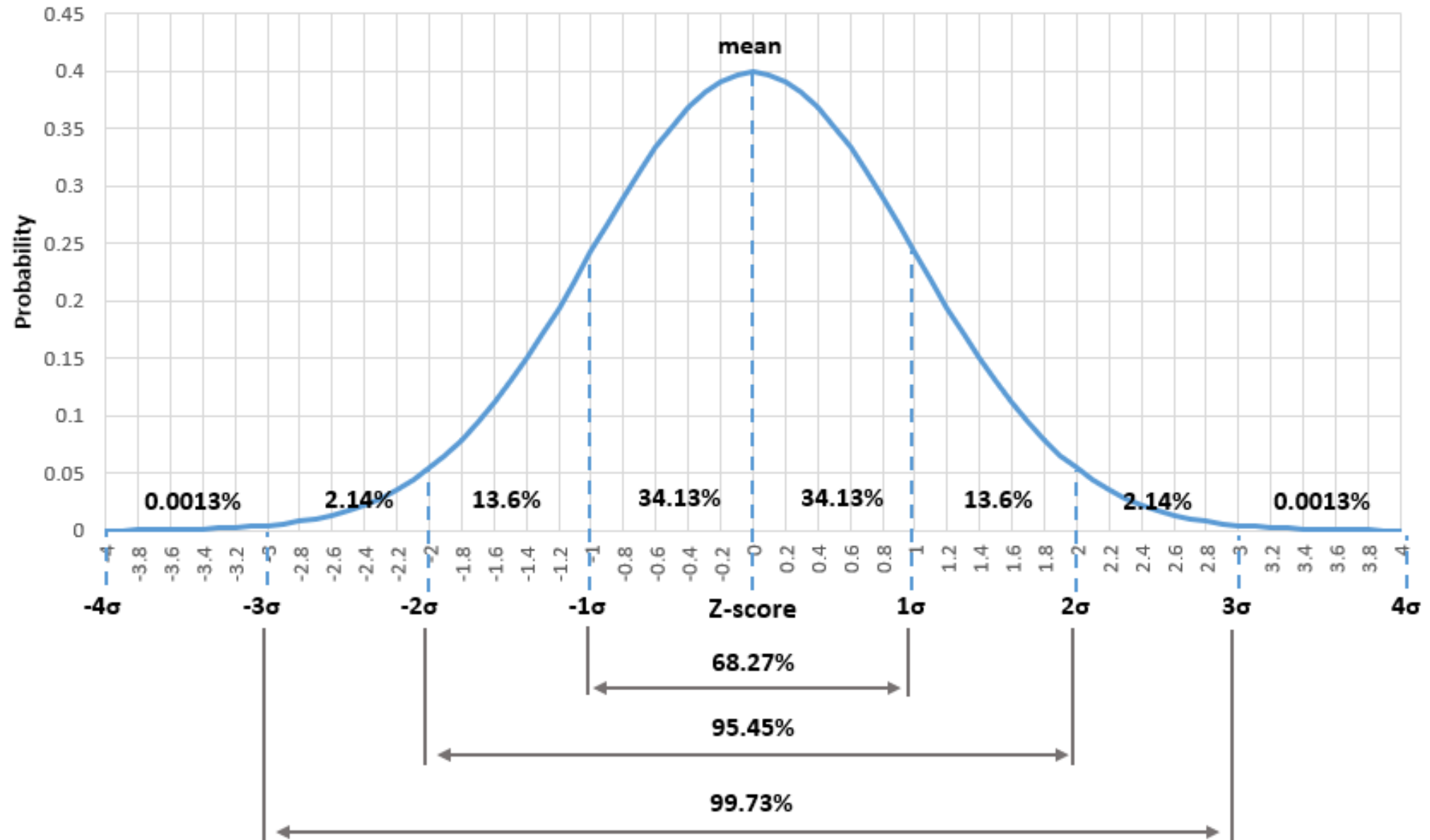


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- Approximately **95.4%** of the area is between  $[\mu - 2\sigma = -2; \mu + 2\sigma = 2]$
- Approximately **99.7%** of the area is between  $[\mu - 3\sigma = -3; \mu + 3\sigma = 3]$



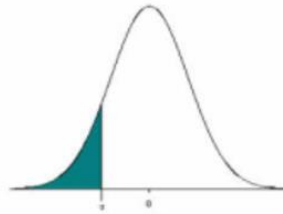
## Z distribution with quantiles



# Why it is so useful the Standard Normal Distribution $Z \sim N(0,1)$ and the standardization of a Normal distribution in a Standard Normal Distribution?

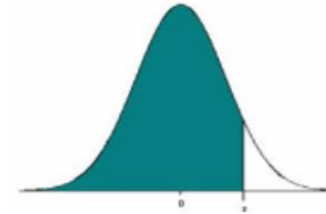
We have the tables of the Standard Normal Distribution  $Z \sim N(0,1)$

Table of Standard Normal Probabilities for Negative Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0015	0.0015	0.0014	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9985	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

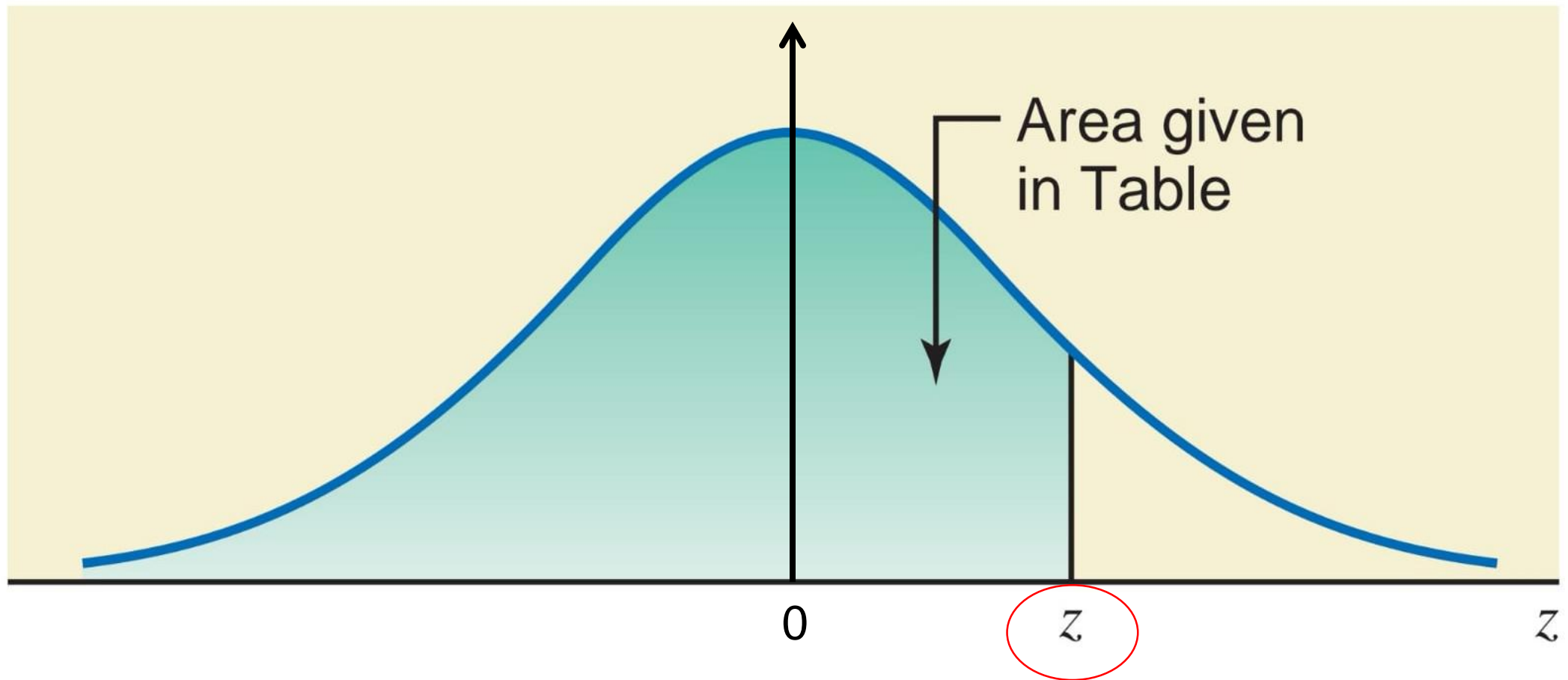


**Why it is so useful the Standard Normal Distribution  $Z \sim N(0,1)$  and the standardization of a Normal distribution in a Standard Normal Distribution?**

We have the tables of the Standard Normal Distribution  $Z \sim N(0,1)$

For each **z-score**, the tables give to us the value of the **area** between  $-\infty$  to  $z$

The area in the tables is also the **probability of having a value lower than  $z$**



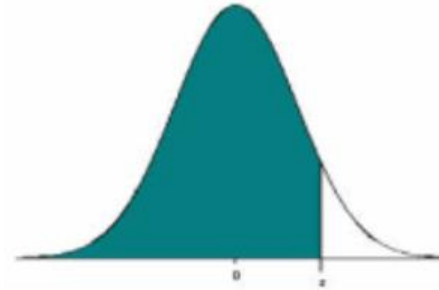
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-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
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-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
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-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

A normal distribution curve is shown with the horizontal axis labeled with 0 and z. The area under the curve to the left of z is shaded teal.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998



**Table of Standard Normal Probabilities for Positive Z-scores**



If we have only the table of Standard Normal Probabilities for Positive Z-scores we can find all the values (also for negative  $z$ )

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964

# How to use the table?

## Positive value of z



Find the value of z in the table.

This is the area between  $[-\infty; z]$  or the **probability lower than z**

Ex.  $P(z < 2.04) = ?$

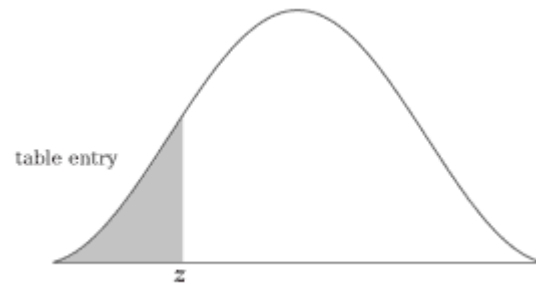
$P(z < 2.04) = 0.9793$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998



# How to use the table?

## Negative value of z



From the condition of probability distribution:

$$\Sigma P(-\infty < Z < +\infty) = 1$$

Z is also symmetric around the mean

You can find the negative value of z taking the value of z in absolute value (or in modulus) doing:

$$1 - |z|$$

Ex.  $P(z < -2.04) = ?$

$$P(z < -2.04) = 1 - P(z < 2.04) = 1 - 0.9793 = 0.0207$$

...in Table for negative Z scores....

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233

### **Exercise 1**

Z is a normally standard distributed variable with mean  $\mu = 0$  and standard deviation  $\sigma = 1$ .

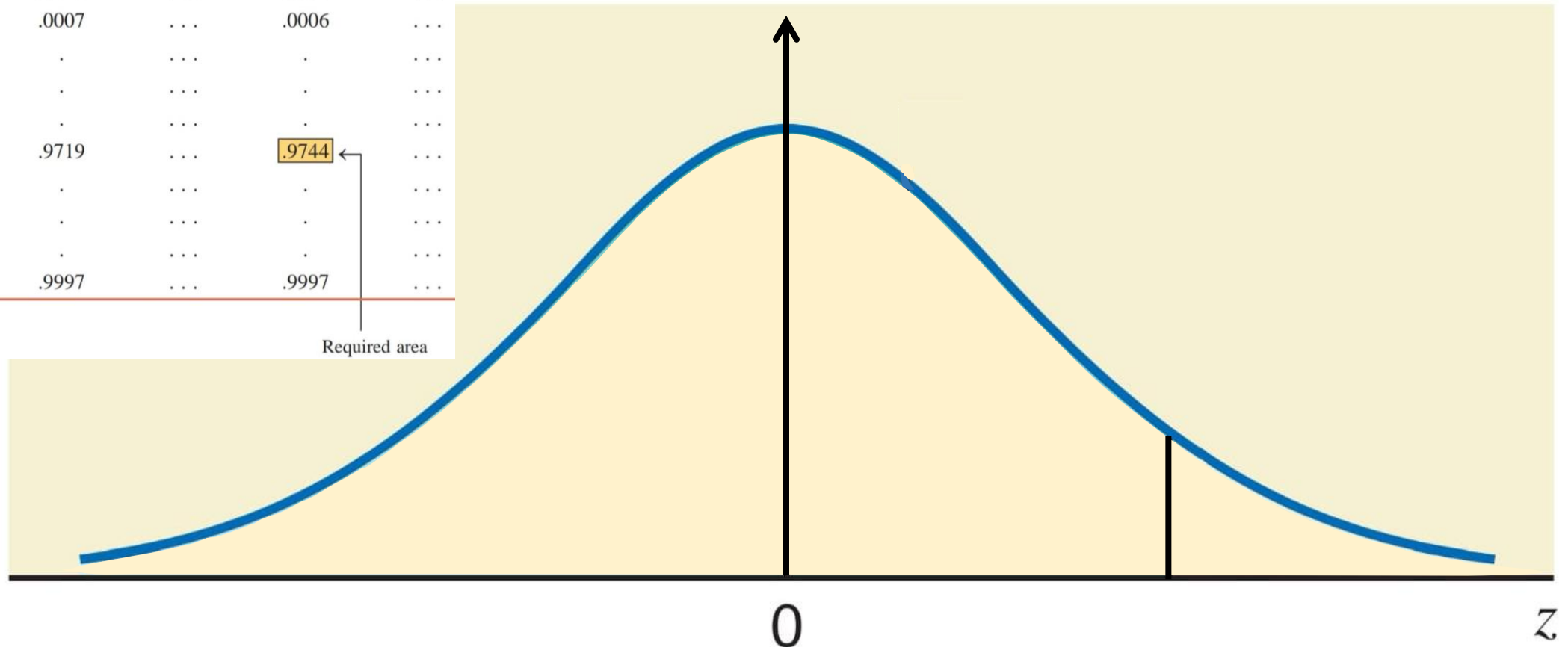
Find

- a.  $P(z < 1.95)$
- b.  $P(z > 2.32)$
- c.  $P(1.19 < z < 2.12)$

**Exercise with Tables of Standard Normal Probabilities for positive Z-scores**

## Exercise 1

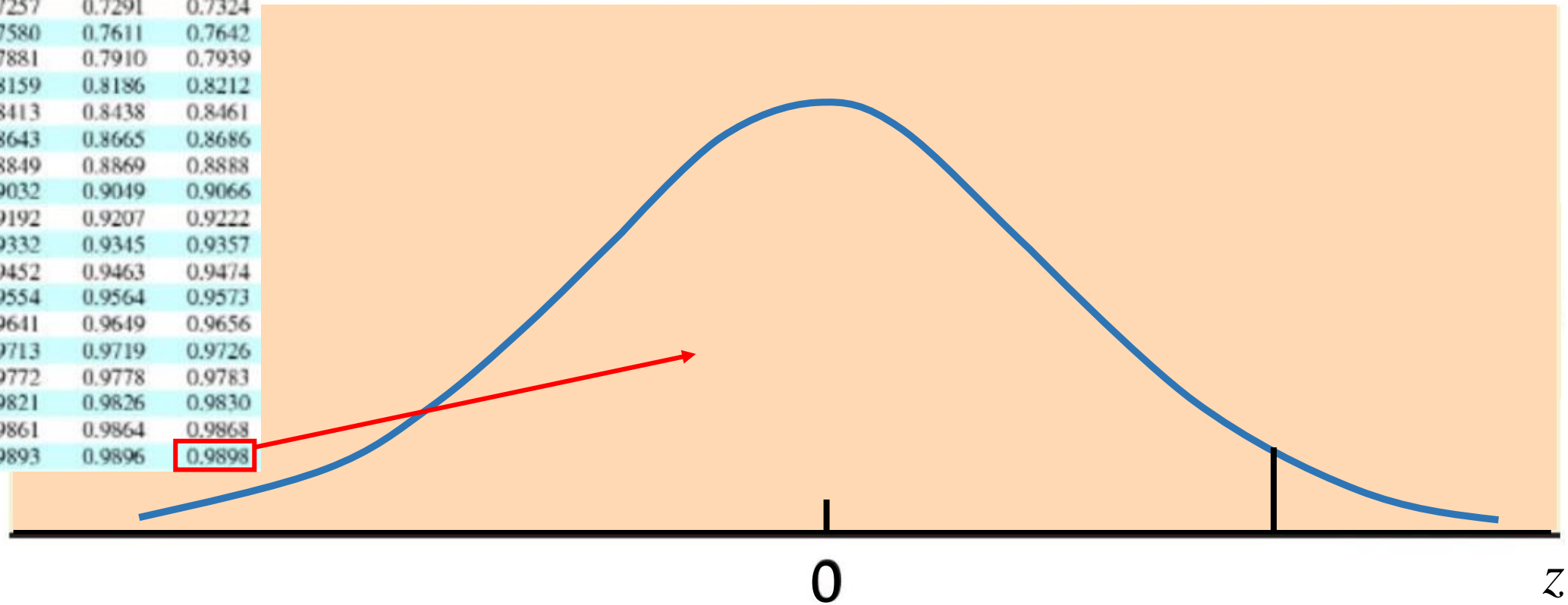
z	.00	.01	...	.05	...
-3.4	.0003	.0003	...	.0003	...
-3.3	.0005	.0005	...	.0004	...
-3.2	.0007	.0007	...	.0006	...
.	.	.	...	.	...
.	.	.	...	.	...
.	.	.	...	.	...
1.9	.9713	.9719	...	.9744	...
.	.	.	...	.	...
.	.	.	...	.	...
.	.	.	...	.	...
3.4	.9997	.9997	...	.9997	...



a.  $P(z < 1.95) = 0.9744 = 97.44\%$

## Exercise 1

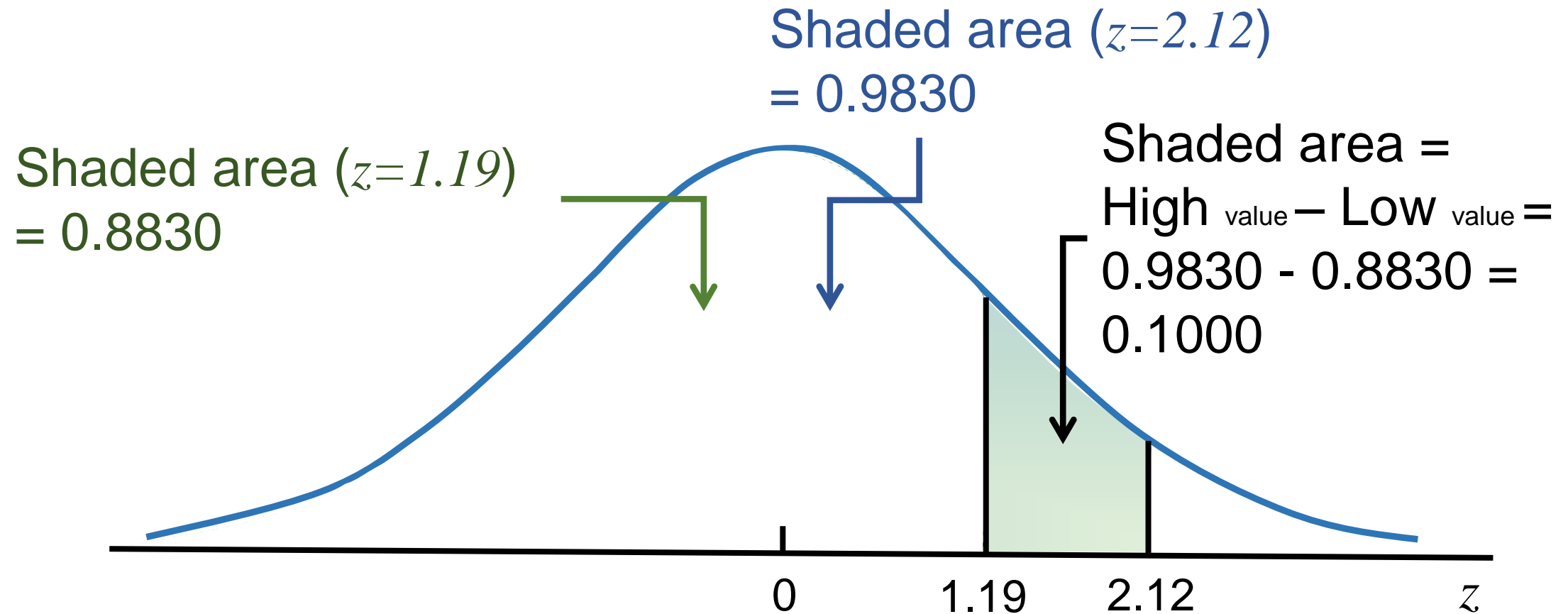
z	0.00	0.01	0.02
0.0	0.5000	0.5040	0.5080
0.1	0.5398	0.5438	0.5478
0.2	0.5793	0.5832	0.5871
0.3	0.6179	0.6217	0.6255
0.4	0.6554	0.6591	0.6628
0.5	0.6915	0.6950	0.6985
0.6	0.7257	0.7291	0.7324
0.7	0.7580	0.7611	0.7642
0.8	0.7881	0.7910	0.7939
0.9	0.8159	0.8186	0.8212
1.0	0.8413	0.8438	0.8461
1.1	0.8643	0.8665	0.8686
1.2	0.8849	0.8869	0.8888
1.3	0.9032	0.9049	0.9066
1.4	0.9192	0.9207	0.9222
1.5	0.9332	0.9345	0.9357
1.6	0.9452	0.9463	0.9474
1.7	0.9554	0.9564	0.9573
1.8	0.9641	0.9649	0.9656
1.9	0.9713	0.9719	0.9726
2.0	0.9772	0.9778	0.9783
2.1	0.9821	0.9826	0.9830
2.2	0.9861	0.9864	0.9868
2.3	0.9893	0.9896	0.9898



b.  $P(z > 2.32) = 0.0102 = 1.02\%$



## Exercise 1



c.  $P(1.19 < z < 2.12) = 0.1000 = 10\%$

### Exercise 2

X is a normally distributed variable with mean  $\mu = 30$  and standard deviation  $\sigma = 4$ . Find

- a.  $P(x < 40)$
- b.  $P(x > 21)$
- c.  $P(30 < x < 35)$

### Exercise with the 2 Tables of Standard Normal Probabilities for positive and negative Z-scores

#### Solution

To solve the point a., b., c. we have to convert the Normal distribution, X in the Normal standard distribution, Z, by standardization:

$$z = \frac{x - \mu}{\sigma}$$

with mean  $\mu = 30$  and standard deviation  $\sigma = 4$ .

- a. For  $x = 40$ , the z-value (or z-score) is:  $z = (40 - 30) / 4 = \mathbf{2.5}$   
Hence  $P(x < 40) = P(z < 2.5) = [\text{area to the left of } 2.5] \text{ (from the Normal standard distribution table)} = 0.9938 = 99.38\%$
- b. For  $x = 21$ , the z-score is:  $z = (21 - 30) / 4 = \mathbf{-2.25}$   
Hence  $P(x > 21) = P(z > -2.25) = [\text{total area}] - [\text{area to the left of } -2.25] = 1 - P(z < -2.25) = 1 - 0.0122 = 0.9878 = 98.78\%$
- c. For  $x = 30$ :  $z = (30 - 30) / 4 = \mathbf{0}$  and for  $x = 35$ :  $z = (35 - 30) / 4 = \mathbf{1.25}$   
Hence  $P(30 < x < 35) = P(0 < z < 1.25) = \text{High value} - \text{Low value} = P(z < 1.25) - P(z < 0) = [\text{area to the left of } z = 1.25] - [\text{area to the left of } 0] = 0.8944 - 0.5 = 0.3944 = 39.44\%$

### Exercise 2bis

X is a normally distributed variable with mean  $\mu = 30$  and standard deviation  $\sigma = 4$ . Find

- a.  $P(x < 40)$
- b.  $P(x > 21)$
- c.  $P(30 < x < 35)$

### Exercise with only the Tables of Standard Normal Probabilities for positive Z-scores

Solution

We have to convert the Normal distribution, X in the Normal standard distribution, Z, by standardization:

$$z = \frac{x - \mu}{\sigma}, \text{ with mean } \mu = 30 \text{ and standard deviation } \sigma = 4.$$

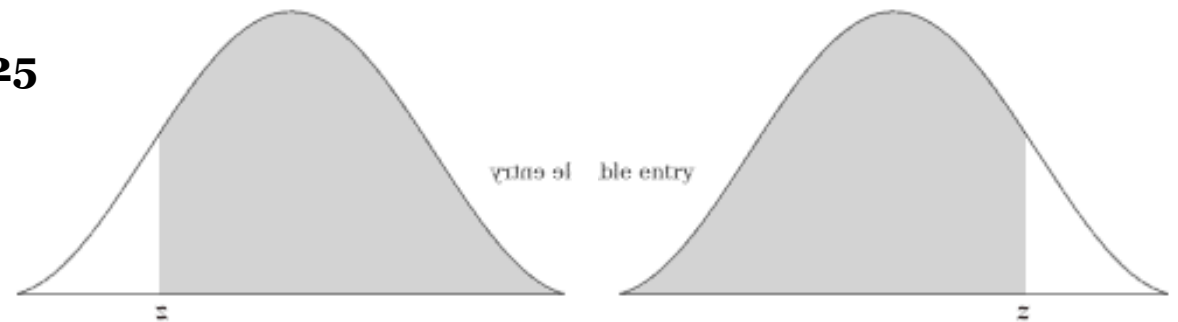
- a.  $P(x < 40) = P(z < 2.5) = 0.9938 = 99.38\%$
- b. For  $x = 21$ , the z-score is:  $z = (21 - 30) / 4 = -2.25$   
Hence  $P(x > 21) = P(z > -2.25)$ .

**The Z function is Symmetric**

$$\text{Hence } P(z > -2.25) = P(z < 2.25)$$

$P(z < 2.25)$  from the table is 0.9878 so,  $P(z > -2.25) = P(z < 2.25) = 98.78\%$

- c.  $P(30 < x < 35) = P(0 < z < 1.25) = P(z < 1.25) - P(z < 0) = 0.8944 - 0.5 = 0.3944 = 39.44\%$



### Exercise 3

A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. What is the probability that a car picked at random is travelling at more than 100 km/hr?

Solution

Let  $X$  be the random variable that represents the speed of cars.

$X$  has  $\mu = 90$  and  $\sigma = 10$ .

We have to find the probability that  $x$  is higher than 100 or  $P(x > 100)$

For  $x = 100$ , the z-score is:  $z = (100 - 90) / 10 = 1$

$$P(x > 90) = P(z > 1) = [\text{total area}] - [\text{area to the left of } z = 1] = 1 - P(z < 1) = 1 - 0.8413 = 0.1587$$

The probability that a car selected at a random has a speed greater than 100 km/hr is the 15.87%

#### Exercise 4

For a certain type of computers, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. John owns one of these computers and wants to know the probability that the length of time will be between 50 and 70 hours.

#### Solution

Let  $x$  be the random variable that represents the length of time.  
It has a mean of 50 and a standard deviation of 15.

We have to find the probability that  $x$  is between 50 and 70 or  $P(50 < x < 70)$

For  $x = 50$  the z-score is:  $z = (50 - 50) / 15 = 0$

For  $x = 70$  the z-score is:  $z = (70 - 50) / 15 = 1.33$  (rounded to 2 decimal places)

$P(50 < x < 70) = P(0 < z < 1.33) = [\text{area to the left of } z = 1.33] - [\text{area to the left of } z = 0] = P(z < 1.33) - P(z < 0) = 0.9082 - 0.5 = 0.4082$ .

The probability that John's computer has a length of time between 50 and 70 hours is the 40,82%.

### Exercise 5

Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

#### Solution

Let  $X$  be the random variable that represents the scores.

$X$  is normally distributed with a mean of 500 and a standard deviation of 100.

The total area under the normal curve represents the total number of students who took the test.

If we multiply the values of the areas under the curve by 100, we obtain percentages.

For  $x = 585$  the z-score is:  $z = (585 - 500) / 100 = \mathbf{0.85}$

The proportion  $P$  of students who scored below 585 is given by:

$$P(z < 0.85) = [\text{area to the left of } z = 0.85] = 0.8023 = 80.23\%$$

Tom scored better than 80.23% of the students who took the test and he will be admitted to this University.

### Exercise 6

The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assembled at this plant in a period of time

- a) less than 19.5 hours?
- b) between 20 and 22 hours?

### Exercise with the 2 Tables of Standard Normal Probabilities for positive and negative Z-scores

Solution

- a) We have to calculate the probability that a car can be assembled in less than 19.8 hours or  $P(x < 19.5)$

The z-score for  $x=19.5$  is:  $z = (19.5-20)/2$ , so we have to calculate from the table of Z the probability:

$$P(z < -0.25) = 0.4013 = 40.13\%$$

- b) We have to calculate the probability that a car can be assembled between 20 and 22 hours or  $P(20 < x < 22)$

The z-score for  $x=20$  is  $z=0$  and for  $x=22$  is  $z=1$ . So,

$$P(0 < z < 1) = P(z < 1) - P(z < 0) = 0.8413 - 0.5 = 0.3413 = 34.13\%$$

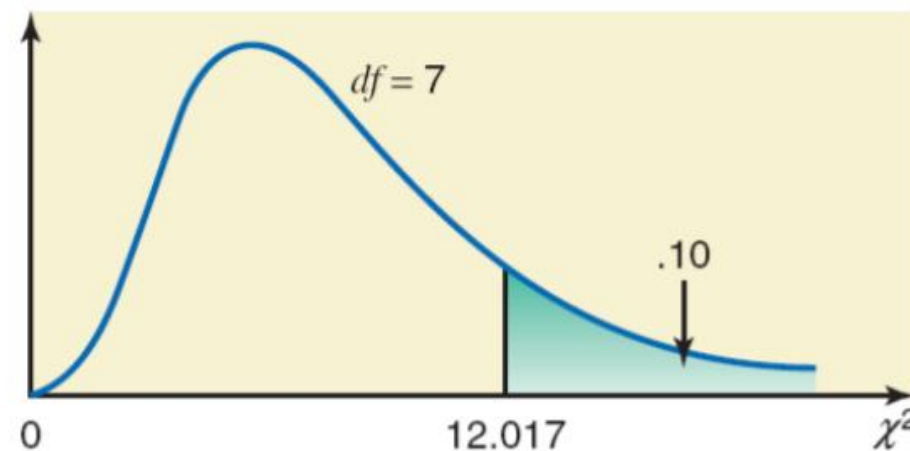
## Chi square distribution, $\chi^2$

Let  $Z_1, Z_2, Z_3, \dots, Z_n$ , be  $n$  i.i.d. random variables  $Z_i \sim N(0,1)$

$$X = \sum_{i=1}^n Z_i^2 \sim \chi_n^2$$

where  $\mathbf{n}$ =degrees of freedom (only parameter)

$$E(X) = n \quad \text{and} \quad V(X) = 2n$$





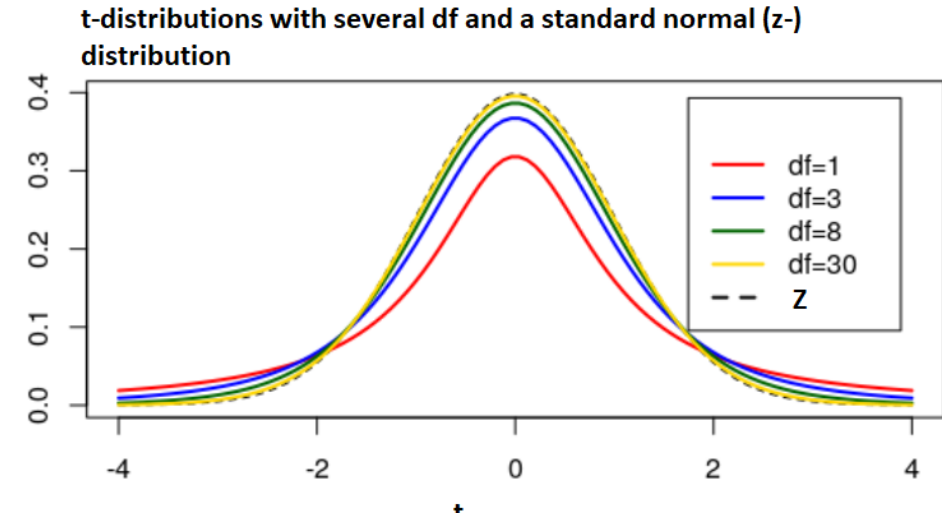
## t-Student distribution, $t_n$

Let  $Z \sim N(0,1)$  and  $Y \sim \chi_n^2$ ,

$$X = \frac{Z}{\sqrt{Y/n}} \sim t_n$$

where  $n$  = degrees of freedom

- The total area under a t-student distribution curve is 1
- Distribution is bell-shaped and symmetric around  $E(X) = 0$
- $V(X) = n/(n-2)$ , for  $n > 2$
- For  $n > 60$ ,  $X$  converges to  $Z \sim N(0,1)$



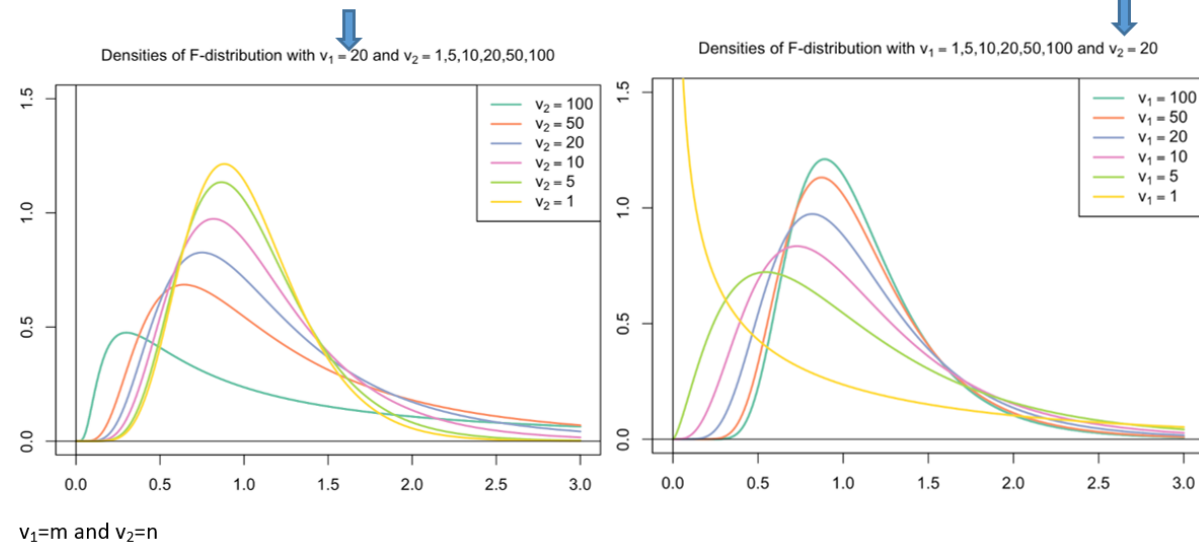
## F distribution, $F_{n,m}$

Let  $\mathbf{X} \sim \chi_m^2$  and  $\mathbf{Y} \sim \chi_n^2$ ,

$$F = \frac{X/m}{Y/n} \sim F_{m,n}$$

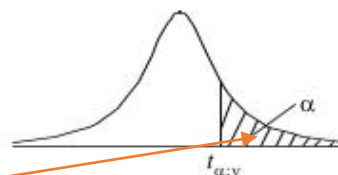
where  $m$  = numerator's degree of freedom,  $n$  = denominator's degree of freedom (2 parameters)

The total area under F distribution curve is 1

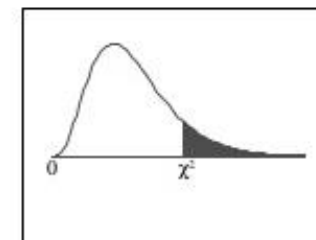


**Table of the Student's  $t$ -distribution**

The table gives the values of  $t_{\alpha;v}$  where  
 $\Pr(T_v > t_{\alpha;v}) = \alpha$ , with  $v$  degrees of freedom

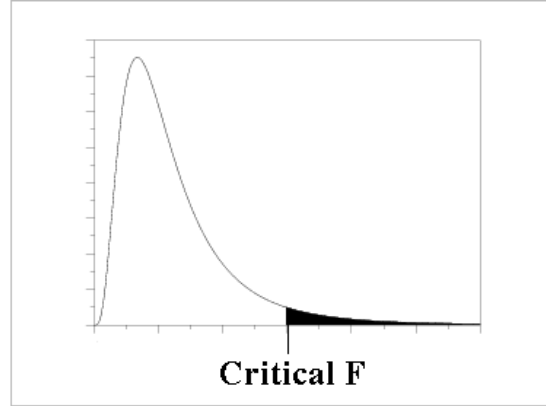


$\alpha$	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
$v$							
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
$\infty$	1.282	1.645	1.960	2.326	2.576	3.090	3.291

**Chi-Square Distribution Table**

The shaded area is equal to  $\alpha$  for  $\chi^2 = \chi^2_{\alpha}$ .

$df$	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.800}$	$\chi^2_{.700}$	$\chi^2_{.600}$	$\chi^2_{.500}$	$\chi^2_{.400}$	$\chi^2_{.300}$	$\chi^2_{.200}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	10.597	12.838	14.860	16.750	18.548	20.278	21.955
2	0.010	0.020	0.051	0.103	0.211	0.599	0.706	0.900	1.385	1.678	2.000	2.278	2.575	2.773	2.945	3.219	3.572
3	0.072	0.115	0.216	0.352	0.584	0.878	1.005	1.213	1.599	1.848	2.179	2.366	2.578	2.746	2.915	3.178	3.420
4	0.207	0.297	0.484	0.711	1.064	1.357	1.485	1.679	2.000	2.202	2.445	2.602	2.746	2.877	2.990	3.158	3.348
5	0.412	0.554	0.831	1.145	1.610	1.938	2.072	2.237	2.501	2.676	2.876	2.990	3.125	3.219	3.291	3.438	3.599
6	0.676	0.872	1.237	1.635	2.204	2.645	2.793	2.945	3.153	3.291	3.485	3.591	3.708	3.772	3.827	3.975	4.127
7	0.989	1.239	1.690	2.167	2.833	3.307	3.455	3.607	3.816	3.953	4.147	4.253	4.370	4.434	4.489	4.637	4.790
8	1.344	1.646	2.180	2.733	3.490	4.041	4.189	4.341	4.550	4.687	4.881	4.987	5.104	5.168	5.223	5.371	5.524
9	1.735	2.088	2.700	3.325	4.168	4.819	4.967	5.119	5.328	5.465	5.659	5.765	5.882	5.946	6.001	6.149	6.302
10	2.156	2.558	3.247	3.940	4.865	5.581	5.729	5.881	6.090	6.227	6.421	6.527	6.644	6.708	6.763	6.911	7.064
11	2.603	3.053	3.816	4.575	5.578	6.304	6.452	6.604	6.813	6.950	7.144	7.250	7.367	7.431	7.486	7.634	7.787
12	3.074	3.571	4.404	5.226	6.304	7.041	7.189	7.341	7.550	7.687	7.881	7.987	8.104	8.168	8.223	8.371	8.524
13	3.565	4.107	5.009	5.892	7.042	7.779	7.927	8.079	8.288	8.425	8.619	8.725	8.842	8.906	8.961	9.109	9.262
14	4.075	4.660	5.629	6.571	7.790	8.527	8.675	8.827	9.036	9.173	9.367	9.473	9.590	9.654	9.709	9.857	10.010
15	4.601	5.229	6.262	7.261	8.547	9.284	9.432	9.584	9.793	9.930	10.124	10.230	10.347	10.411	10.466	10.614	10.767
16	5.142	5.812	6.908	7.962	9.312	10.049	10.197	10.349	10.558	10.695	10.889	11.005	11.122	11.186	11.241	11.389	11.542
17	5.697	6.408	7.564	8.672	10.085	10.822	10.970	11.122	11.331	11.468	11.662	11.778	11.895	11.959	12.014	12.162	12.315
18	6.265	7.015	8.231	9.390	10.865	11.602	11.750	11.902	12.111	12.248	12.442	12.558	12.675	12.739	12.794	12.942	13.095
19	6.844	7.633	8.907	10.117	11.651	12.388	12.536	12.688	12.897	13.034	13.228	13.344	13.461	13.525	13.580	13.728	13.881
20	7.434	8.260	9.591	10.851	12.443	13.180	13.328	13.480	13.689	13.826	14.020	14.136	14.253	14.317	14.372	14.520	14.673
21	8.034	8.897	10.283	11.591	13.240	13.977	14.125	14.277	14.486	14.623	14.817	14.933	15.050	15.114	15.169	15.317	15.470
22	8.643	9.542	10.982	12.338	14.041	14.778	14.926	15.078	15.287	15.424	15.618	15.734	15.851	15.915	15.970	16.118	16.271
23	9.260	10.196	11.689	13.091	14.848	15.585	15.733	15.885	16.094	16.231	16.425	16.541	16.658	16.722	16.777	16.925	17.078
24	9.886	10.856	12.401	13.848	15.659	16.396	16.544	16.696	16.905	17.042	17.236	17.352	17.469	17.533	17.588	17.736	17.889
25	10.520	11.524	13.120	14.611	16.473	17.210	17.358	17.510	17.719	17.856	18.050	18.166	18.283	18.347	18.402	18.550	18.703
26	11.160	12.198	13.844	15.379	17.292	18.029	18.177	18.329	18.538	18.675	18.869	18.985	19.102	19.166	19.221	19.369	19.522
27	11.808	12.879	14.573	16.151	18.114	18.851	18.999	19.151	19.360	19.497	19.691	19.807	19.924	19.988	20.043	20.191	20.344
28	12.461	13.565	15.308	16.928	18.939	19.676	19.824	19.976	20.185	20.322	20.516	20.632	20.749	20.813	20.868	21.016	21.169
29	13.121	14.256	16.047	17.708	19.768	20.505	20.653	20.805	21.014	21.151	21.345	21.461	21.578	21.642	21.697	21.845	21.998
30	13.787	14.953	16.791	18.493	20.599	21.336	21.484	21.636	21.845	21.982	22.176	22.292	22.409	22.473	22.528	22.676	22.829
40	20.707	22.164	24.433	26.509	29.051	31.566	31.801	32.036	32.351	32.566	32.851	33.066	33.281	33.406	33.531	33.776	34.021
50	27.991	29.707	32.357	34.764	37.689	40.486	40.721	40.956	41.271	41.486	41.771	41.986	42.201	42.326	42.451	42.746	43.001
60	35.534	37.485	40.482	43.188	46.459	49.332	49.567	49.802	50.117	50.332	50.617	50.832	51.047	51.172	51.297	51.592	51.847
70	43.275	45.442	48.758	51.739	55.329	58.332	58.567	58.802	59.117	59.332	59.617	59.832	60.047	60.172	60.297	60.592	60.847
80	51.172	53.540	57.153	60.391	64.278	67.332	67.567	67.802	68.117	68.332	68.617	68.832	69.047	69.172	69.297	69.592	69.847
90	59.196	61.754	65.647	69.126	73.291	76.406	76.641	76.876	77.191	77.406	77.691	77.906	78.121	78.246	78.371	78.666	78.921
100	67.328	70.065	74.222	77.929	82.358	85.527	85.762	86.007	86.322	86.537	86.822	87.037	87.252	87.377	87.502	87.797	88.052

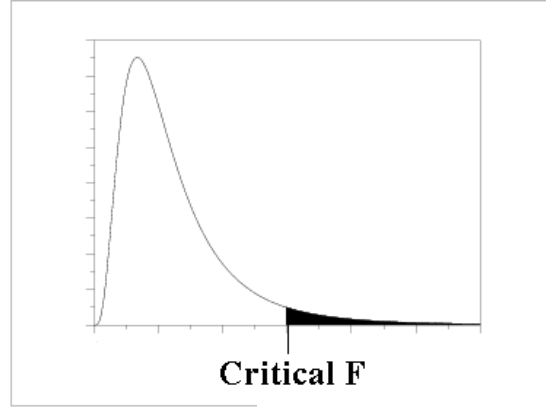


F - Distribution ( $\alpha = 0.05$  in the Right Tail)

Denominator Degrees of Freedom $df_2$	Numerator Degrees of Freedom $df_1$	Numerator Degrees of Freedom								
		1	2	3	4	5	6	7	8	9
1		161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2		18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385
3		10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123
4		7.7086	9.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	6.9988
5		6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725
6		5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990
7		5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767
8		5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881
9		5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789
10		4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204
11		4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962
12		4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964
13		4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144
14		4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458
15		4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876
16		4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377
17		4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943
18		4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563
19		4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227
20		4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928
21		4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660
22		4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419
23		4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201
24		4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002
25		4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821
26		4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655
27		4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501
28		4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360
29		4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229
30		4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107
40		4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240
60		4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401
120		3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588
$\infty$		3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799

F - Distribution ( $\alpha = 0.05$  in the Right Tail)

Denominator Degrees of Freedom $df_2$	Numerator Degrees of Freedom $df_1$	10	12	15	20	24	30	40	60	120	$\infty$
1		241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31
2		19.396	19.413	19.429	19.446	19.454	19.462	19.471	19.479	19.487	19.496
3		8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5720	8.5494	8.5264
4		5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6877	5.6581	5.6281
5		4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.3650
6		4.0600	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7		3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8		3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9		3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10		2.9782	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11		2.8536	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.4480	2.4045
12		2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.3410	2.2962
13		2.6710	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14		2.6022	2.5342	2.4630	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15		2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16		2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17		2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.0107	1.9604
18		2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19		2.3779	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.8780
20		2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21		2.3210	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.8657	1.8117
22		2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8894	1.8380	1.7831
23		2.2747	2.2036	2.1282	2.0476	2.0050	1.9605	1.9139	1.8648	1.8128	1.7570
24		2.2547	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.7896	1.7330
25		2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.7110
26		2.2197	2.1479	2.0716	1.9898	1.9464	1.9010	1.8533	1.8027	1.7488	1.6906
27		2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28		2.1900	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29		2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30		2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40		2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60		1.9926	1.9174	1.8364	1.7480	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120		1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.4290	1.3519	1.2539
$\infty$		1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.3940	1.3180	1.2214	1.0000



F - Distribution ( $\alpha = 0.01$  in the Right Tail)

F - Distribution ( $\alpha = 0.01$  in the Right Tail)

Denominator Degrees of Freedom $df_2$	Numerator Degrees of Freedom $df_1$	Numerator Degrees of Freedom								
		1	2	3	4	5	6	7	8	9
1		4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
2		98.503	99.000	99.166	99.249	99.299	99.333	99.356	99.374	99.388
3		34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345
4		21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659
5		16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158
6		13.745	10.925	9.7795	9.1483	8.7459	8.4661	8.2600	8.1017	7.9761
7		12.246	9.5466	8.4513	7.8466	7.4604	7.1914	6.9928	6.8400	6.7188
8		11.259	8.6491	7.5910	7.0061	6.6318	6.3707	6.1776	6.0289	5.9106
9		10.561	8.0215	6.9919	6.4221	6.0569	5.8018	5.6129	5.4671	5.3511
10		10.044	7.5594	6.5523	5.9943	5.6363	5.3858	5.2001	5.0567	4.9424
11		9.6460	7.2057	6.2167	5.6683	5.3160	5.0692	4.8861	4.7445	4.6315
12		9.3302	6.9266	5.9525	5.4120	5.0643	4.8206	4.6395	4.4994	4.3875
13		9.0738	6.7010	5.7394	5.2053	4.8616	4.6204	4.4410	4.3021	4.1911
14		8.8616	6.5149	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297
15		8.6831	6.3589	5.4170	4.8932	4.5556	4.3183	4.1415	4.0045	3.8948
16		8.5310	6.2262	5.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804
17		8.3997	6.1121	5.1850	4.6690	4.3359	4.1015	3.9267	3.7910	3.6822
18		8.2854	6.0129	5.0919	4.5790	4.2479	4.0146	3.8406	3.7054	3.5971
19		8.1849	5.9259	5.0103	4.5003	4.1708	3.9386	3.7653	3.6305	3.5225
20		8.0960	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567
21		8.0166	5.7804	4.8740	4.3688	4.0421	3.8117	3.6396	3.5056	3.3981
22		7.9454	5.7190	4.8166	4.3134	3.9880	3.7583	3.5867	3.4530	3.3458
23		7.8811	5.6637	4.7649	4.2636	3.9392	3.7102	3.5390	3.4057	3.2986
24		7.8229	5.6136	4.7181	4.2184	3.8951	3.6667	3.4959	3.3629	3.2560
25		7.7698	5.5680	4.6755	4.1774	3.8550	3.6272	3.4568	3.3239	3.2172
26		7.7213	5.5263	4.6366	4.1400	3.8183	3.5911	3.4210	3.2884	3.1818
27		7.6767	5.4881	4.6009	4.1056	3.7848	3.5580	3.3882	3.2558	3.1494
28		7.6356	5.4529	4.5681	4.0740	3.7539	3.5276	3.3581	3.2259	3.1195
29		7.5977	5.4204	4.5378	4.0449	3.7254	3.4995	3.3303	3.1982	3.0920
30		7.5625	5.3903	4.5097	4.0179	3.6990	3.4735	3.3045	3.1726	3.0665
40		7.3141	5.1785	4.3126	3.8283	3.5138	3.2910	3.1238	2.9930	2.8876
60		7.0771	4.9774	4.1259	3.6490	3.3389	3.1187	2.9530	2.8233	2.7185
120		6.8509	4.7865	3.9491	3.4795	3.1735	2.9559	2.7918	2.6629	2.5586
$\infty$		6.6349	4.6052	3.7816	3.3192	3.0173	2.8020	2.6393	2.5113	2.4073

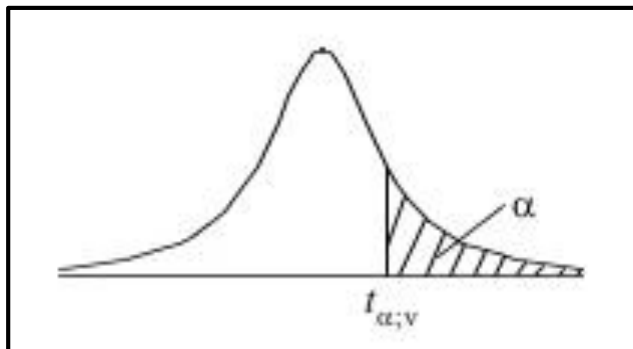
Denominator Degrees of Freedom $df_2$	Numerator Degrees of Freedom $df_1$	10	12	15	20	24	30	40	60	120	$\infty$
1		6055.8	6106.3	6157.3	6208.7	6234.6	6260.6	6286.8	6313.0	6339.4	6365.9
2		99.399	99.416	99.433	99.449	99.458	99.466	99.474	99.482	99.491	99.499
3		27.229	27.052	26.872	26.690	26.598	26.505	26.411	26.316	26.221	26.125
4		14.546	14.374	14.198	14.020	13.929	13.838	13.745	13.652	13.558	13.463
5		10.051	9.8883	9.7222	9.5526	9.4665	9.3793	9.2912	9.2020	9.1118	9.0204
6		7.8741	7.7183	7.5590	7.3958	7.3127	7.2285	7.1432	7.0567	6.9690	6.8800
7		6.6201	6.4691	6.3143	6.1554	6.0743	5.9920	5.9084	5.8236	5.7373	5.6495
8		5.8143	5.6667	5.5151	5.3591	5.2793	5.1981	5.1156	5.0316	4.9461	4.8588
9		5.2565	5.1114	4.9621	4.8080	4.7290	4.6486	4.5666	4.4831	4.3978	4.3105
10		4.8491	4.7059	4.5581	4.4054	4.3269	4.2469	4.1653	4.0819	3.9965	3.9090
11		4.5393	4.3974	4.2509	4.0990	4.0209	3.9411	3.8596	3.7761	3.6904	3.6024
12		4.2961	4.1553	4.0096	3.8584	3.7805	3.7008	3.6192	3.5355	3.4494	3.3608
13		4.1003	3.9603	3.8154	3.6646	3.5868	3.5070	3.4253	3.3413	3.2548	3.1654
14		3.9394	3.8001	3.6557	3.5052	3.4274	3.3476	3.2656	3.1813	3.0942	3.0040
15		3.8049	3.6662	3.5222	3.3719	3.2940	3.2141	3.1319	3.0471	2.9595	2.8684
16		3.6909	3.5527	3.4089	3.2587	3.1808	3.1007	3.0182	2.9330	2.8447	2.7528
17		3.5931	3.4552	3.3117	3.1615	3.0835	3.0032	2.9205	2.8348	2.7459	2.6530
18		3.5082	3.3706	3.2273	3.0771	2.9990	2.9185	2.8354	2.7493	2.6597	2.5660
19		3.4338	3.2965	3.1533	3.0031	2.9249	2.8442	2.7608	2.6742	2.5839	2.4893
20		3.3682	3.2311	3.0880	2.9377	2.8594	2.7785	2.6947	2.6077	2.5168	2.4212
21		3.3098	3.1730	3.0300	2.8796	2.8010	2.7200	2.6359	2.5484	2.4568	2.3603
22		3.2576	3.1209	2.9779	2.8274	2.7488	2.6675	2.5831	2.4951	2.4029	2.3055
23		3.2106	3.0740	2.9311	2.7805	2.7017	2.6202	2.5355	2.4471	2.3542	2.2558
24		3.1681	3.0316	2.8887	2.7380	2.6591	2.5773	2.4923	2.4035	2.3100	2.2107
25		3.1294	2.9931	2.8502	2.6993	2.6203	2.5383	2.4530	2.3637	2.2696	2.1694
26		3.0941	2.9578	2.8150	2.6640	2.5848	2.5026	2.4170	2.3273	2.2325	2.1315
27		3.0618	2.9256	2.7827	2.6316	2.5522	2.4699	2.3840	2.2938	2.1985	2.0965
28		3.0320	2.8959	2.7530	2.6017	2.5223	2.4397	2.3535	2.2629	2.1670	2.0642
29		3.0045	2.8685	2.7256	2.5742	2.4946	2.4118	2.3253	2.2344	2.1379	2.0342
30		2.9791	2.8431	2.7002	2.5487	2.4689	2.3860	2.2992	2.2079	2.1108	2.0062
40		2.8005	2.6648	2.5216	2.3689	2.2880	2.2034	2.1142	2.0194	1.9172	1.8047
60		2.6318	2.4961	2.3523	2.1978	2.1154	2.0285	1.9360	1.8363	1.7263	1.6006
120		2.4721	2.3363	2.1915	2.0346	1.9500	1.8600	1.7628	1.6557	1.5330	1.3805
$\infty$		2.3209	2.1847	2.0385	1.8783	1.7908	1.6964	1.5923	1.4730	1.3246	1.0000



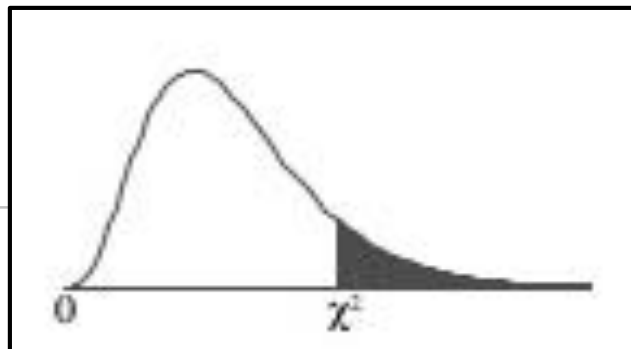
## Exercise 7

- Find the value of  $t$  for 24 df and 0.05 area in the right tail (*one-sided*);
- Find the value of  $t$  for 24 df and 0.05 in the *two-sided* area ;
- Find the probability that a r.v.  $t$  for 24 df assumes a value greater than 2.5;
- Find the probability that a r.v.  $t$  for 24 df assumes a value greater than 1.5;
- Find the probability that a r.v.  $\chi^2$  for 3 df assumes a value greater than or equal to 7;

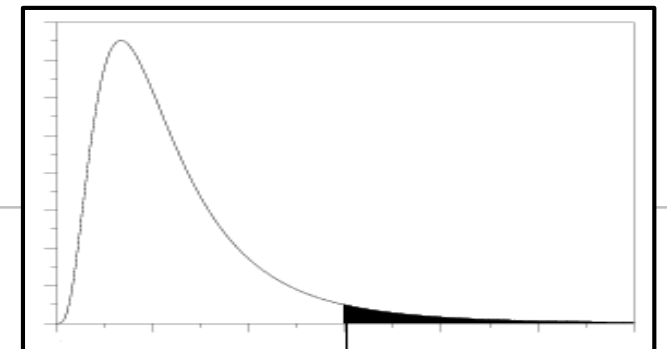
Student's T



$\chi^2$



F



## Exercise 7

a. Find the value of **t** for **24** df and **0.05** area in the right tail (*one-sided*)

## Exercise 7

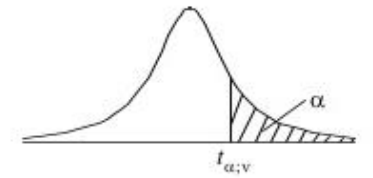
a. Find the value of **t** for **24** df and **0.05** area in the right tail (*one-sided*)

Solution

a. We have to find the 95% percentile of the t di Student with 24 df :  $t_{24,0.05}$ .  
From the table the value is: 1.711

**Table of the Student's  $t$ -distribution**

The table gives the values of  $t_{\alpha;v}$  where  
 $\Pr(T_v > t_{\alpha;v}) = \alpha$ , with  $v$  degrees of freedom



$\alpha$	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
$v$							
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
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26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
$\infty$	1.282	1.645	1.960	2.326	2.576	3.090	3.291



## Exercise 7

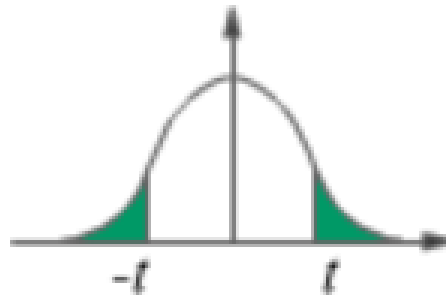
b. Find the value of **t** for **24** df and **0.05** in the *two-sided* area;

## Exercise 7

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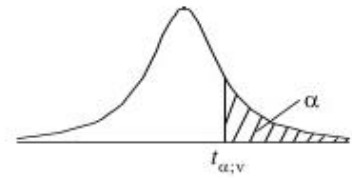
Solution

b. We have to find the 2.5% e 97.5% percentile that border the two-sided area that leaves in the tail a probability of 5%.  
From the table  $t_{24,0.025} = 2.064$  and  $t_{24,0.975} = -2.064$



**Table of the Student's *t*-distribution**

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5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
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## Exercise 7

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## Exercise 7

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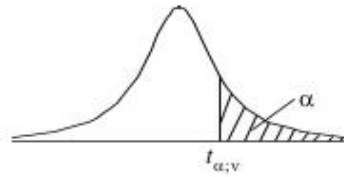
Solution

We have t

If  $X \sim t_{24}$  then,  $P(X > 2.5) = 0.01$

**Table of the Student's *t*-distribution**

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## Exercise 7

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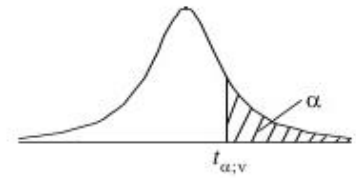
Solution

We have t

If  $X \sim t_{24}$  then,  $0.05 \leq P(X > 1.5) \leq 0.1$

**Table of the Student's *t*-distribution**

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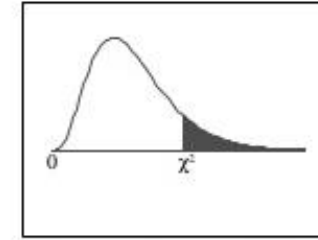


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## Exercise 7

e. Find the **probability** that a r.v.  $\chi^2$  for **3** df assumes a value greater than or equal to **7**





The shaded area is equal to  $\alpha$  for  $\chi^2 = \chi^2_{\alpha}$ .

## Exercise 7

e. Find the **probability** that a r.v.  $\chi^2$  for **3** df assumes a value greater than or equal to **7**

Solution

We have  $\chi^2$

e. If  $X \sim \chi^2_3$ , then  $0.05 \leq P(X > 7) \leq 0.1$

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.800}$	$\chi^2_{.700}$	$\chi^2_{.625}$	$\chi^2_{.500}$	$\chi^2_{.400}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169