Exercise Class in Mathematics

BAE

Preliminary Exercises

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October 6 2022

Exercise 1.
Compute the following limits:

$a)\ \lim_{n\to+\infty} \dfrac{1}{\log(n)\left(1-\cos\left(\frac{1}{n}\right)\right)}$,

$b)\ \lim_{n\to+\infty} \dfrac{e^{\frac{1}{n}}-1}{\ln(n+1)-\ln(n)}$,

$c)\ \lim_{n\to+\infty} \left(\dfrac{n+1}{n}\right)^{2n}$,

$d)\ \lim_{n\to+\infty} \left(\dfrac{n^2+n}{n^2-n+2}\right)^{n}$,

$e)\ \lim_{n\to+\infty} n\cdot\sin\left(\dfrac{3}{n}\right)$,

$f)\ \lim_{n\to+\infty} n\cdot\tan\left(\dfrac{1}{n}\right)$,

$g)\ \lim_{n\to+\infty} \dfrac{\sin\left(\frac{1}{n}\right)}{\sin\left(\frac{3}{n}\right)}$,

$h)\ \lim_{n\to+\infty} \dfrac{\tan^2\left(\frac{1}{n}\right)}{1-\cos\left(\frac{1}{n}\right)}$,

$i)\ \lim_{n\to+\infty} \dfrac{1-\cos\left(\frac{3}{n}\right)}{\sin\left(\frac{3}{n^2}\right)}$,

$j)\ \lim_{n\to+\infty} (n-\sin(n))$,

Exercise 2.
Evaluate the convergence or divergence of the following series. If convergent, determine the sum

$a)\ \sum_{n=0}^{+\infty} \dfrac{2^n+4^n}{8^n}$,

$b)\ \sum_{n=0}^{+\infty} \sin^n\left(\dfrac{\pi}{6}\right)$

$c)\ \sum_{n=1}^{+\infty} \dfrac{2^{2n+1}}{3^{2n}}$,

$d)\ \sum_{n=2}^{+\infty} \dfrac{2^{-2n+1}}{3^{n-2}}$

$e)\ \sum_{n=0}^{+\infty} \dfrac{6^n+2^n}{5^n}$

Exercise 3.
Discuss the behavior of the following geometric series, and then discuss for which values of $\alpha \in \mathbb{R}$ their sum is equal to $\frac{1}{3}$.

$$a)\ \sum_{n=0}^{+\infty}(\log \alpha)^n \ \ \alpha \in (0, +\infty), \qquad b)\ \sum_{n=0}^{+\infty}\frac{1}{(1+\alpha)^n}$$

Exercise 4.
Discuss the behavior of the following geometric series as x changes in $\mathbb{R}$

$$a)\ \sum_{n=0}^{+\infty}\Big(\frac{1}{1+x}\Big)^n, \qquad b)\ \sum_{n=0}^{+\infty}(-1)^n\frac{x^n}{2^n}$$

Exercise 5.
Verify, by applying the definition, the following limits

$$a)\ \lim_{x\to 0}\frac{9^x-1}{3^x-1}=2, \qquad b)\ \lim_{x\to+\infty}\frac{-2x}{x+1}=-2$$
$$c)\ \lim_{x\to 4}(2-\sqrt{x})=0, \qquad d)\ \lim_{x\to 1}\frac{1}{x-1}=\infty$$
$$e)\ \lim_{x\to\infty}(x^3-1)=\infty,$$

Exercise 6.
Calculate the following limits

a) $\lim\limits_{x\to 0}\dfrac{1-\sqrt{1+\sin x}}{x}$ *(rationalization + notable limit)*

b) $\lim\limits_{n\to+\infty} n\cdot\sin\left(e^{-n}\right)$ *$(x = lnt)$*

c) $\lim\limits_{x\to+\infty}\dfrac{1-\cos\frac{1}{x}}{x\sin\left(\pi-\frac{1}{x}\right)}$ *$(\frac{1}{x}=t)$*

d) $\lim\limits_{x\to 1}\dfrac{\log_3 x}{x-1}$ *$(x-1=y)$*

e) $\lim\limits_{x\to+\infty} x^{\frac{1}{x}}$ *$(\dfrac{1}{x}=y)$*

f) $\lim\limits_{x\to 0^+} x^{\ln(1+3x)}$ *$(\ z=e^{\ln z}\ con\ z>0)$*