Exercise Class in Mathematics

BAE

Preliminary Exercises

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Esercizio 1.
Solve the following absolute value equations:

$$\begin{array}{llll} \text{(i)} & |3x-5| = 2x+1; & \text{(ii)} & 4|x-1| = 1; \\ \text{(iii)} & |x^2-1| = 8; & \text{(iv)} & |2x+1| + |-x+1| = x+4. \end{array}$$

Esercizio 2.
Solve the following absolute value inequalities

$$\begin{array}{llll} \text{(i)} & |2x+5| < -3; & \text{(ii)} & |x^2-4| + |x^2-1| > 1; \\ \text{(iii)} & \left|\dfrac{x+1}{2-x}\right| > 2. & & \end{array}$$

Esercizio 3.
Solve the following irrational equations

$$\begin{array}{llll} \text{(i)} & \sqrt{x^2-3x+2} = 2-x; & \text{(ii)} & \sqrt[3]{2x+x^3+1} = 1+x; \\ \text{(iii)} & \sqrt{6x+3} + x = 4; & \text{(ii)} & 2\sqrt{2+x} - \sqrt{x-3} = 4. \end{array}$$

Esercizio 4.
Solve the following irrational inequalities

$$\text{(i)} \quad \sqrt{4x^2+5x-6} < 4x-3; \quad \text{(ii)} \quad \sqrt{x^2-4x-21} > x-3;$$

Esercizio 5.
Solve the following logarithmic equations

$$\begin{array}{llll} \text{(i)} & \log_3(x^2+2x) = 1; & \text{(ii)} & \log_2(x-2) - \log_2(8-x) = \log_2 x - 3; \\ \text{(iii)} & 2(\log_2 x)^2 + 5\log_2 x - 3 = 0. & & \end{array}$$

Esercizio 6.
Solve the following logarithmic inequalities

$$\begin{array}{llll} \text{(i)} & \log_{11}(2-x) > \log_{11}(x+2); & \text{(ii)} & \log_3(2x-3) - \log_3(x+1) < 2; \\ \text{(iii)} & \log^2 x - 7\log x + 12 < 0. & & \end{array}$$

Esercizio 7.
Solve the following exponential equation through logarithms

$$7^{x+1} + 2 \cdot 7^x = 11$$

Esercizio 8.
Evaluate the value of the indicated function, exploiting the given informations

i) $\sin\alpha = \dfrac{7}{25}$ and $0 < \alpha < \dfrac{\pi}{2}$; $\cos\alpha$?

ii) $\sin\alpha = -\dfrac{9}{41}$ and $\alpha \in$ fourth quadrant; $\cos\alpha$?

iii) $\cos\alpha = \dfrac{3}{4}$ and $\dfrac{3}{2}\pi < \alpha < 2\pi$; $\tan\alpha$?

Esercizio 9.
Evaluate the value of the indicated function, exploiting the given informations

$$\text{(i)} \quad \sin\Big(2\pi - \frac{\pi}{3}\Big); \quad \text{(ii)} \quad \cos\Big(2\pi - \frac{\pi}{6}\Big);$$
$$\text{(iii)} \quad \tan\Big(-\frac{\pi}{4}\Big).$$

Esercizio 10.

		T	F
(a)	If $\sin\alpha < 0$ and $\cos\alpha < 0$, then α belongs to the fourth quadrant	☐	☐
(b)	If $\cos\alpha > 0$, then $\sin\alpha > 0$.	☐	☐
(c)	If $\sin\alpha = -\dfrac{8}{9}$ then α belongs to the third or to the fourth quadrant	☐	☐
(d)	If $\sin\alpha = \cos\alpha$, then it can only be $\alpha = \dfrac{\pi}{4}$	☐	☐
(e)	There is no angle α such that $\cos\alpha = \dfrac{5}{4}$	☐	☐
(f)	If $-1 \leq \cos\alpha \leq 1$, then $1 \leq \cos^2\alpha \leq 1$	☐	☐

Esercizio 11.
Evaluate the value of the following expressions

$$\text{(i)} \quad \arccos\Big(-\frac{\sqrt{2}}{2}\Big); \qquad \text{(ii)} \quad \arcsin\Big(\frac{\sqrt{3}}{2}\Big);$$
$$\text{(iii)} \quad \arcsin\frac{1}{2} + \arccos\Big(-\frac{\sqrt{3}}{2}\Big).$$