

Exercise Class in Mathematics

BAE

Preliminary Exercises

Teacher: Prof Davide Pirino

Teaching Assistants: Alessio Fiorentini & Isabella Valdivia

September 22 2022

Esercizio 1.

Determine the domain and range of the following functions

$$\begin{array}{ll} \text{(i)} & y = \sqrt{x+4}; \quad \text{(ii)} \quad y = |x^2 - 4| + 1; \\ \text{(iii)} & y = -\sqrt{4-x^2} \quad \text{(iv)} \quad y = \begin{cases} x+1 & x < -2 \\ -2x-3 & x \geq -2 \end{cases}. \end{array}$$

Esercizio 2.

Determine the domain of the following functions

$$\begin{array}{ll} \text{(i)} & y = 3x^2 - 4x + 7; \quad \text{(ii)} \quad y = \frac{1}{2x^2 + 5x - 3}; \\ \text{(iii)} & y = \sqrt[3]{\frac{x}{5x-3}}; \quad \text{(iv)} \quad y = \sqrt{\frac{x+1}{x^2-8x}}; \\ \text{(v)} & y = \sqrt{3x+2} + \sqrt{4x}; \quad \text{(vi)} \quad y = \ln(x^2 - 4x - 12); \\ \text{(vii)} & y = \frac{1}{\ln x + 1}; \quad \text{(viii)} \quad y = \sqrt{\ln(x+3)}; \\ \text{(x)} & y = \frac{1}{2^{x+4} - 2}; \quad \text{(xi)} \quad y = \sqrt{2^x - 4^x}; \\ \text{(xii)} & y = \frac{1}{\ln^2 x - 2 \ln x + 1}; \quad \text{(xiii)} \quad y = \ln(1 - \sqrt{1-2x}). \end{array}$$

Esercizio 3.

Establish if the following functions are even, odd or neither

$$\begin{array}{ll} \text{(i)} & y = 3x^2 + 2x - 1; \quad \text{(ii)} \quad y = x^2 - |5x|; \\ \text{(iii)} & y = \frac{\sqrt{7-x^2}}{x}; \quad \text{(iv)} \quad y = \frac{x^3 - 1}{1 - x^2}; \\ \text{(v)} & y = \frac{e^x + e^{-x}}{x^2}; \quad \text{(vi)} \quad y = \frac{|x| + x^2}{2x}; \end{array}$$

Esercizio 4.

For each of the following functions sketch a graph and establish whether they are increasing, decreasing, non increasing or non decreasing functions. If none of these cases, determine the intervals where the function is increasing or decreasing

$$\begin{array}{ll}
 \text{(i)} & y = \sqrt{x}; \\
 \text{(ii)} & y = |2x|; \\
 \text{(iii)} & y = 2^{-x}; \\
 \text{(iv)} & y = \begin{cases} 2 & x \geq 0 \\ x + 2 & -2 < x < 0; \\ -1 & x \leq -2 \end{cases} \\
 \text{(iii)} & y = \begin{cases} 2x - 1 & x \leq 2 \\ 7 - x^2 & x > 2 \end{cases}; \\
 \text{(iv)} & y = x^2 - 3x - 10;
 \end{array}$$

Esercizio 5.

Establish if the following functions are injective, surjective (on the set \mathbb{R} of Real numbers), or bijective

$$\begin{array}{ll}
 \text{(i)} & y = 2x - 1; \\
 \text{(ii)} & y = x^2 - 2x; \\
 \text{(iii)} & y = \frac{x}{x^2 + 1}; \\
 \text{(iv)} & y = \frac{x + 3}{x - 4}.
 \end{array}$$

Esercizio 6.

After having sketched the graph of the following functions, establish if they are injective, surjective (on the set \mathbb{R} of Real numbers). Moreover determine the Range of the function

$$\begin{array}{ll}
 \text{(i)} & y = x^2 - 2x; \\
 \text{(ii)} & y = 1 + |x - 2|; \\
 \text{(iii)} & y = \begin{cases} 1 + 2x & x < 0 \\ 1 & 0 \leq x \leq 2. \\ \frac{x}{2} & x > 2 \end{cases} .
 \end{array}$$

Esercizio 7.

Establish if the following functions are invertible. If they are, determine their inverse function, if they aren't, restrict their domain, and evaluate the inverse in the restricted domain

$$\begin{array}{ll}
 \text{(i)} & y = 1 - 3x; \\
 \text{(ii)} & y = x^2 + x; \\
 \text{(iii)} & y = \frac{2x - 1}{x - 3}; \\
 \text{(iv)} & y = 1 + \sqrt{1 + x}; \\
 \text{(v)} & y = \sqrt{9 - x^2}.
 \end{array}$$

Esercizio 8.

Write the equation of the line that passes through the point $C = (-2; 1)$ and that is parallel to the line that passes through the points $A(4; -3)$ and $B = (-5; 0)$

Esercizio 9.

For which values of k the line of equation $(k + 2)x + (k + 3)y - 1 = 0$ is

- i) parallel to the x -axis;
- ii) parallel to the y -axis;
- iii) parallel to the line of equation $x - 2y = 0$
- iv) perpendicular to the line of equation $4x - 2y + 1 = 0$.