Exercise Class in Mathematics

BAE

Sixth Exercise class

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Exercise 1.
Use the definition of the derivative to compute the derivative of the following functions at the given point x_0

$$a)\ y = x^4 - 5x^3 - 1 \ \text{ at } x_0 = 0, \qquad b)\ y = \frac{x^3 - 3x + 1}{x} \ \text{ at } x_0 = -1,$$

$$c)\ y = \log(x+2) \ \text{ at } x_0 = 1.$$

Exercise 2.
Compute the derivatives of the following functions

$$a)\ y = x^4 - 5x^3 - 1, \qquad b)\ y = \frac{x^3}{x^2+1},$$

$$c)\ y = \frac{e^x}{e^x+1}, \qquad d)\ y = 3x^2 e^{-x}$$

Exercise 3.
For each of the following functions, compute the equation of the tangent line at the given point x_0.

$$a)\ y = x^3 e^{2x-2} \ \text{ at } x_0 = 1, \qquad b)\ y = \ln(x^5 + 3x + 4) \ \text{ at } x_0 = 0,$$

$$c)\ y = \sqrt{x^2+4} \ \text{ at } x_0 = 0.$$

Exercise 4.
For each of the following functions, say if they are continuous and differentiable, and if not, identify the nature of discontinuity and non-differentiability points

$$a)\ y = x\sqrt[3]{x^3 - x}, \qquad b)\ y = |x^2 - 2x|,$$

$$c)\ y = \frac{|x^2 - x|}{x}.$$

Exercise 5.
For each of the following functions, compute the first order Taylor approximation $P(x)$ at the point x_0. Evaluate also the error (absolute and in percentage) that is made by approximating the function $f(x_1)$ with $P(x_1)$.

$a)$ $y = \sqrt{x+1}$, at $x_0 = 0$, the error at $x_1 = 0.1$ $b)$ $y = \dfrac{x+1}{x}$ at $x_0 = -1$, the error at $x_1 = -0,9$

Exercise 6.
Determine the intervals in which the functions are increasing and decreasing

$$a)\ y = \frac{x^2-1}{x}, \qquad b)\ y = x\sqrt{2x+1}$$

$$c)\ y = \log\left(\frac{x^2+4}{x^2-4}\right).$$

Exercise 7.
Determine the intervals in which the functions are concave and convex

$$a)\ y = (x-1)e^x, \qquad b)\ y = \frac{\ln(x)}{\ln(x)-1}$$

$$c)\ y = \frac{x}{\sqrt{x^2+1}}.$$

Exercise 8.
Compute the stationary points of the following functions and determine if these are local maxima, local minima or inflection points with horizontal tangents

$$a)\ y = \frac{x^2-3}{x-5}, \qquad b)\ y = xe^{-x^2}$$

$$c)\ y = \log(2x-x^2).$$

Exercise 9.
Use De L'Hopital rule to compute the following limits

$$a)\ \lim_{x\to 0} \frac{x\cos(x)-\sin(x)}{x^3}, \qquad b)\ \lim_{x\to 0} \frac{x-\ln(1+x)}{x}$$

$$c)\ \lim_{x\to -\infty} xe^x.$$

Exercise 10.
For each of the following functions determine, if possible

(a) the domain

(b) the sign

(c) the asymptotes

(d) the intervals in which functions are increasing and decreasing

(e) the intervals where the functions are concave and convex

(f) local maxima, local minima and inflection points

Finally use the information collected above to sketch the graph of the functions.

$$a)\ y = \frac{3x^2}{x^2+1}, \qquad b)\ y = x - \sqrt{x^2 - 2x}$$

$$c)\ y = \frac{\log(x)}{2 - \log(x)}.$$