**BAE Math 2 Exercises**

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**Exercise 1.** Compute the following definite integrals:

$a)$ $\displaystyle\int_{-1}^{1} (x^4 - 3x^3 + 2x + 5)\, dx;$

$b)$ $\displaystyle\int_{0}^{1} \frac{x}{\sqrt{x+1}}\, dx;$

$c)$ $\displaystyle\int_{0}^{\pi/2} x\cos(x)\, dx;$

$d)$ $\displaystyle\int_{0}^{\pi/4} \frac{x}{\cos^2(x)}\, dx;$

$e)$ $\displaystyle\int_{\pi/4}^{\pi/2} \sin^3(x)\, dx;$

$f)$ $\displaystyle\int_{-1}^{0} x^2(1+x)^8\, dx;$

$g)$ $\displaystyle\int_{0}^{\pi/2} \frac{\sin(x)}{1+\cos^2(x)}\, dx;$

$h)$ $\displaystyle\int_{0}^{1} \frac{e^{\arctan(x)}}{1+x^2}\, dx;$

$i)$ $\displaystyle\int_{0}^{1} \frac{x^5}{\sqrt{2-x^3}}\, dx;$

$j)$ $\displaystyle\int_{1}^{e} \frac{\log(\sqrt{x})}{x}\, dx;$

**Exercise 2.** Compute the following limits, if they exist:

$a)$ $\displaystyle\lim_{x\to 0} \frac{\int_0^x e^{t^2}\, dt}{x};$

$b)$ $\displaystyle\lim_{x\to 0} \frac{\int_0^x e^{t^2}\sin(t^4 - t^3)\, dt}{\sin(x)};$

$c)$ $\displaystyle\lim_{x\to 1} \frac{1}{x^2-1}\int_1^x t^3 e^{\sqrt{t}}\, dt;$

$d)$ $\displaystyle\lim_{x\to 0} \frac{1}{\sin^2(x)}\int_0^x \frac{t}{\cos(t)}\, dt;$

**Exercise 3.** For each of the following functions describe the domain, determine the intervals of monotonicity and find the local maximum points and the local minimum points, if any:

$$a) \quad f(x) = \int_{-1}^{x} \frac{1 - e^{t^3}}{t^2 + 1}\, dt; \qquad b) \quad f(x) = x^3 e^{x^6} - \int_{0}^{x^3} e^{t^2}\, dt;$$

**Exercise 4.** For each function $f$ compute the area between the curve $y = f(x)$ and the $x$-axis in the given interval:

$$a) \quad f(x) = x^3 - x \qquad [0, 2]; \qquad b) \quad f(x) = \frac{1}{\sqrt{x} - 3} \qquad [1, 4];$$

**Exercise 5.** Determine if the following functions satisfiy the hypothesis of the *integral mean value theorem* on the given interval and, if so, find all the points of the interval that satisfy the mean value property.

$$a) \quad f(x) = 2x - 3x^2 \qquad [0, 1]\,; \qquad b) \quad f(x) = \frac{2}{\sqrt{x}} \qquad [1, 9]$$

**Exercise 6.** Compute the following improper integrals:

$$a) \quad \int_{1}^{+\infty} \frac{4}{x^5}\, dx; \qquad b) \quad \int_{3}^{4} \frac{2}{\sqrt{x - 3}}\, dx;$$

$$c) \quad \int_{0}^{1} \log(x)\, dx; \qquad d) \quad \int_{1}^{+\infty} \log(x)\, dx;$$

**Exercise 7.** Find the horizontal asymptotes of the following functions, if they exist:

$$a) \quad f(x) = \int_{1}^{x} \frac{5}{t^2 + 1}\, dt; \qquad b) \quad f(x) = x \cdot \int_{0}^{1/x} e^{t^2}\, dt;$$