BAE Math 2 Exercises

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Exercise 1. Compuite the rank of the following matrices:

a) $\begin{pmatrix} -6 & 9 \\ 2 & -3 \end{pmatrix}$ *b)* $\begin{pmatrix} -1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{pmatrix}$ *d)* $\begin{pmatrix} -3 & 3 & 1 & 35 & -49 & 1 & 1 \\ 7 & -7 & 1 & -32 & 49 & 0 & 7 \\ 0 & 0 & 5 & -32 & 49 & 0 & 7 \end{pmatrix}$

e) $\begin{pmatrix} 3 & 1 & 2 & 1 \\ 0 & 1 & 5 & 4 \\ 1 & 0 & 0 & -3 \\ -2 & 0 & 0 & 6 \\ 1 & -1 & 0 & 0 \end{pmatrix}$ *f)* $\begin{pmatrix} 2 & 1 & 3 & 9 & -2 & 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 9 & -2 & 0 & 1 & 1 & 1 \\ 2 & 1 & 3 & 9 & -2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 3 & 9 & -2 & 1 & 1 & 1 & 1 \end{pmatrix}$

Exercise 2. Compuite the rank of the following matrices depending on the value of the parameter $k \in \mathbb{R}$:

$$A_k = \begin{pmatrix} -k & 2k & 3k \\ 1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \qquad B_k = \begin{pmatrix} 3 & 3k & k & 2 \\ 0 & k-1 & -1 & 1 \\ k & 1 & 0 & 1 \end{pmatrix}$$

Exercise 3. For each of the following collections of vectors, determine if the vectors are linearly dependent or independent, and in case they are linearly dependent, find a subcollection of linearly independent vectors out of them:

$$a) \quad \mathbf{v_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} -2 \\ 4 \end{pmatrix};$$

$$b) \quad \mathbf{v_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 3 \\ 0 \end{pmatrix};$$

$$c) \quad \mathbf{v_1} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix};$$

$$d) \quad \mathbf{v_1} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix};$$

$$e) \quad \mathbf{v_1} = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 2 \\ -2 \\ -1 \\ 0 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{v_4} = \begin{pmatrix} 2 \\ -2 \\ -2 \\ 0 \end{pmatrix};$$

$$f) \quad \mathbf{v_1} = \begin{pmatrix} 4 \\ -1 \\ 4 \\ 0 \\ 2 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 2 \\ -2 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \\ 1 \end{pmatrix};$$

$$g) \quad \mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 5 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 0 \\ 3 \\ 0 \\ 0 \\ -3 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v_4} = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix};$$

Exercise 4. Determine if the following linear systems admit solutions and if so, find them:

$$a)\quad \begin{cases} x_1 + 3x_3 = 1 \\ x_1 + x_2 - x_3 = -1 \\ x_1 - x_2 = 3 \end{cases} ; \qquad b)\quad \begin{cases} x_1 - 2x_2 + 3x_3 = 2 \\ x_1 - 8x_2 + 5x_3 = 1 \\ x_1 + x_2 + 2x_3 = 3 \end{cases} ;$$

$$c)\quad \begin{cases} 2x_1 - 3x_2 + 2x_3 = 5 \\ 2x_1 + 7x_2 - 2x_3 = 3 \\ x_1 - 4x_2 + 2x_3 = 3 \end{cases} ; \qquad d)\quad \begin{cases} x_1 - 2x_2 + 2x_3 + x_4 = 1 \\ 2x_1 - 4x_2 + 3x_3 - x_4 = 3 \end{cases} ;$$

Exercise 5. Discuss the number of solutions of the following linear system depending on the values of the real parameter $k \in \mathbb{R}$:

$$a)\quad \begin{cases} kx_2 + x_3 = k \\ 2x_1 + x_2 - kx_3 = 0 \\ x_1 - x_2 - x_3 = 0 \end{cases} ; \qquad b)\quad \begin{cases} x_1 + kx_2 + x_3 = -1 \\ 2x_1 + 2x_2 = 1 \\ 3x_1 + 3kx_2 - x_3 = 2k \end{cases} ;$$

$$c)\quad \begin{cases} x_1 + kx_2 + x_3 = k \\ x_1 + x_2 + kx_3 = k \\ kx_3 = 2 \end{cases} ; \qquad d)\quad \begin{cases} kx_2 + kx_3 = -2 \\ kx_1 + (k-1)x_3 = k \\ 2kx_1 + kx_2 + x_3 = 0 \end{cases} ;$$