BAE Math 2 Exercise

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1) Determine if the following linear systems admit solutions and if so, find them:

$a)$
$$\begin{cases} x_1 + 3x_3 = 1 \\ x_1 + x_2 - x_3 = -1 \\ x_1 - x_2 = 3 \end{cases} \quad ; \qquad$$
$b)$
$$\begin{cases} x_1 - 2x_2 + 3x_3 = 2 \\ x_1 - 8x_2 + 5x_3 = 1 \\ x_1 + x_2 + 2x_3 = 3 \end{cases} \quad ;$$

$c)$
$$\begin{cases} 2x_1 - 3x_2 + 2x_3 = 5 \\ 2x_1 + 7x_2 - 2x_3 = 3 \\ x_1 - 4x_2 + 2x_3 = 3 \end{cases} \quad ; \qquad$$
$d)$
$$\begin{cases} x_1 - 2x_2 + 2x_3 + x_4 = 1 \\ 2x_1 - 4x_2 + 3x_3 - x_4 = 3 \end{cases} \quad ;$$

2) Discuss the number of solutions of the following linear system depending on the values of the real parameter $k \in \mathbb{R}$:

$a)$
$$\begin{cases} kx_2 + x_3 = k \\ 2x_1 + x_2 - kx_3 = 0 \\ x_1 - x_2 - x_3 = 0 \end{cases} \quad ; \qquad$$
$b)$
$$\begin{cases} x_1 + kx_2 + x_3 = -1 \\ 2x_1 + 2x_2 = 1 \\ 3x_1 + 3kx_2 - x_3 = 2k \end{cases} \quad ;$$

$c)$
$$\begin{cases} x_1 + kx_2 + x_3 = k \\ x_1 + x_2 + kx_3 = k \\ kx_3 = 2 \end{cases} \quad ; \qquad$$
$d)$
$$\begin{cases} kx_2 + kx_3 = -2 \\ kx_1 + (k-1)x_3 = k \\ 2kx_1 + kx_2 + x_3 = 0 \end{cases} \quad ;$$

3) Calculate the eigenvalues and the eigenvectorsof the following matrices and establish their definiteness

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & 1 & 1 \\ -3 & 0 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

$$C = \begin{pmatrix} 2 & -3 & 0 \\ -1 & 0 & 0 \\ -1 & 1 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 4 & 6 & 0 \\ -3 & -5 & 0 \\ -3 & -6 & -5 \end{pmatrix}$$

$$E = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad F = \begin{pmatrix} 2 & 0 & 0 \\ -3 & -1 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

$$G = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \qquad H = \begin{pmatrix} -7 & -3 & -16 \\ -2 & 0 & -2 \\ 6 & 3 & 15 \end{pmatrix}$$

4) Identify the domain of the following functions

    a) $f(x,y) = \log(1 - x^2 - y^2);$ b) $f(x,y) = \sqrt{x^2 + 4y^2 - 4}$

    c) $f(x,y) = e^{\frac{3}{x-2y}};$ d) $f(x,y) = \dfrac{3x + 2y}{4x^2 - y^2}$

    e) $f(x,y) = e^{\frac{y}{\sqrt{x^2-4}}};$ f) $f(x,y) = \dfrac{\sin xy^2}{\sqrt{1 - 4x^2 - 9y^2}}$

5) Write the level curves of the following functions:

    a) $f(x,y) = \log(1 - x^2 - y^2);$ b) $f(x,y) = \sqrt{x^2 + 4y^2 - 4}$

    c) $f(x,y) = e^{\frac{3}{x-2y}};$ d) $f(x,y) = \dfrac{1}{4x^2 + y^2}$

6) Compute the partial derivatives and write the gradient of the following functions:

    a) $f(x,y) = 3x^2y^3 - 2x^2y^4;$ b) $f(x,y) = e^{x^2y}$

    c) $f(x,y) = \log(1 - x^2 - y^2);$ d) $f(x,y) = \arctan(2x^3y^2)$

    e) $f(x,y) = x^2\sin(2x - 3y);$