MATHEMATICS
Time allowed 1 Hour

Penalties: ☐ ☐

A.Y. 2022/2023

EXAM REGULATIONS.

Any kind of electronic device (calculators, smartphones, smartwatches etc..) is FORBIDDEN. Put all your devices in the location indicated by the examiners. If an attendee violates this rule (even if the device is switched off or in offline mode), she/he will be immediately expelled from the exam session and her/his exam will be invalidated.

Examiners will mark only THIS A3 paper SHOWING THE UNIVERSITY LOGO on the upper left corner. However, any other additional papers (drafts, notes, scribbles or anything else) MUST be delivered to the examiners as a proof that you solved the problems by yourself.

WARNINGS: if, during the exam correction phase, the examiners should find a suspicion of cheating, the student can be summoned for an oral exam to verify the truthfulness of what has been written.
Make sure that you indicate all necessary steps to solve the exercises. If a justification of your result is missing the exercise will not be marked.

PENALTIES: Each examiner can, UNQUESTIONABLY, assign a penalty if she/he considers that two participants have communicated with each other (in any way). One penalty does not imply anything for the valuation of the exam. However, should an attendee be given two penalties, she/he will be immediately expelled from the exam session and her/his exam will be invalidated.

SUGGESTION: Remember to always double check the consistency of your results. EVEN IF CORRECT, inconsistent statements will result in a negative impact on the final grade of the exam. Use a clear and clean handwriting. Unclear or ambiguous sentences may result in a negative impact on the final grade of the exam.

By signing below you fully accept the exam regulations.

MATRICOLA ..... Lastname ..... Name .....

The exam is divided in 2 parts. Part 1 (6 exercizes) refers to questions from 1 to 6. This part amounts to 20 points. You must get at least 16 points in this part. Part 2 (2 exercizes) refers to questions 7 and 8. This part amounts to 12 points. **Examiners will mark Part 2 only if you get 16 points or more in Part 1.** If you get less than 16 points in Part 1, you have not passed the exam. To pass the exam you must get at least 16 points in Part 1 AND 18 points or more in total.

If you intend to withdraw please tick the box below and sign next.

☐ Withdrawn Signature .....

1. Find the condition on the parameters a, b, and variable x for which the following infinte sum converges:

$$\sum_{n=0}^{\infty}(ax)^n(bx)^n$$

2. Find the value of parameter a for which the limit of the function, $f(x)=\begin{cases}a+2x & x<-5\\ -12+x^2 & x\geq -5\end{cases}$, with $x\to -5$, exists.

3. Find the turning points for $f(x) = \sin(x)\cos(x)$.

4. Compute all eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} -1 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

5. Compute the following integral

$$\int \frac{\ln(x)+1}{x\sqrt{\ln(x)}}dx$$

6. Indicate the number of solutions of the following linear system as the parameter k changes:

$$\begin{cases} x + y & = 1 \\ kx + 2ky + z & = 2 \\ x + 2y + kz & = 2 \end{cases}$$

7. Consider the function rule

$$f(x) = \frac{\sin^2(x-1)}{|1-x|-1}$$

Find the domain and the vertical and horizontal asymptotes of the function and discuss it (zeroes, critical points, monotonicity) within the subdomain $x_l < x < x_r$, where x_l, x_r define the innermost vertical asymptotes. Provide below all necessary steps to solve the exercise.

8. Consider the function $f(x, y) = \ln(xy + 4)$. Answer the following questions:

   (a) Compute and plot the domain;

   (b) Compute and plot the level curves L_c and specify for which values of c they exist;

   (c) Compute all stationary points and identify their nature.

   Provide below all necessary steps to solve the exercise.