Mathematics 2, AY 2022-2023
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Additional Exercises n.6

1 Functions of several variables: Gradient and Hessian matrix

Ex 1. For each of the following function

   i. Compute the domain

   ii. Compute the gradient and specify in which subset of the domain the gradient exists.

   iii. Compute the Hessian matrix and specify in which subset of the domain the Hessian matrix exists.

   iv. Compute the tangent plane at the given point $\mathbf{p}$

1. $f(x,y) = 3x^2 + y^3 - 3x + y,$ $\mathbf{p} = (1,1)$

2. $f(x,y) = e^{xy} + (x - y)^2,$ $\mathbf{p}(0,1)$

3. $f(x,y) = \ln(4x^3 - 3y),$ $\mathbf{p} = (e, e^3)$

4. $f(x,y) = xy - \ln(x),$ $\mathbf{p} = (1,0)$

5. $f(x,y) = x\ln(y) + 2x^2 + 1,$ $\mathbf{p} = (1,1)$

6. $f(x,y) = xe^y - \ln\left(\dfrac{x}{y}\right),$ $\mathbf{p} = (1,1)$

7. $f(x,y) = x\sqrt{y^2 + x^2}$

8. $f(x,y) = \dfrac{x}{y} - \sqrt{y^2 - x^2},$ $\mathbf{p} = (0,2)$

9. $f(x,y) = xy\sqrt{1 - x^2 - y^2},$ $\mathbf{p} = \left(\tfrac{1}{2}, \tfrac{1}{2}\right)$

10. $f(x,y) = e^{x-y}(x^2 - 2y^2),$ $\mathbf{p} = \left(1, \tfrac{1}{2}\right)$

11. $f(x,y) = (x^2 + y^2)e^{-(x^2+y^2)},$ $\mathbf{p} = (0,0)$

12. $f(x,y) = \ln(x^2 + y^2 + 2x + 2)$

13. $f(x,y) = \ln\left(\dfrac{4y - y^2}{x^2 + 1}\right),$ $\mathbf{p} = (1,1)$

14. $f(x,y) = e^{(xy-x)},$ $\mathbf{p} = (1,1)$

15. $f(x,y) = x^2 + 2xy + \ln(x^2 - y),$ $\mathbf{p} = (0, -e^2)$

2 Unconstrained Optimization

Ex 2. Find and classify all stationary points of the following functions

1. $f(x,y) = x^2 + y^2 + xy - 2x - y$

2. $f(x,y) = (3x + 1)y^3 + x^2y$

3. $f(x,y) = x^3 - y^2 - 3x + 2y$

4. $f(x,y) = (x - 1)^4 - y^2$

5. $f(x,y) = e^{x-y}(x^2 - 2y^2)$

6. $f(x,y) = x^3 + y^3 - 3xy$

7. $f(x,y) = ye^x - 3x - y + 5$

8. $f(x,y) = xe^y - x - 1$

9. $f(x,y) = x\ln(y) + 2x^2 + 1$

10. $f(x,y) = x^3 - x^2y + y^2$

11. $f(x,y) = 2(x^4 + y^4 + 1) - (x + y)^2$

12. $f(x,y) = \sqrt{x^2 + y^2}$

13. $f(x,y) = e^{x^2 - y^2 - x + 2y}$

14. $f(x,y) = \ln(x^2 + y^2 + 3xy + 3)$

15. $f(x,y) = xy + x^2\ln(y)$

16. $f(x,y) = xy\sqrt{1 - x^2 - y^2}$

17. $f(x,y) = (x + y)e^{-(x^2+y^2)}$

18. $f(x,y) = \ln(x^2 + y^2 + 2x + 2)$

19. $f(x,y) = \ln\left(\dfrac{4y - y^2}{x^2 + 1}\right)$

20. $f(x,y) = e^{(xy-x)}$