BAE Math 2 Exercises

Teacher: Prof. Katia Colaneri

Teaching Assistants: Alessio Fiorentino & Isabella Valdivia

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Exercise 1. Compupite the partial derivatives and write the gradient of the following functions:

$a)\quad f(x,y) = 3x^2y^3 - 2x^2y^4$

$b)\quad f(x,y) = e^{x^2y}$

$c)\quad f(x,y) = \log(1 - x^2 - y^2)$

$d)\quad f(x,y) = \arctan(2x^3y^2)$

$e)\quad f(x,y) = x^2\sin(2x - 3y)$

$f)\quad f(x,y) = \dfrac{xy}{x^2 + y + 2}$

$g)\quad f(x,y) = \dfrac{xy}{x^2 + y + 2}$

$h)\quad f(x,y) = \dfrac{5}{y^3} + \sqrt{x^2 + y^4 + 1}$

$i)\quad f(x,y) = e^y \log\left(\dfrac{x^2}{y}\right)$

$j)\quad f(x,y) = y\tan\left(\dfrac{y}{x}\right)$

$k)\quad f(x,y) = \dfrac{\sqrt{y}}{x} e^{\frac{x}{y}}$

$l)\quad f(x,y) = \cos^2\left(\dfrac{x^2+1}{y}\right)$

$m)\quad f(x,y) = (\sqrt{x^2 + 5})\,\log(y)$

$n)\quad f(x,y) = \log\left(\dfrac{x^3 + \cos(x^2)}{y^2 + 3}\right)$

Exercise 2. For each function f determine if the given point P is a stationary point:

a) $f(x,y) = 3x^5y^3 + x^2y^2 - 3xy^2 + 11$ $\qquad P = (0,0)$

b) $f(x,y) = x^5 e^{x+y^2}$ $\qquad P = (0,3)$

c) $f(x,y) = \dfrac{x^2}{y^4} - \dfrac{y}{x^2}$ $\qquad P = (1,-1)$

d) $f(x,y) = x^2 - 6xy + 9y^2 - 9$ $\qquad P = (3,1)$

Exercise 3. For each function f write the Cartesian equation of the tangent plane to the graph of f at the given points, if possible:

a) $f(x,y) = 3y^3 - 2x^2y + 2xy + \sqrt{x}$ $\qquad P_1 = (1,-1) \qquad P_2 = (1,0) \qquad P_3 = (0,1)$

b) $f(x,y) = 5ye^x + \log(y) + 1$ $\qquad P_1 = (0,1) \qquad P_2 = (0,e) \qquad P_3 = (2,-1)$

Exercise 4. For each of the following functions find all the stationary points and determine what kind of points they are:

a) $f(x,y) = x^3 - 3xy + 6y$

b) $f(x,y) = x^2 - 3y^2 + y - 6$

c) $f(x,y) = x^2 - 2xy + 5y^2 + 3y$

d) $f(x,y) = 3x^2 + 3x^2y + xy$

e) $f(x,y) = x^3 - xy + y^2$

f) $f(x,y) = y^3 + 2x^6 + 3y^2 + 3x^2 - 5$

e) $f(x,y) = 2y^3 + 3y^2 - 12y + 12x^2y$

f) $f(x,y) = (x^2 + 6x)\sqrt{y^2 + 2}$

g) $f(x,y) = e^{x^2 - 3y^2}$

h) $f(x,y) = y\sin(x)$