

## Computations with limits

Let  $f$  and  $g$  be functions such that

$$\lim_{x \rightarrow x_0} f(x) = l, \quad \lim_{x \rightarrow x_0} g(x) = m$$

with  $l$  and  $m$  both finite. Then

$$\lim_{x \rightarrow x_0} f(x) + g(x) = l + m$$

$$\lim_{x \rightarrow x_0} f(x) - g(x) = l - m$$

$$\lim_{x \rightarrow x_0} f(x) \cdot g(x) = l \cdot m$$

$$\text{if } m \neq 0, \quad \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \frac{l}{m}; \quad \text{if } m = 0 \text{ and } l \neq 0, \quad \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \infty$$

$$\lim_{x \rightarrow x_0} [f(x)]^p = l^p,$$

$$\lim_{x \rightarrow x_0} a^{f(x)} = a^l$$

$$\text{if } l > 0, \quad \lim_{x \rightarrow x_0} [f(x)]^{g(x)} = l^m$$

## Intuitive and Notable limits

$$\lim_{x \rightarrow +\infty} a^x = \begin{cases} +\infty, & \text{if } a > 1 \\ 0, & \text{if } 0 < a < 1, \end{cases}$$

$$\lim_{x \rightarrow -\infty} a^x = \begin{cases} 0, & \text{if } a > 1 \\ +\infty, & \text{if } 0 < a < 1, \end{cases}$$

$$\lim_{x \rightarrow +\infty} x^p = \begin{cases} +\infty, & \text{if } p > 0 \\ 0, & \text{if } p < 0, \end{cases}$$

$$\lim_{x \rightarrow +\infty} \log x = +\infty,$$

$$\lim_{x \rightarrow 0^+} \log x = -\infty,$$

$$\lim_{x \rightarrow 0^+} x^p \log x = 0, \quad p > 0,$$

$$\lim_{x \rightarrow +\infty} \frac{\log x}{x^p} = 0, \quad p > 0,$$

$$\lim_{x \rightarrow +\infty} \frac{x^p}{a^x} = 0, \quad p > 0, a > 1,$$

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^x = e,$$

$$\lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x}\right)^x = e,$$

$$\lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log a},$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a, \quad a > 0,$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1,$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}.$$

### List of undetermined forms

$$+\infty - \infty, \quad 0 \cdot \infty, \quad \frac{\infty}{\infty}, \quad \frac{0}{0}, \quad 0^0, \quad \infty^0, \quad 1^\infty.$$