MATHEMATICS 1
ADDITIONAL EXERCISES N. 1

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1. Review on Equations and Inequalities

(1) Solve the following equations:
   (a) $(1-3x)(x+2) = (2x+1)\left(\frac{1}{3}x + \frac{2}{3}\right) - \frac{x^2-4}{3}$
   (b) $\frac{(x-1)(x+2)}{3} - \frac{2}{5}(x^2-4x) = \frac{8(1-3x)}{15}$
   (c) $(x-2)(x+2) = \frac{8}{3}x - 5$
   (d) $\frac{7x-1}{x^2+4x+4} + \frac{4}{x^2-4} - \frac{3x^2+5x-2}{x^3-4x+2x^2-8} = 0$
   (e) $\frac{2}{x-2} + \frac{3x}{x^2-4} + 1 = \frac{16}{3x-6} - \frac{x+5}{3x^2-12}$
   (f) $x^3(x^4+1) + \frac{2}{x} = \frac{x^4+33}{16x}$
   (g) $\left(\frac{1}{x+3}\right)^2 - \frac{7}{x+3} - 18 = 0$
   (h) $\sqrt{x^2+4} = x+1$
   (i) $2x-5 = \sqrt{4x^2-x-1}$
   (j) $\sqrt{2x+3} - 1 = \sqrt{8x+5}$
   (k) $\sqrt{7x+1} - 2\sqrt{x+1} = -1$

(2) Solve the following equations with exponentials:
   (a) $3^{2x+1} = 1$
   (b) $e^{2x+1} = e^{\frac{1}{x}}$
   (c) $e^{2x} + 3e^x - 4 = 0$
   (d) $\frac{e^2x+1}{e^x-1} + \frac{e^x+1}{2} = 1$
   (e) $\frac{1}{e^{2x+2}} - \frac{1}{e^x} + 1 = 0$

(3) Solve the following equations with logarithms:
   (a) $\log(3x+4) = 0$
   (b) $\log(x+1) = \log(2x)$

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(c) $2\log(x-1)+\log(x-2)=0$
(d) $\log^2(2x+1)-4\log(2x+1)+3=0$

(4) Solve the following trigonometric equations:
(a) $\sin(3x)=\frac{1}{2}$
(b) $2\sin(x)-\sqrt{3}=0$
(c) $\cos(2x)=\cos(x)$
(d) $4\sin^2(x)-1=0$
(e) $2\sin^2(x)+\sqrt{3}\sin(x)=0$
(f) $\cos^2(x)+\cos(x)=0$
(g) $\cos^2(x)-3\cos(x)+2=0$
(h) $\sin(x)-\cos(x)=0$
(i) $\sin(x)-\sqrt{3}\cos(x)=0$

(5) Solve the following inequalities:
(a) $(2x-5)(x+9)-(x+3)(x-9)<0$
(b) $\frac{x-2}{2}-\frac{5}{3}>\frac{x}{2}-\frac{2x+x^2}{3}$
(c) $(2x^2-8)(3x+5)\leq 0$
(d) $x^3-x^2-6x<0$
(e) $\frac{x^2}{x-2}>x+1$
(f) $\frac{x}{x-2}-2\geq\frac{-x+3}{x+1}$
(g) $\frac{x^2+5x+6}{x^2-4x+4}\leq 1$
(h) $\frac{x(x-1)(x^2-3)}{(x+1)(x-4)}$

(6) Solve the following inequalities with exponentials:
(a) $3^{2x+1}>1$
(b) $2^{2-x}\leq 8$
(c) $2e^{x+3}>5$
(d) $\frac{e^x-1}{x-1}\leq 0$
(e) $(e^x-2)(e^{-x}+1)\geq 0$
(f) $\frac{3e^{2x}}{4-e^x}\geq 1$

(7) Solve the following inequalities with logarithms:
(a) $\log(x^2-4)\geq 0$
(b) $\log(x+1)\leq\log(x-1)$
(c) $\log(4x^2-1)+\log(4-x)\leq 0$
(d) $\log^2(x+2)-2\log(x+2)-3\leq 0$
(e) $\log(2x+1)-\log(3-x)<2$
(f) $\log^2(1-3x)-13\log(1-3x)+36\leq 0$

(8) Solve the following trigonometric inequalities:
  (a) $4\sin^2(x) - 1 > 0$
  (b) $\sin(x) - \sqrt{3}\cos(x) > 0$
  (c) $\sin(x) + \cos(x) + 1 < 0$
  (d) $\dfrac{2\sin^2(x) - 1}{\cos(x)} < 0$
  (e) $\dfrac{\sin(x)}{\sin(x) + 1} > 1$
  (f) $\dfrac{\sin(2x) + \cos(x)}{\cos(2x) + 1} \geq 0$
(9) Solve the following equalities and inequalities with absolute values:
  (a) $|x^2 + 3x| = -2$
  (b) $|x^2 + 3x| > -2$
  (c) $|7x + 3| = 2 - x$
  (d) $|2x - 5| = x$
  (e) $\left|\dfrac{x^2 + 2}{x + 2}\right| \leq 2$

2. Equation of a line

(1) Draw, in a Cartesian plane, the lines with equations:
  (a) $x = 1$
  (b) $y = -3$
  (c) $x + y + 1 = 0$
  (d) $3x + 4y - 1 = 0$
  (e) $y = \frac{1}{2}x + \frac{1}{4}$
  (f) $2x - 3y = 6$
(2) Compute the equations of the lines through points:
  (a) $A = (1, 1)$, $B = (1, 3)$
  (b) $A = (1, 1)$, $B = (3, 1)$
  (c) $A = (-1, 5)$, $B = (2, 0)$
(3) Compute the point-slope equations of the lines:
  (a) $P = (1, -1)$, $m = \frac{1}{2}$
  (b) $P = (0, 3)$, $m = -3$
  (c) $P = (-2, 0)$, $m = 0$

3. Set theory and Logic

(1) For each of the following sets, provide a representation in mathematics terms
  (a) the set of all natural numbers that are multiple of 2;
  (b) the set of all integer numbers between -1 and 3 included;
  (c) the set of all rational numbers between -1 and 3 included;

   (d) the set of all real numbers between -1 and 3 included;
(2) Given $A = \{a, b, c, d\}$, say if the following propositions are true or false
   (a) $b \in A$
   (b) $\{a, b\} \in A$
   (c) $\{a, b\} \subset A$
   (d) $d \notin A$
   (e) $\emptyset \subset A$
(3) Let U be the universal set, A and B subsets of U. Determine, in each of the following cases $A \cup B$, $A \cap B$, A^c, B^c, $(A \cap B)^c$
   (a) $U = \{-2, -1, 0, 1, 2\}$, $A = \{-1, 0\}$, $B = \{-2, 0, 1\}$
   (b) $U = \mathbb{N}$, $A = \{n \in \mathbb{N} : n \leq 10\}$, $B = \{n \in \mathbb{N} : n \geq 2\}$
   (c) $U = \mathbb{Z}$, $A = \{z \in \mathbb{Z} : 0 \leq z \leq 3\}$, $B = \{z \in \mathbb{Z} : -7 \leq zeq - 1\}$
   (d) $U = \mathbb{R}$, $A = \{x \in \mathbb{R} : x^2 \leq 4\}$, $B = \{x \in \mathbb{R} : x^2 > 1\}$
(4) Given sets A and B compute $A \setminus B$ and $B \setminus A$. Do you get the same set?
   (a) $A = \{-3, -2, -1, 4, 5, 6\}$, $B = \{-3, -2, 0, 2, 6, 8\}$
   (b) $A = \{x \in \mathbb{R} : -4 \leq x \leq 4\}$, $B = \{x \in \mathbb{R} : x > 1\}$
(5) Translate in mathematical terms the following statements
   (a) Every number in the set A has the opposite in A
   (b) There is only one number in A which is odd
   (c) There are at least two numbers in A whose sum is equal to zero
(6) For each of the following pair of propositions say if $P \Rightarrow Q$, $Q \Rightarrow P$, $P \Leftrightarrow Q$
   (a) P: $x^2 = 4$; Q: $x = 2$
   (b) P: $x^3 = 8$, Q: $x = 2$
   (c) P: x is larger than 3; Q: x^2 is larger than 9
   (d) P: x and y are negative; Q: $x + y$ is negative
   (e) P: $(x + 1)^2(y - 2) > 0$; Q: $y > 2$
(7) Say which of the following sets is open, closed, not open nor closed, bounded, unbounded
   (a) $\{x \in \mathbb{R} : -4 \leq x \leq 4\}$
   (b) $\{x \in \mathbb{R} : x > 1\}$
   (c) $\{x \in \mathbb{R} : x^2 > 3\}$
   (d) $\{x \in \mathbb{R} : 2 \leq x^2 \leq 4\}$
   (e) $\{x \in \mathbb{R} : -1 \leq x \leq 5\}$
(8) Say which of the following sets is open, closed, not open nor closed, bounded, unbounded
   (a) $\{x \in \mathbb{R} : -4 \leq x \leq 4\}$.
   (b) $\{x \in \mathbb{R} : x > 1\}$.
   (c) $\{x \in \mathbb{R} : x^2 > 3\}$.
   (d) $\{x \in \mathbb{R} : 2 < x^2 \leq 4\}$.
   (e) $\{x \in \mathbb{R} : -1 < x < 5\}$.