MATHEMATICS 1
ADDITIONAL EXERCISES N. 2

KATIA COLANERI

Notation: log stands for the natural logatithm (i.e. the logarithm with the basis e)

1. Domain and Range

(1) Compute the domain of the following functions

$$
\begin{aligned}
(a)&\quad f(x)=\frac{5-2x}{x^2}\\
(b)&\quad f(x)=\frac{x+2}{x^2+x+1}\\
(c)&\quad f(x)=\frac{3x-4}{5-x^2}\\
(d)&\quad f(x)=\sqrt{(x-2)(x-1)}\\
(e)&\quad f(x)=\sqrt{4x^2-9}\\
(f)&\quad f(x)=\sqrt[3]{\frac{x}{x-3}}\\
(g)&\quad f(x)=\frac{x+4}{\sqrt[3]{x^2-1}}\\
(h)&\quad f(x)=\sqrt{x}+\sqrt{x+1}\\
(i)&\quad f(x)=\frac{x+2}{x-\sqrt{x+2}}\\
(j)&\quad f(x)=\frac{1}{|x+1|-|x|}\\
(k)&\quad f(x)=\sqrt{\frac{x}{x+7}}
\end{aligned}
$$

Katia Colaneri, Department of Economics and Finance, University of Rome Tor Vergata, Via Columbia 2, 00133 Rome, Italy.

E-mail address: katia.colaneri@uniroma2.it.

$$(l) \quad f(x) = \frac{\cos(x)}{2\sin(x) - 1}$$

$$(m) \quad f(x) = e^{\sqrt{3x^2-4}}$$

$$(n) \quad f(x) = \frac{\log(1-x)}{2x-1}$$

$$(o) \quad f(x) = \frac{\sqrt{3-x}}{\log(x^2+1)}$$

$$(p) \quad f(x) = \frac{\sqrt{e^{2x}+e^x+1}}{\sqrt{e^{2x}-1}}$$

$$(q) \quad f(x) = \log\left(\frac{x+3}{x-1}\right)$$

$$(r) \quad f(x) = \log(x+3) - \log(x-1)$$

$$(s) \quad f(x) = \sqrt{\frac{\log(1-x)}{\log(x^2-9)}}$$

$$(t) \quad f(x) = \log\left(\frac{x^2+4}{1-4x^2}\right)$$

(2) Compute the Range of the following functions

$$(a) \quad f(x) = \frac{5}{x^2}$$

$$(b) \quad f(x) = x^2 - 3x + 1$$

$$(c) \quad f(x) = \sqrt{(x-2)(x-1)}$$

$$(d) \quad f(x) = \sqrt{4x^2-9}$$

$$(e) \quad f(x) = \sqrt{x} + \sqrt{x+1}$$

$$(f) \quad f(x) = e^{\sqrt{3x^2-4}}$$

$$(g) \quad f(x) = \log(x+3)$$

$$(h) \quad f(x) = \frac{1}{x^2+5}$$

$$(i) \quad f(x) = x^4 + x^2 - 3$$

2. Even/Odd functions

(1) For each of the following functions say if they are even, odd or neither

$$
\begin{aligned}
(a) \quad & f(x) = \frac{2x^2+1}{4x^4} \\
(b) \quad & f(x) = \frac{1}{x^3+x} \\
(c) \quad & f(x) = \frac{x^3}{x^4+1} \\
(d) \quad & f(x) = \sqrt{25-x^2} \\
(e) \quad & f(x) = \frac{x^4-x^3}{x^4+1} \\
(f) \quad & f(x) = |x+5| \\
(g) \quad & f(x) = \log(x+3) \\
(h) \quad & f(x) = \frac{x^3}{4|x|+3}
\end{aligned}
$$

3. Increasing/decreasing functions

(1) Show, using the definition, that the following functions are increasing/decreasing on the indicated interval
   (a) $f(x) = x^2 + 4$, is strictly increasing on $I = (0, +\infty)$
   (b) $f(x) = \frac{1}{x-3}$, is strictly decreasing on $I = (-\infty, 3)$
   (c) $f(x) = \sqrt{x+1}$, is strictly increasing on $I = (-1, +\infty)$
   (d) $f(x) = -x^3 - 1$ is strictly decreasing on $\mathbb{R}$

4. Injective/Surjective/Bijective functions

(1) For each of the following functions say if they are injective, surjective, bijective
   (a) $f : [0, +\infty) \to \mathbb{R}$, such that $f(x) = \sqrt{x}$
   (b) $f : [0, +\infty) \to [0, +\infty)$, such that $f(x) = \sqrt{x}$
   (c) $f : \mathbb{R} \to \mathbb{R}$, such that $f(x) = x^3 + 1$
   (d) $f : \mathbb{R} \to \mathbb{R}$, such that $f(x) = -x^2 - x + 2$
   (e) $f : \left(-\infty, \frac{9}{4}\right] \to \mathbb{R}$, such that $f(x) = -x^2 - x + 2$
   (f) $f : \mathbb{R} \to [0, 1]$, such that $f(x) = 1 - \sin^2(x)$
   (g) $f : \mathbb{R} \setminus \{1\} \to \mathbb{R} \setminus \{0\}$, such that $f(x) = \frac{1}{x-1}$