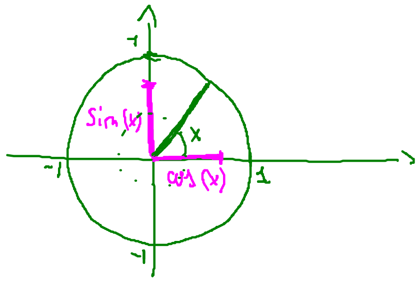
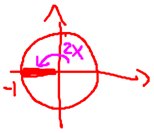
Practice - Math 1 ②

0) $$\frac{\sin(2x)+\cos(x)}{\cos(2x)+1} \geq 0$$



$\cos(2x)+1 \neq 0$



$\cos(2x) \neq -1 \rightarrow 2x \neq \pi + 2k\pi \quad k \in \mathbb{Z}$

$\cos(x \pm 2\pi) = \cos(x)$

$\sin(x \pm 2\pi) = \sin(x)$

$\cos^2(x) + \sin^2(x) = 1$

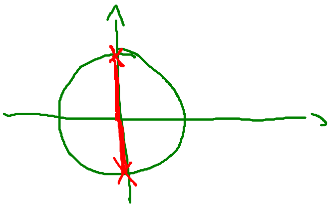
Useful relations:

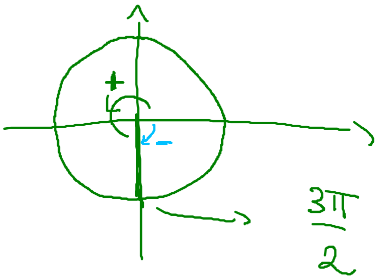
$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

C.E.

$\boxed{x \neq \frac{\pi}{2} + k\pi} \quad k \in \mathbb{Z}$





$\frac{3\pi}{2}, \; -\frac{\pi}{2},$

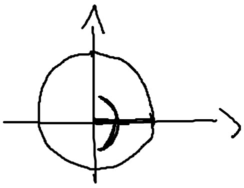
$x \in [0, 2\pi] \qquad [-\pi, \pi]$

$$\frac{2\sin(x)\cos(x)+\cos(x)}{\cos(2x)+1} \geq 0$$

$$\frac{\cos(x)\left[2\sin(x)+1\right]}{\cos(2x)+1} \geq 0$$

	0		$\pi/2$		π		$\frac{7\pi}{6}$		$\frac{3\pi}{2}$		$\frac{11\pi}{6}$		2π
C.E.			×						×				
A $\cos(x)$		+	×	−		−		−	×	+		+	
B $2\sin(x)+1$		+	×	+		+		−	×	−		+	
C $\cos(2x)+1$		+	×	+		+		+	×	+		+	
		+	×	−		−		+	×	−		+	

A $\cos(x) \geq 0$

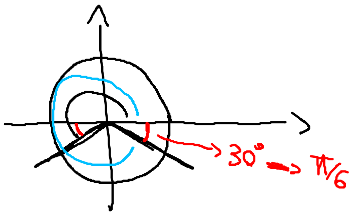


$0 \leq x \leq \pi/2$

$\frac{3\pi}{2} \leq x \leq 2\pi$

B $2\sin(x) + 1 \geq 0$

$\sin(x) \geq -\frac{1}{2}$

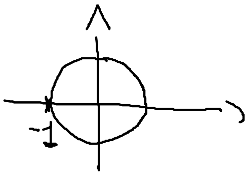


$0 \leq x \leq \frac{7\pi}{6}$

$\frac{11\pi}{6} \leq x \leq 2\pi$

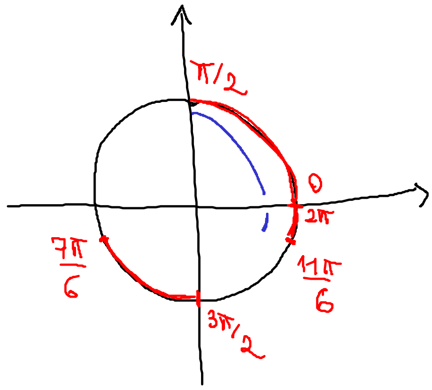
C $\cos(2x) + 1 \geq 0$

$\underbrace{\cos(2x)} \geq -1$



$\forall x \in \mathbb{R}$

$$\boxed{\begin{aligned} &0 + 2k\pi \leq x < \frac{\pi}{2} + 2k\pi \\ &\frac{7\pi}{6} + 2k\pi \leq x < \frac{3\pi}{2} + 2k\pi \\ &\frac{11\pi}{6} + 2k\pi \leq x \leq 2\pi + 2k\pi \end{aligned}}$$



NOT NECESSARY BUT USEFUL TO CHECK

$$\boxed{\begin{aligned} &-\frac{\pi}{6} + 2k\pi \leq x < \frac{\pi}{2} + 2k\pi \\ &\frac{7\pi}{6} + 2k\pi \leq x < \frac{3\pi}{2} + 2k\pi \end{aligned}}$$

$+2\pi \rightarrow \frac{11\pi}{6} + 2k\pi \leq x < \frac{5\pi}{2} + 2k\pi$ the same thing

$$\boxed{0 + 2k\pi \leq x < \frac{\pi}{2} + 2k\pi}$$

$$\frac{11\pi}{6} - 2\pi + 2k\pi \leq x \leq 2\pi - 2\pi + 2k\pi$$

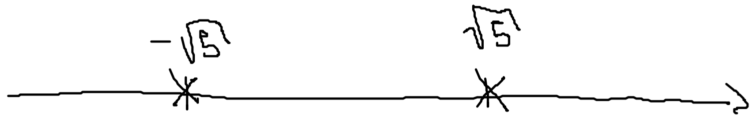
$$\boxed{-\frac{\pi}{6} + 2k\pi \leq x \leq 0 + 2k\pi}$$

Find the domain of this function

1) $$f(x)=\frac{3x-4}{5-x^2}$$

Denominator $\neq 0$

$5-x^2\neq 0 \qquad x^2\neq 5 \qquad x\neq \pm\sqrt{5}$

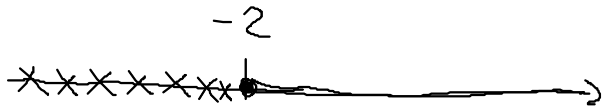


$$\boxed{\{x\in\mathbb{R} : x\neq \pm\sqrt{5}\}}$$

2) $$f(x)=\frac{\sqrt{x+2}}{e^x}$$

Arg. SQ. ROOT ≥ 0

$x+2\geq 0 \qquad x\geq -2$



$$\boxed{\{x\in\mathbb{R} : x\geq -2\}}$$

Denom. $\neq 0$

$e^x\neq 0 \qquad \mathbb{R}\xrightarrow{e}\mathbb{R}^+$

$\forall\, x\in\mathbb{R}$

For every

$e^0=1$
$e^1=e$
$\log(1)=0$
$\log(0)$ FORBIDDEN
$\log(-a)$ FORBIDDEN $a>0$

3) $$f(x)=\frac{\sqrt{x^2-x}}{\log(x)}$$

$\mathbb{R}^+\xrightarrow{\log}\mathbb{R}$

Arg. of the LOG >0

1 $\boxed{x>0}$

Denom. $\neq 0$

$\log(x)\neq 0$

$e^{\log(x)}\neq e^0$

2 $\boxed{x \neq 1}$

Arg. SQ. ROOT ≥ 0

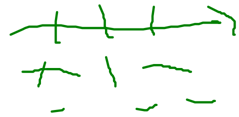
$x^2 - x \geq 0$

~~$x^2 \geq x$~~

~~$x \geq 1$~~

WRONG!!

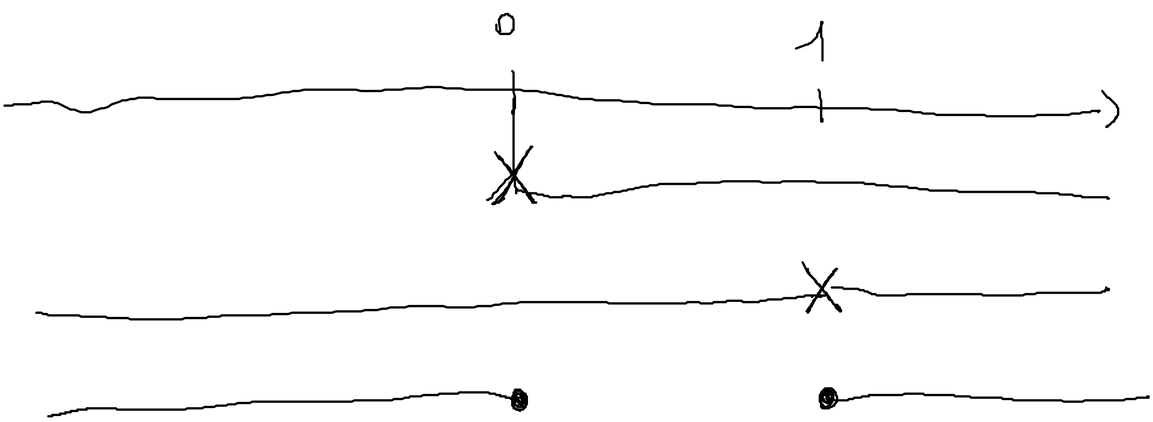
$(x - \_)(x - \_)(x - \_) \geq 0$



$x(x-1) \geq 0$

	0		1	
x	$-$		$+$	$+$
$(x-1)$	$-$		$-$	$+$
	$+$		$-$	$+$

3 $\boxed{x \leq 0 \vee x \geq 1}$

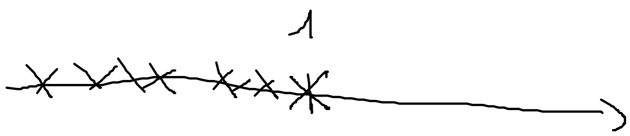


1 $x > 0$

2 $x \neq 1$

3 $x \leq 0 \vee x \geq 1$

$\boxed{x > 1}$



$$\boxed{\{x \in \mathbb{R} : x > 1\}}$$

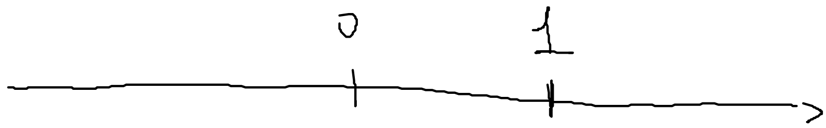
4) $f(x) = \sqrt{x \log(x)}$

Arg of LOG > 0

$$\boxed{x > 0}$$

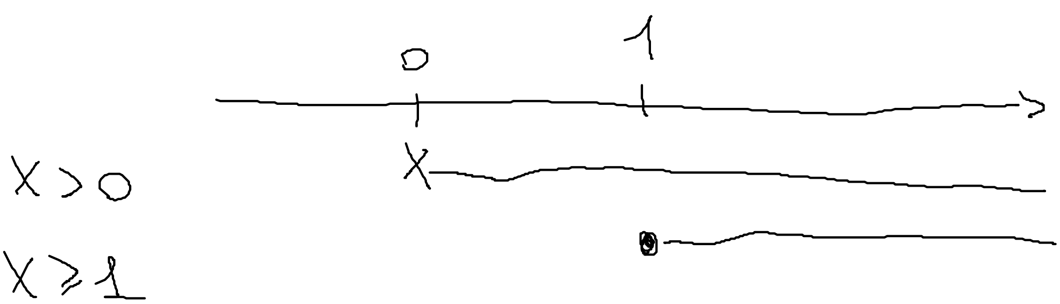
Arg SQ. ROOT ≥ 0

$$x \log(x) \geq 0$$



x	$-$	$+$	•	$+$
$\log(x)$	xxxxxxxxxxx	$-$	•	$+$
	FORBIDDEN	$-$		$+$

$$\boxed{x \geq 1}$$



$$\boxed{x \geq 1}$$

$$\boxed{\{x \in \mathbb{R} : x \geq 1\}}$$

A $\quad e^x = y \quad (x>0) \qquad y>1 \qquad (e \cdot e \cdot e = y > 1)$

B $\quad e^x = y \quad (x<0) \qquad 0<y<1 \qquad \left(\frac{1}{e \cdot e \cdot e \ldots} = y < 1\right) \quad (y>0)$

$\log(e^x) = \log y \qquad x = \log y \qquad$ A

$x>0 \qquad y>1$

B $\quad x = \log y$

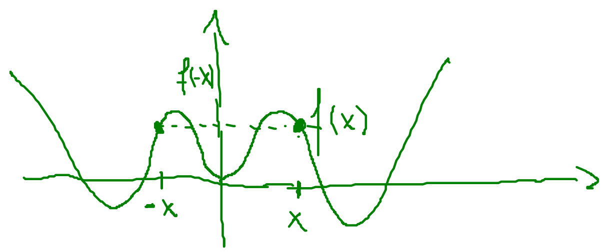
$x<0 \qquad 0<y<1$

5) Determine if the following functions are EVEN, ODD or neither.

a) $f(x) = x^2 + \sin(x)$

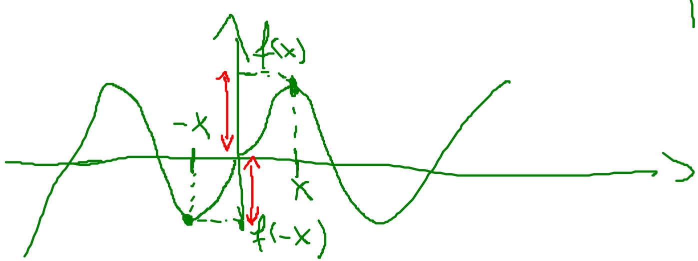
EVEN

$f(-x) = f(x)$

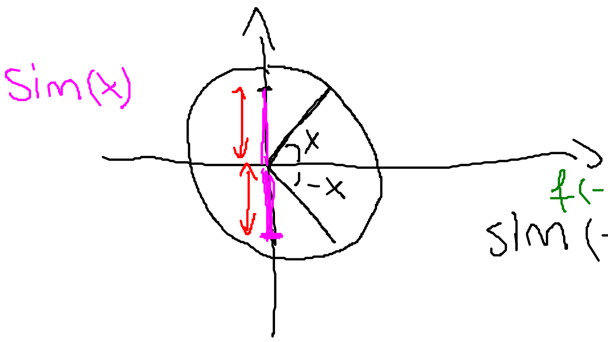


ODD

$f(-x) = -f(x)$



$$f(-x) = (-x)^2 + \sin(-x) = x^2 - \sin(x)$$



$f(-x) = -f(x)$

$\sin(-x) = -\sin(x)$

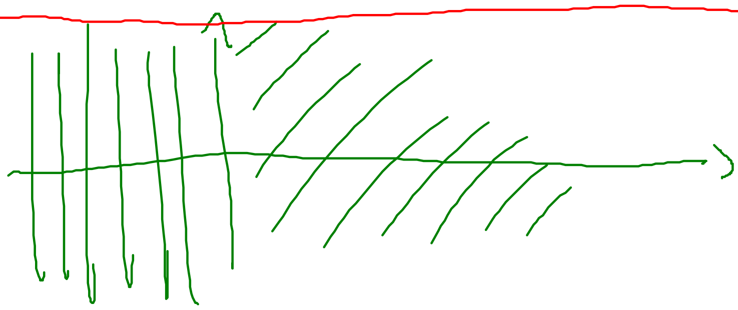
$$f(-x) = x^2 - \sin(x) \stackrel{?}{=} f(x) = x^2 + \sin(x)$$

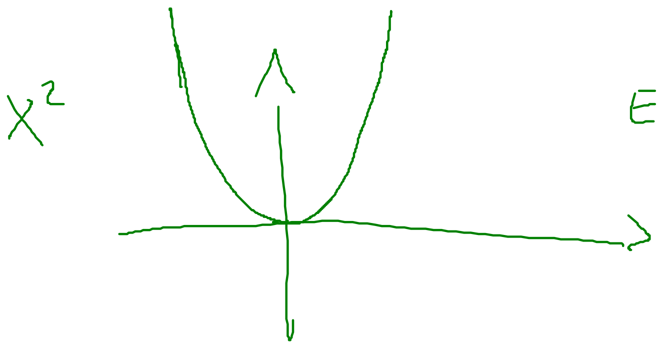
NOT EVEN

$$f(-x) = x^2 - \sin(x) \stackrel{?}{=} -f(x) = -x^2 - \sin(x)$$

NOT ODD

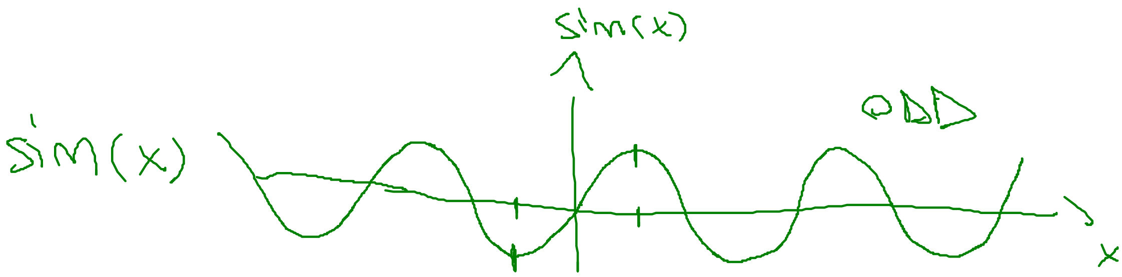
$\boxed{f(x) \text{ is neither EVEN nor ODD}}$





EVEN

$f(-x) = x^2 = f(x)$



ODD

ODD + EVEN = m EVEN m ODD

EVEN + EVEN = EVEN

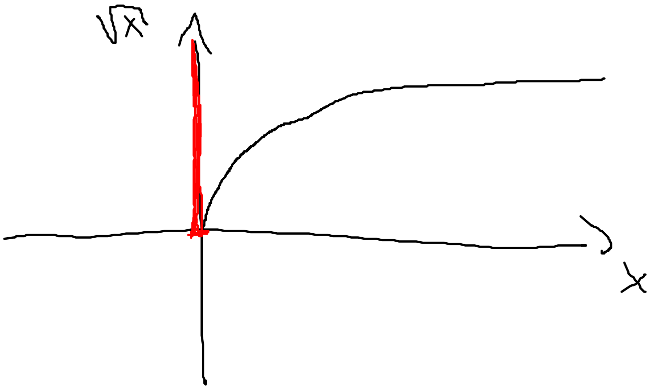
ODD + ODD = ODD

b)
$$f(x) = \frac{2x^2+1}{4x^4} \qquad f(-x) = \frac{2(-x)^2+1}{4(-x)^4} = \frac{2x^2+1}{4x^4}$$

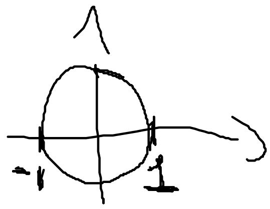
EVEN!

6) Determine the range of the following functions

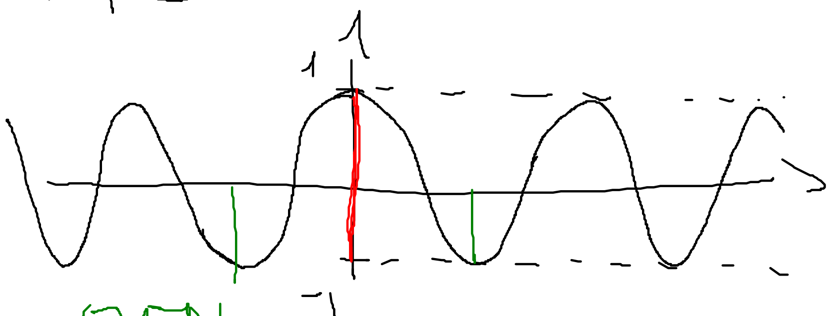
a) $f(x) = \sqrt{x}$ $\longrightarrow$ $\boxed{f(x) \geq 0}$



b) $f(x) = \cos(x)$



$-1 \leq f(x) \leq 1$



$\cos(-x) = \cos(x)$ EVEN

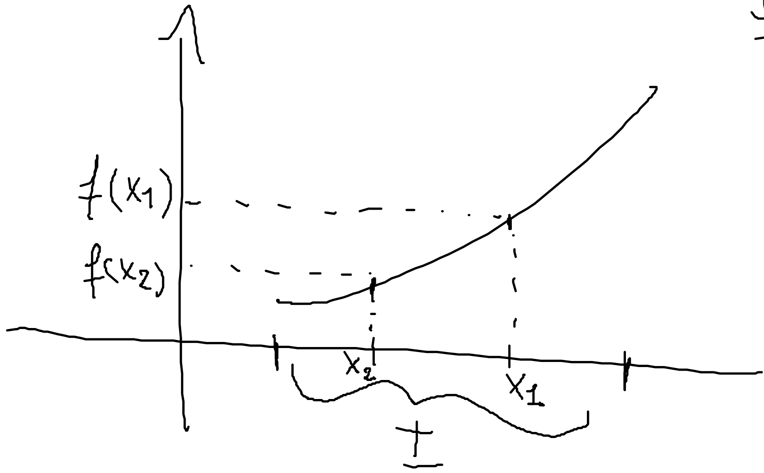
7) Prove that $f(x)$ is increasing in a certain interval I

Def.

$$\forall x_1, x_2 \in I : x_1 > x_2 \Rightarrow f(x_1) \geq f(x_2)$$

$$f(x_1) > f(x_2)$$

strictly



strictly pos.
$>$
strictly neg.
$<$

a) $f(x) = 2x + 1$ $\quad I = x \in \mathbb{R}$

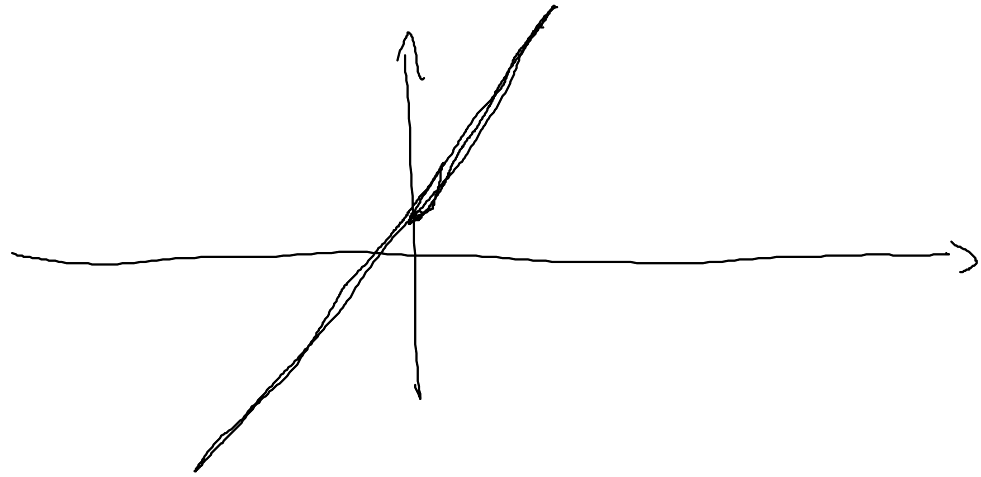
$$x_1, x_2 \quad x_1 > x_2 \Rightarrow \underline{x_1 - x_2} > 0$$

$$f(x_1) - f(x_2) \geq 0 \Leftrightarrow f(x_1) \geq f(x_2)$$

$$f(x_1) - f(x_2) = 2x_1 + 1 - (2x_2 + 1) = \underset{>0}{2}\,\overset{>0}{(x_1 - x_2)} > 0$$

$$\Downarrow$$

$$f(x_1) > f(x_2)$$ strictly increasing ✓



b)

$$f(x) = \sqrt{x+1} \qquad I = (0, +\infty) \qquad x_1, x_2 \qquad x_1 > x_2$$

$$f(x_1) - f(x_2) = \sqrt{x_1+1} - \sqrt{x_2+1} =$$

$$= \frac{\sqrt{x_1+1} - \sqrt{x_2+1}}{\left(\sqrt{x_1+1} + \sqrt{x_2+1}\right)} \cdot \left(\sqrt{x_1+1} + \sqrt{x_2+1}\right) =$$

$$= \frac{\left(\sqrt{x_1+1}\right)^2 - \left(\sqrt{x_2+1}\right)^2}{\sqrt{x_1+1} + \sqrt{x_2+1}} = \frac{x_1 + \cancel{1} - x_2 - \cancel{1}}{\sqrt{x_1+1} + \sqrt{x_2+1}} =$$

$$= \frac{\overbrace{x_1 - x_2}^{>0}}{\underbrace{\underset{>0}{\sqrt{x_1+1}} + \underset{>0}{\sqrt{x_2+1}}}_{>0}} > 0 \qquad f(x_1) > f(x_2)$$

strictly increasing