MATHEMATICS 1
ADDITIONAL EXERCISES N. 3

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Notation: log stands for the natural logatithm (i.e. the logarithm with the basis e)

1. Inverse function, composite functions and plots

(1) For each of the following functions, say if they are invertible and if so, compute the inverse.

$$\begin{aligned}
&(a) \quad f:\mathbb{R}\to\mathbb{R} \quad f(x)=4-3x\\
&(b) \quad f:\mathbb{R}\to\mathbb{R} \quad f(x)=4-x^2\\
&(c) \quad f:[0,+\infty)\to(-\infty,4] \quad f(x)=4-x^2\\
&(d) \quad f:\mathbb{R}\setminus\{-1\}\to\mathbb{R}\setminus\{0\} \quad f(x)=\frac{1}{x+1}\\
&(e) \quad f:\mathbb{R}\to[5,+\infty) \quad f(x)=x^2+5\\
&(f) \quad f:(-\infty,2]\to[0,+\infty) \quad f(x)=\sqrt{8-2x}\\
&(g) \quad f:\mathbb{R}\to(0,+\infty) \quad f(x)=e^{2x+1}\\
&(h) \quad f:\mathbb{R}\setminus\{2\}\to\mathbb{R}\setminus\{1\} \quad f(x)=\frac{x+1}{x-2}\\
&(i) \quad f:(3,+\infty)\to\mathbb{R} \quad f(x)=\log(x-3)
\end{aligned}$$

(2) Given the following plots of functions $g(x)$, draw, if possible:
- the inverse function
- $|g(x)|$
- $g(x+2)$
- $g(x)-3$

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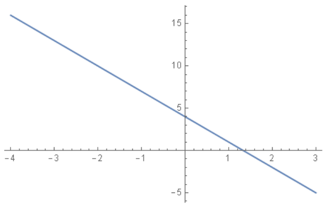

FIGURE 1. 2 (a)

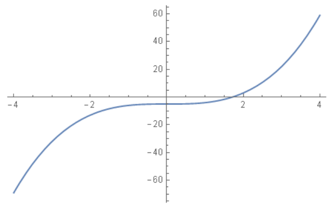

FIGURE 2. 2 (b)

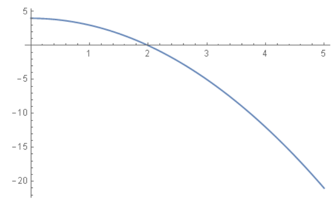

FIGURE 3. 2 (c)

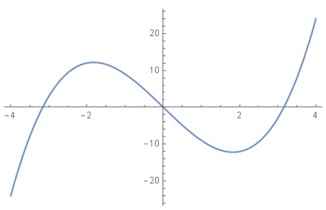

FIGURE 4. 2 (d)

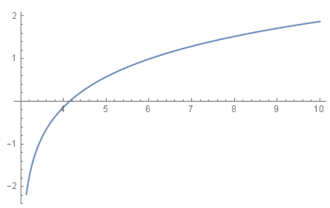

FIGURE 5. 2 (e)

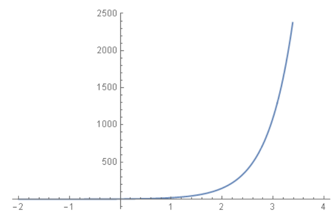

FIGURE 6. 2 (f)

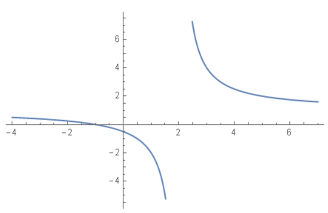

FIGURE 7. 2 (g)

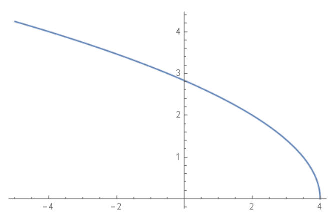

FIGURE 8. 2 (h)

(3) For each of the following pair of functions f and g, compute $f(g(x))$ and $g(f(x))$ and specify their domain and range

$$(a) \quad f:\mathbb{R}\to\mathbb{R} \quad f(x)=2x+1, \quad g:\mathbb{R}\to[0,+\infty) \quad g(x)=x^2$$
$$(b) \quad f:\mathbb{R}\to\mathbb{R} \quad f(x)=x^3, \quad g:\mathbb{R}\setminus 1\to\mathbb{R}\setminus\{0\} \quad g(x)=\frac{1}{x-1}$$
$$(c) \quad f:\mathbb{R}\to(0,+\infty) \quad f(x)=e^x, \quad g:\mathbb{R}\to\mathbb{R} \quad g(x)=3x+5$$
$$(d) \quad f:(0,+\infty)\to\mathbb{R} \quad f(x)=\log(x), \quad g:\mathbb{R}\to[-1,+\infty) \quad g(x)=x^2-1$$
$$(e) \quad f:[-1,1]\to[0,1] \quad f(x)=\sqrt{1-x^2}, \quad g:(-1,+\infty)\to\mathbb{R} \quad g(x)=\log(x+1)$$

2. Sequences

(1) Prove the following limits using the definition.

$$
\begin{aligned}
(a)&\quad \lim_{n\to\infty} \frac{n+5}{n^2} = 0\\
(b)&\quad \lim_{n\to\infty} \frac{2n+1}{n} = 2\\
(c)&\quad \lim_{n\to\infty} \frac{n^2+n+1}{n} = +\infty\\
(d)&\quad \lim_{n\to\infty} \sqrt{n} = +\infty\\
(e)&\quad \lim_{n\to\infty} \frac{1-n^2}{n^2} = -1\\
(f)&\quad \lim_{n\to\infty} 1-n^3 = -\infty\\
(g)&\quad \lim_{n\to\infty} \frac{5}{n+3} = 0\\
(h)&\quad \lim_{n\to\infty} \frac{n^2+2n}{3n^2+1} = \frac{1}{3}\\
(i)&\quad \lim_{n\to\infty} e^n = +\infty\\
(j)&\quad \lim_{n\to\infty} e^{-n} = 0\\
(k)&\quad \lim_{n\to\infty} \log(n+1) = 0\\
(l)&\quad \lim_{n\to\infty} \log\left(\frac{n}{n+1}\right) = 0\\
(m)&\quad \lim_{n\to\infty} e^{\frac{n}{n+1}} = e
\end{aligned}
$$

(2) Show that the following limits do not exist

$$
\begin{aligned}
(a)&\quad \lim_{n\to\infty} \cos(n\pi)\\
(b)&\quad \lim_{n\to\infty} \frac{(-1)^n n}{n+1}\\
(c)&\quad \lim_{n\to\infty} n^{(-1)^n}\\
(d)&\quad \lim_{n\to\infty} (-2)^n\\
(e)&\quad \lim_{n\to\infty} (-n)^n
\end{aligned}
$$

(3) Compute the following limits using the Absolute value Theorem or the Comparison Theorem.

$$(a) \quad \lim_{n\to\infty} \frac{\cos(n)}{n^2}$$

$$(b) \quad \lim_{n\to\infty} \frac{(-1)^n n + 1}{1 - n^2}$$

$$(c) \quad \lim_{n\to\infty} \frac{3 + \sin(n)}{n + 4}$$

$$(d) \quad \lim_{n\to\infty} \frac{3}{n\cos(n\pi)}$$

$$(e) \quad \lim_{n\to\infty} \frac{3}{n\cos(n) + 2n}$$

$$(f) \quad \lim_{n\to\infty} \frac{n + n(-1)^n}{n^2}$$

$$(g) \quad \lim_{n\to\infty} \frac{2n\cos(n\pi)}{n + n^2}$$