MATHEMATICS 1
ADDITIONAL EXERCISES N. 4

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Notation: log stands for the natural logarithm (i.e. the logarithm with the basis e)

1. Sequences

(1) Compute the following limits

(a) $\lim_{n\to\infty} \frac{(n+1)^2}{n^2+1}$

(b) $\lim_{n\to\infty} \frac{n^2+5n+1}{3n+7}$

(c) $\lim_{n\to\infty} \frac{2n^2-3n-4}{\sqrt{n^4+4}}$

(d) $\lim_{n\to\infty} n-\sqrt{2n^2+5n}$

(e) $\lim_{n\to\infty} \sqrt{5n+6}-\sqrt{2n+1}$

(f) $\lim_{n\to\infty} n(\sqrt{n^2+1}-n)$

(g) $\lim_{n\to\infty} n+\sqrt[3]{1-n^3}$

(h) $\lim_{n\to\infty} \frac{n+1}{n^2+1}$

(i) $\lim_{n\to\infty} \frac{n^4+5}{n^5+7n-1}$

(j) $\lim_{n\to\infty} \frac{1-n}{\sqrt{n}+1}$

(k) $\lim_{n\to\infty} \frac{n+(-1)^n}{n-(-1)^n}$

(l) $\lim_{n\to\infty} \sqrt{n+2}-\sqrt{n-1}$

(m) $\lim_{n\to\infty} \sqrt{n^2+1}-\sqrt{n}$

(n) $\lim_{n\to\infty} n\sqrt{\frac{1}{n+1}}$

(o) $\lim_{n\to\infty} e^n-2^n$

(p) $\lim_{n\to\infty} 3^n+4^n-5^n$

(q) $\lim_{n\to\infty} \frac{2^{n+1}-4^{n-1}}{3^n}$

(r) $\lim_{n\to\infty} \frac{2^{n+1}+1}{3^n+1}$

(s) $\lim_{n\to\infty} n-\log(n)$

(t) $\lim_{n\to\infty} \frac{(n3^n+n^5+\sin(n))n}{(3^n+2^n)n^2}$

(u) $\lim_{n\to\infty} \frac{\log^2(n)}{n}$

(v) $\lim_{n\to\infty} \frac{n^4+5}{n^5+7n-1}$

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(w) $\lim_{n\to\infty} 3^{n+1} - 3^{\sqrt{n^2-1}}$

(x) $\lim_{n\to\infty} \dfrac{5^n - n^5}{4^n + n^6}$

(y) $\lim_{n\to\infty} \dfrac{(n^2+1)\log(n)}{n^3}$

(z) $\lim_{n\to\infty} \dfrac{n! + 2^n}{(n+1)!}$

(2) Compute the following limits

(a) $\lim_{n\to\infty} \left(\dfrac{n+1}{n+3}\right)^n$

(b) $\lim_{n\to\infty} \left(\dfrac{3n+1}{3n-3}\right)^{n-1}$

(c) $\lim_{n\to\infty} \left(\dfrac{n+1}{n}\right)^{n^2}$

(d) $\lim_{n\to\infty} \left(\dfrac{n^2+1}{n^2-1}\right)^n$

(e) $\lim_{n\to\infty} \left(1 + \dfrac{1}{2^n}\right)^{n^2}$

(f) $\lim_{n\to\infty} \left(1 + 3^{-n}\right)^n$

(g) $\lim_{n\to\infty} \left(1 + \dfrac{1}{\log(n)}\right)^n$

(h) $\lim_{n\to\infty} (2n+1)\log\left(\dfrac{2n+1}{2n-3}\right)$

(i) $\lim_{n\to\infty} \left(1 + e^{-n}\right)^{n!}$

(j) $\lim_{n\to\infty} n^3 \log\left(1 + 3^{-n}\right)$

(k) $\lim_{n\to\infty} n\log\left(\dfrac{n+1}{n+3}\right)$

(l) $\lim_{n\to\infty} n\left(e^{\frac{1}{n^2}-1}\right)$

(m) $\lim_{n\to\infty} n^2\left(e^{\frac{1}{n}-1}\right)$

(n) $\lim_{n\to\infty} n\left(e^{\frac{n+1}{n-1}-1}\right)$

(o) $\lim_{n\to\infty} 2n\left(e^{\frac{3}{n}-1}\right)$

(p) $\lim_{n\to\infty} (n^2+1)\left(e^{\frac{2}{n^2}-1}\right)$

(q) $\lim_{n\to\infty} n^2 \sin\left(\dfrac{2}{n^2}\right)$

(r) $\lim_{n\to\infty} n\sin\left(\dfrac{1}{n+1}\right)$

(s) $\lim_{n\to\infty} \dfrac{\sin\left(\frac{1}{n^2}\right)}{1-\cos\left(\frac{1}{n}\right)}$

(t) $\lim_{n\to\infty} \dfrac{e^{2+\frac{2}{n}} - 1}{\sin\left(\frac{1}{n}\right)}$

(u) $\lim_{n\to\infty} \dfrac{\log\left(\frac{n+2}{n-1}\right)}{\left(1-\cos\left(\frac{1}{2n}\right)\right)}$

(v) $\lim_{n\to\infty} \dfrac{\log\left(\frac{n^2-1}{n^2+1}\right)}{\left(1-\cos\left(\frac{2}{n}\right)\right)}$

(w) $\lim_{n\to\infty} \dfrac{x^3+3x+1}{\sin\left(\frac{2}{3n^3}\right)}$

(x) $\lim_{n\to\infty} \dfrac{5^n - n^5}{\log(1+5^{-n})}$

(y) $\lim_{n\to\infty} \dfrac{(n^2+1)}{n\sin\left(\frac{1}{2n}\right)}$

(z) $\lim_{n\to\infty} \dfrac{n! + 2^n}{\log\left(1 + \frac{1}{(n+1)!}\right)}$

2. Series

(1) Compute the following series (if they exist)

(a) $\displaystyle\sum_{n=0}^{+\infty}\left(\cos\left(\frac{\pi}{3}\right)\right)^n$ (b) $\displaystyle\sum_{n=0}^{+\infty}\left(\cos\left(\frac{\pi}{4}\right)\right)^n$

(c) $\displaystyle\sum_{n=0}^{+\infty}\left(\cos\left(\frac{\pi}{6}\right)\right)^n$ (d) $\displaystyle\sum_{n=0}^{+\infty}\left(\cos\left(\frac{4\pi}{3}\right)\right)^n$

(e) $\displaystyle\sum_{n=0}^{+\infty}\frac{\left(\sin\left(\frac{\pi}{4}\right)\right)^n+1}{(\sqrt{2})^n}$ (f) $\displaystyle\sum_{n=0}^{+\infty}\frac{\left(\sin\left(\frac{\pi}{3}\right)\right)^n+2}{(-3)^n}$

(g) $\displaystyle\sum_{n=0}^{+\infty}\frac{2^n+4^n}{5^n}$ (h) $\displaystyle\sum_{n=0}^{+\infty}\frac{3^n+5^n}{4^n}$

(i) $\displaystyle\sum_{n=0}^{+\infty}\frac{2^n-5^n}{7^n}$ (j) $\displaystyle\sum_{n=0}^{+\infty}\frac{e^n+1}{e^{2n}}$

(k) $\displaystyle\sum_{n=0}^{+\infty}\frac{3^{n+2}}{5^n}$ (l) $\displaystyle\sum_{n=0}^{+\infty}\left(-\frac{2}{3}\right)^{n-3}$

(m) $\displaystyle\sum_{n=1}^{+\infty}\left(-\frac{1}{3}\right)^n$ (n) $\displaystyle\sum_{n=2}^{+\infty}\left(\sin\left(\frac{\pi}{3}\right)\right)^n$

(o) $\displaystyle\sum_{n=2}^{+\infty}(1-\log(2))^n$ (p) $\displaystyle\sum_{n=1}^{+\infty}\frac{e^n}{3^n}$

(q) $\displaystyle\sum_{n=1}^{+\infty}\frac{e^n}{2^n}$ (r) $\displaystyle\sum_{n=1}^{+\infty}(-\sqrt{2})^n$

(2) Say for which values of the parameter a the following series exist.

(a) $\displaystyle\sum_{n=0}^{+\infty}(2a+1)^n$ (b) $\displaystyle\sum_{n=0}^{+\infty}\left(a^2+3a+2\right)^n$

(c) $\displaystyle\sum_{n=0}^{+\infty}(1+\log(a))^n$ (d) $\displaystyle\sum_{n=0}^{+\infty}\left(\frac{a+1}{a}\right)^n$

(e) $\displaystyle\sum_{n=0}^{+\infty}(a^2-1)^n$ (f) $\displaystyle\sum_{n=0}^{+\infty}(\cos(a))^n$

(g) $\displaystyle\sum_{n=0}^{+\infty}\left(\frac{2a+1}{a}\right)^n$ (h) $\displaystyle\sum_{n=0}^{+\infty}\left(\frac{3^a}{2}\right)^n$