MATHEMATICS 1

Mock Test

November 26th, 2021, A.Y. 2021/2022

This test will be discussed in class on December 3rd 2021

Any kind of electronic device (calculators, smartphones, smartwatches etc..) is FORBIDDEN. Put all your devices in the location indicated by the examiners. If an attendee violates this rule (even if the device is switched off or in offline mode), she/he will be immediately expelled from the exam session and her/his exam will be invalidated.

Remember to always double check the consistency of your results. EVEN IF CORRECT, inconsistent statements will result in a negative impact on the final grade of the exam.

MARKS: you get 3 points for each correct answer, 0 points for unanswered questions, -1 points for each incorrect answers. Open questions are not penalized.

Time for the test: 1 hour and 15 minutes

MATRICOLA ..... Lastname ..... Name .....

1. Compute (if it exists) the sum of the series

$$\sum_{n=1}^{+\infty} \frac{2^n - 3^n}{5^n}$$

☐ A. $-\frac{5}{6}$ ☐ B. $-\frac{5}{56}$

☐ C. $\frac{55}{24}$ ☐ D. $\frac{25}{6}$

2. Compute the limit of the following sequence

$$\lim_{n\to+\infty} \frac{2n - \sqrt{n^2 - 3}}{n - \sqrt{3 + 4n}}$$

☐ A. 2 ☐ B. -2

☐ C. 1 ☐ D. -1

3. Compute the domain and the range of the function:

$$f(x) = 1 - \sqrt{\log(x + 5)}$$

☐ A. $D = (-5, +\infty)\,, \quad R = (-\infty, 0]$

☐ B. $D = (-4, +\infty)\,, \quad R = (0, \infty)$

☐ C. $D = (-5, +\infty)\,, \quad R = [0, \infty)$

☐ D. $D = (-4, +\infty)\,, \quad R = (-\infty, 1]$

4. Consider the function

$$f(x) = \frac{1}{x^2 - 1}$$

Then,

☐ A. The function is injective in its domain

☐ B. The range of the function is $\mathbb{R} \setminus \{0\}$

☐ C. The function is even

☐ D. The function is strictly increasing in its domain

5. Compute the first order Taylor approximation $P(x)$ at point $x_0 = 0$ of the function $f(x)$ and evaluate the error R which is made by approximating $f(x_1)$ with $P(x_1)$ for $x_1 = 0.1$

$$f(x) = \sqrt{10x + 1}$$

☐ A. $P(x) = 5x + 1, \qquad R = 1.5 - \sqrt{2}$

☐ B. $P(x) = 10x + 1, \qquad R = 2 - \sqrt{2}$

☐ C. $P(x) = \dfrac{5}{10x + 1}, \qquad R = \dfrac{5}{\sqrt{2}} - \sqrt{2}$

☐ D. $P(x) = \dfrac{10}{10x + 1}, \qquad R = \dfrac{10}{\sqrt{2}} - \sqrt{2}$

6. Consider the following function:

$$f(x) = \frac{\sqrt{x^2 + 1}}{x - 2}$$

Then,

☐ A. $y = 1$ is the unique horizontal asymptote

☐ B. $y = 1$ and $y = -1$ are both horizontal asymptotes

☐ C. $y = 0$ and $y = 2$ are both horizontal asymptotes

☐ D. $f(x)$ has no horizontal asymptotes

7. Consider the following function:

$$f(x) = \frac{x}{\log x}$$

Then,

☐ A. The maximum of $f(x)$ in the interval $[2, 3]$ is achieved for $x = 2$

☐ B. The maximum of $f(x)$ in the interval $[2, 3]$ is achieved for $x = 3$

☐ C. The maximum of $f(x)$ in the interval $[2, 3]$ is achieved for $x = e$

☐ D. $f(x)$ has no maximum in $[2, 3]$

8. Consider the following function:

$$f(x) = \log x - x^2$$

Then,

- ☐ A. $f(x)$ has two local maxima at $x = -\sqrt{\frac{1}{2}}$ and $x = \sqrt{\frac{1}{2}}$ and no local minima
- ☐ B. $f(x)$ has two local maxima at $x = -\sqrt{\frac{1}{2}}$ and $x = \sqrt{\frac{1}{2}}$ and one local minimum at $x = 0$
- ☐ C. $f(x)$ has one local maximum at $x = \sqrt{\frac{1}{2}}$ and no local minima
- ☐ D. $f(x)$ has one local maximum at $x = \sqrt{\frac{1}{2}}$ and an inflection point with horizontal tangent at $x = 0$

9. Compute all intervals of concavity and convexity and all inflection points of the function

$$f(x) = e^{2x} - 2e^x + 5x$$

- ☐ A. Concave: $(-\infty, \log(1/2))$, Convex: $(\log(1/2), +\infty)$
- ☐ B. Convex for all $x \in \mathbb{R}$
- ☐ C. Concave: $(-\infty, \log(2))$, Convex: $(\log(2), +\infty)$
- ☐ D. Concave: $(-\infty, 0)$, Convex: $(0, +\infty)$

10. Prove the following limit using the definition:

$$\lim_{n\to\infty} e^{\frac{2n-1}{n}} = e^2$$

11. Provide the statement of Fermat Theorem

12. Say if the function is continuous and identify all discontinuity points (if any)

$$f(x) = \frac{1 - e^x}{\sin(x)}$$