Portfolio Theory
Portfolio Costruction

Ugo Pomante
Professor of «Finance & Banking»
Tor Vergata University

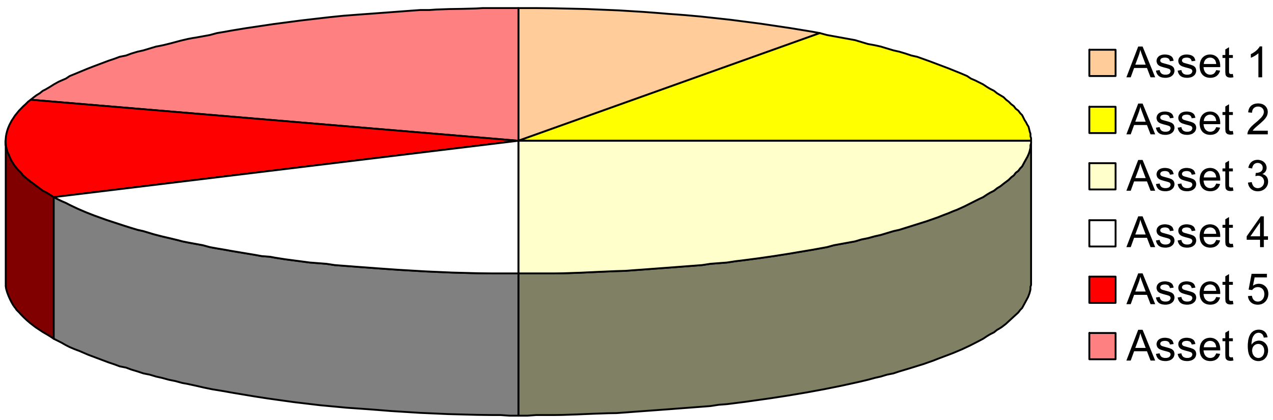
Introduction to “Portfolio Theory” and “Portfolio Construction”

The purpose of this section is to show which are the main stage necessary to built a Portfolio

Portfolio construction: Is it easy?

At a first glimpse it might appear that building a portfolio is easy:

- ➢ you have just to aggregate different assets (bond, stocks, commodities, real estate)



Portfolio construction: Is it easy? The answer is NO

…..Unfortunately building a GOOD portfolio that is able to satisfy the needs/expectations of an investor is HARD.

In fact, in order to construct a good portfolio, you have to make many difficult decisions.

Portfolio Construction: Problems to be solved

- Selection of Financial Markets where "to invest the money" (Bonds, Equities, Commodities, etc…) : Which? How many?
- Estimation of future trend of the markets selected.
- Construction of an optimization model that returns the optimal portfolio (that returns the optimal weights of the Financial Market) in the long run.
- Development of an evaluation model that it is able to verify that the portfolio selected satisfies the needs of the investor.
- Development of a "market timing" model, useful in order to make tactical changes to the portfolio composition, in order to anticipate bull/bear trends.
- Selection of the best financial products for every financial market.

A mistake? Very dangerous!

- Adverse selection of Financial Markets (too much risky or poorly performing markets)
- Error in Estimation of future trend of the markets selected or overconfidence in the ability of prediction.
- Construction of a weak optimization model.
- Incapacity to verify that the portfolio selected satisfies the needs of the investor.
- Errors in "market timing" decisions (increase of the equity weight at the beginning of a *bear trend*).
- Adverse selection of products for every financial market (poor performance or high costs).

Don’t under-estimate the process of portfolio construction

- Mistakes can be “deadly”.
- So, it is necessary to have:
  - skilled human resources;
  - good IT procedures;
  - consistent models of Portfolio Construction.

Well-Organized procedures

- Institutional investors (pension funds, mutual funds, etc…) organize the process of portfolio construction on stages...
- …where every phase is able either to create (extra-performance) or destroy value (under-performance);
- There are three main stages.

Stages of the “Portfolio Construction”

1. Strategic Asset Allocation

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2. Tactical Asset Allocation

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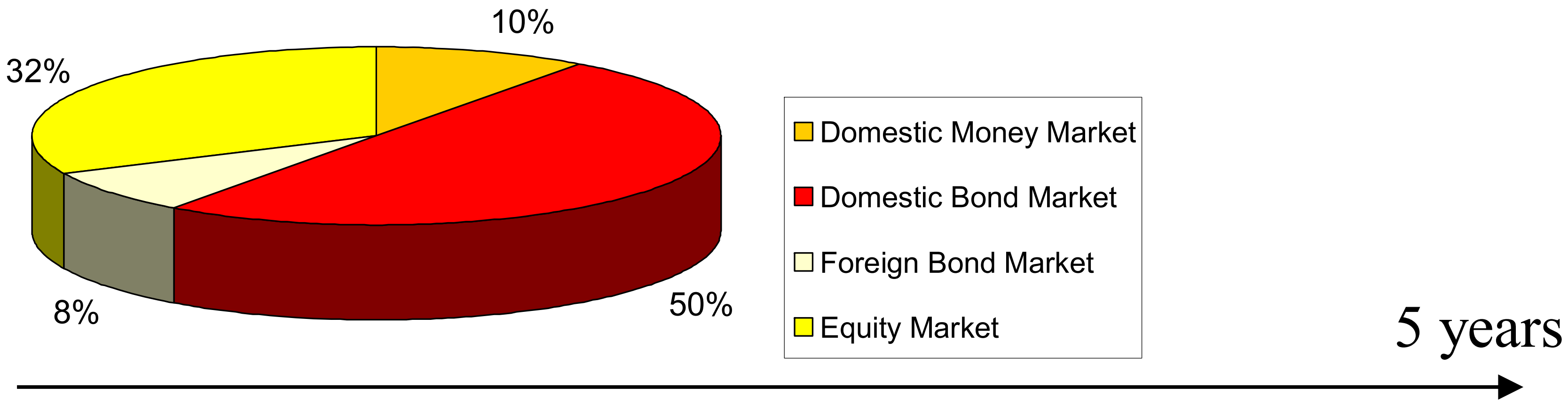
3. Stock-Bond/Fund Selection

Stage 1: Strategic Asset Allocation (SAA)

- **Strategic Asset Allocation** is:
  - the portfolio composed by financial markets (or asset classes)…..
  - ….. that the investor must hold in the long run (all the investment horizon).

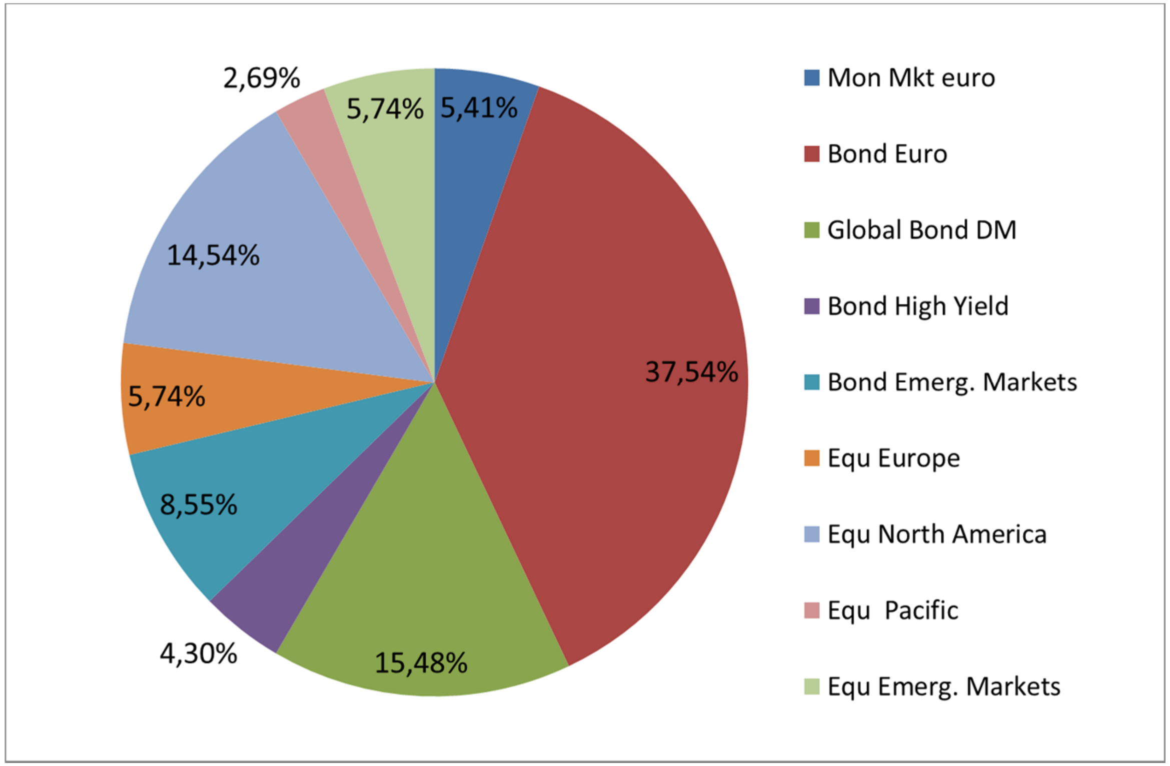
Strategic Asset Allocation: Example

- The investor has a 5-years investment horizon.
- The Asset Manager builds a portfolio, composed by financial markets (*that is supposed to be coherent with the risk tolerance of the investor*).



"On average" the portfolio composition is expected to be this one in the next five years.

ASSET CLASSES	Strategic Weight
Mon Mkt euro	5,41%
Bond Euro	37,54%
Global Bond DM	15,48%
Bond High Yield	4,30%
Bond Emerg. Markets	8,55%
Equ Europe	5,74%
Equ North America	14,54%
Equ Pacific	2,69%
Equ Emerg. Markets	5,74%
PORTFOLIO	**100,00%**

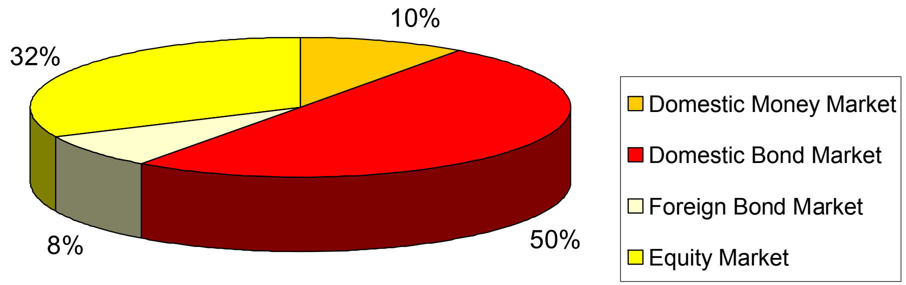


Stage 2: Tactical Asset Allocation (TAA)

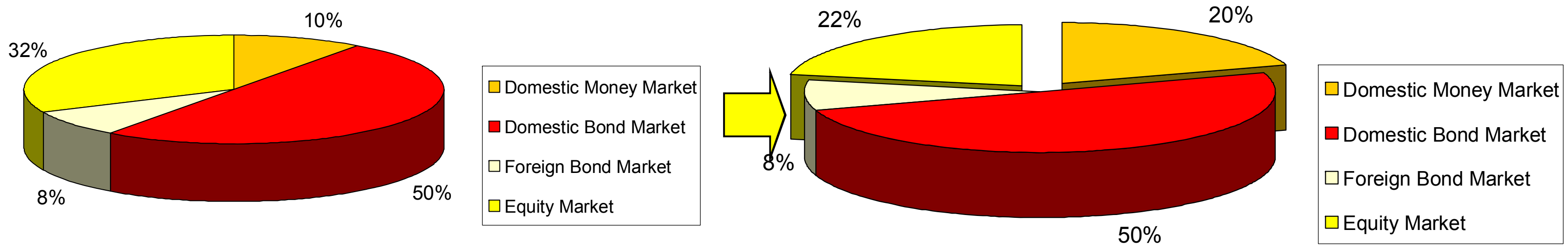
- **Tactical Asset Allocation** is:
  - The change made to the strategic composition in order to anticipate bull/bear trends.
  - ….. that the investor must hold in the short run (next 1-3 months).

Tactical Asset Allocation: Example

- The SAA is the following:



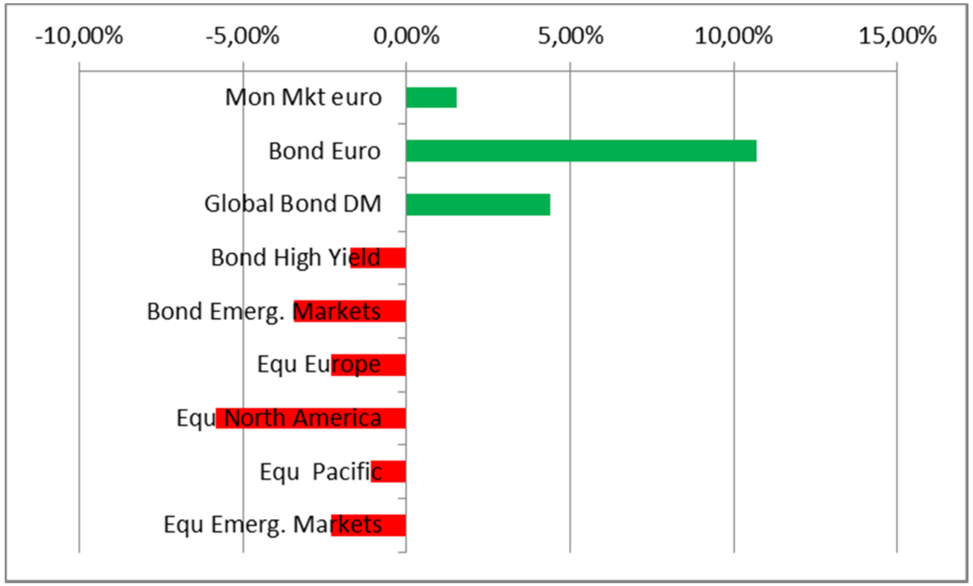
- But the Asset Manager has the expectation that in the next 3 months the Equity Market will decrease.
- So, for the next 3 months he suggests the following changes in the portfolio composition:



After three months, the tactical portfolio will be dismantled (and the strategic portfolio will be resumed)…..may be we will create e new tactical solution.

From Strategic to Tactical Asset Allocation:

ASSET CLASSES	Strategic Weight	Tactical Weights	Delta
Mon Mkt euro	5,41%	6,95%	1,54%
Bond Euro	37,54%	48,22%	10,68%
Global Bond DM	15,48%	19,88%	4,40%
Bond High Yield	4,30%	2,58%	-1,72%
Bond Emerg. Markets	8,55%	5,13%	-3,42%
Equ Europe	5,74%	3,45%	-2,30%
Equ North America	14,54%	8,72%	-5,82%
Equ Pacific	2,69%	1,61%	-1,08%
Equ Emerg. Markets	5,74%	3,45%	-2,30%
PORTFOLIO	**100,00%**	**100,00%**	

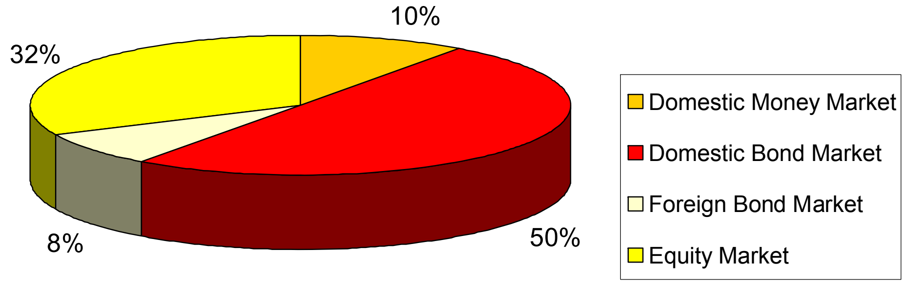


Stage 3: Stock-Bond/Fund Selection

- **Stock-Bond/Fund Selection** is:
  - the process to select the best product for every market in the portfolio.
  - You can (alternatively):
    - o directly select stocks & bonds (stock-bond selection);
    - o indirectly select stocks & bonds, identifying the best fund managers (fund selection).

Stock-Bond/Fund Selection : Example (1/2)

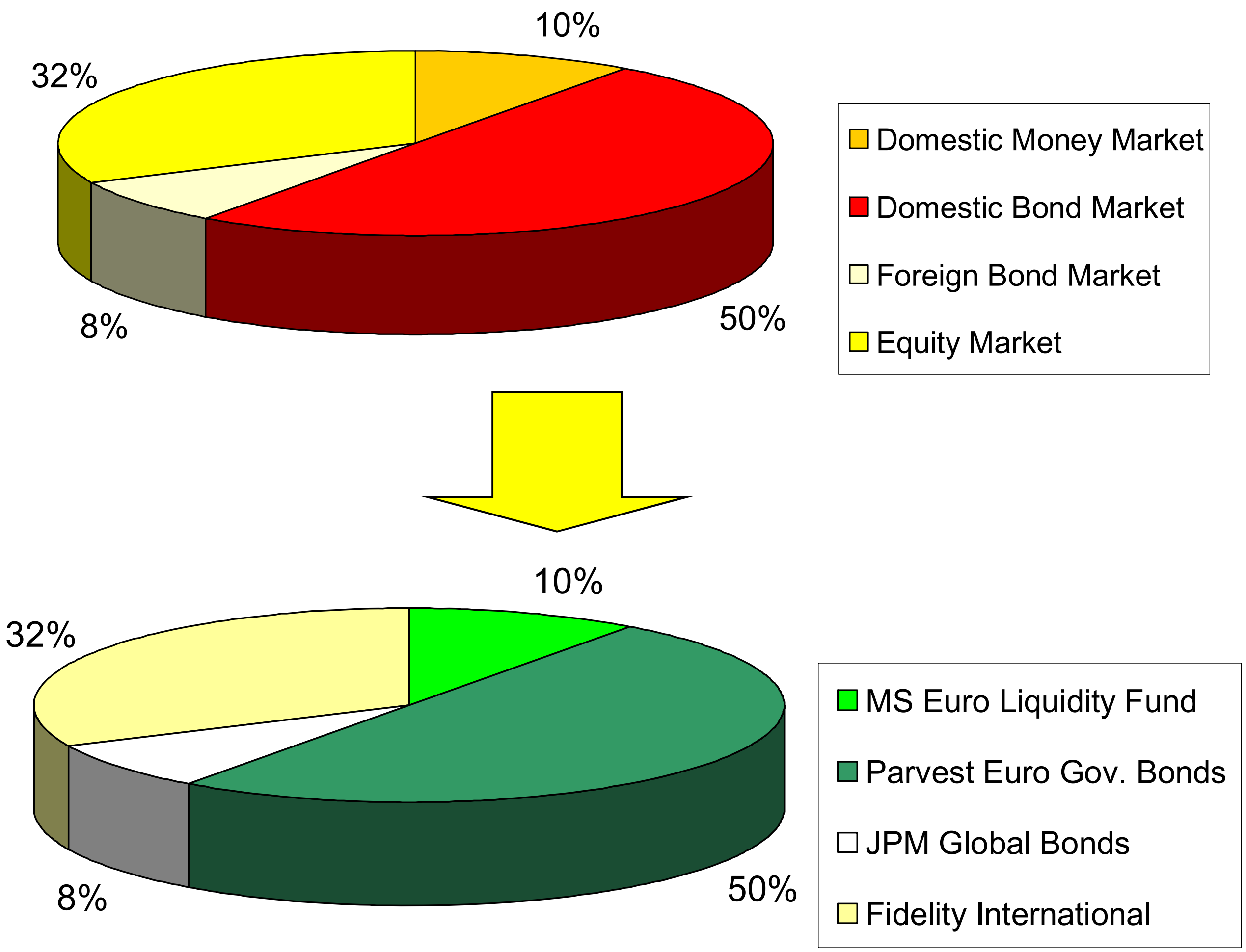
- A French Pension Fund has the following SAA:



- The board of directors does not have the ability of directly selecting the stocks/bonds….
- so the Pension Fund identifies, for every market, the fund managers that are supposed to be the best ones:

Markets (Asset Classes)	**Funds selected**
Domestic Money Market	MS Euro Liquidity Fund
Domestic Bond Market	Parvest Euro Gov. Bonds
Foreign Bond Market	JPM Global Bonds
Equity Market	Fidelity International

Stock-Bond/Fund Selection : Example (2/2)



From Markets....

.....to Products.

Norway's oil fund

https://www.nbim.no/en/

About the fund

Norway's oil fund, or the Government Pension Fund Global which is its official name, was created after we discovered oil in the North Sea. The fund was set up to shield the economy from ups and downs in oil revenue. It also serves as a financial reserve and as a long-term savings plan so that both current and future generations get to benefit from our oil wealth.

In 1969, one of the world's largest offshore oilfields was discovered off Norway. Suddenly we had a lot of oil to sell, and the country's economy grew dramatically. It was decided early on that revenue from oil and gas should be used cautiously in order to avoid imbalances in the economy. In 1990, the Norwegian parliament passed legislation to support this, creating what is now the Government Pension Fund Global, and the first money was deposited in the fund in 1996. As the name suggests, it was decided that the fund should only be invested abroad.

Oil revenue has been very important for Norway, but one day the oil will run out. The aim of the fund is to ensure that we use this money responsibly, think long-term and so safeguard the future of the Norwegian economy.

Market value

Investment returns, capital inflows and exchange rates affect the fund's market value.

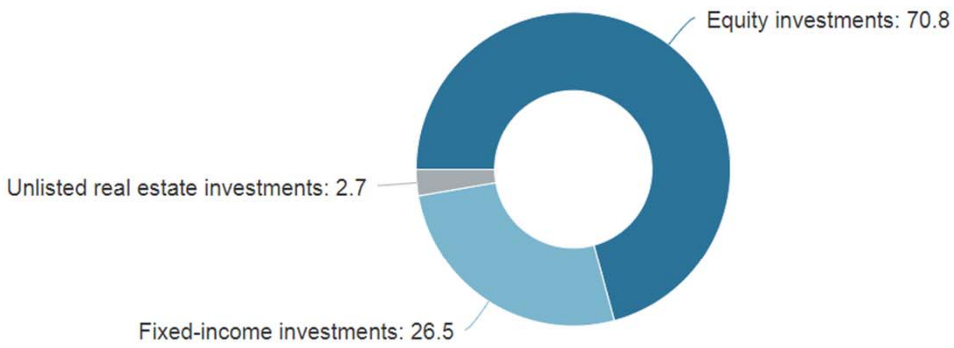
Current market value

10 359 billion NOK

Download to Excel

Asset allocation

At the end of 2019, the fund was invested with 70.8 percent in equities, 2.7 percent in unlisted real estate and 26.5 percent in fixed income.



At the end of 3Q 2019 the fund's equity investments had a market value of 6,729 billion kroner, while the market value of the fixed-income investments was 2,744 billion kroner. The unlisted real estate investments had a market value of 268 billion kroner.

https://www.nbim.no/en/the-fund/holdings/holdings-as-at-31.12.2019/?fullsize=true

As previously seen, a portfolio is first an allocation of markets and is then transformed into an allocation of products.

Moreover, forecasting (strategic and tactical) concerns markets, not products.

For all this, in the portfolio construction it assumes great importance:

Return/Risk of Financial Markets ESTIMATION

From Investor's Preferences to Financial Markets Evaluation

- It is well known that Investors:
  - love return;
  - hate risk (are *risk adverse*).

So, if we want to build a portfolio that best suit the investor's preferences, we need to measure *risk-return* of single Markets and of Portfolios of Markets.

Risk-Return analysis of Financial Markets

The Financial Markets

Analysis of the following Asset Classes.

ASSET CLASSES:

- Money Market EMU
- Bond Market EMU
- Global Bond Market Dev. Mkts
- Corporate Bonds High Yield
- Em Mkts Bonds
- Equity Market Europe
- Equity Market North America
- Equity Market Pacific
- Equity Emerging Markets

MARKET INDEXES: BENCHMARK

- JPM Euro 3 months
- JPM EMU All Mat
- JPM Global
- ML Global HY
- JPM EMBI + Composite
- MSCI Europe
- MSCI North America
- MSCI Pacific
- MSCI Emerging Markets

These are fictious (or artificial) baskets of stocks/bonds which composition is a **good proxy** of the composition of a market.
Thanks to the ability to replicate the composition, the index performance is an excellent proxy for the market performance

Historical series of annual returns

Thanks to the market indexes we have a set of time series returns of financial markets:

Years/Markets	Money Market EMU	Bond Market EMU	Global Bond Dev. Mkts	Bond High Yield	Bond Emerg. Markets	Equ Europe	Equ North America	Equ Pacific	Equ Emerg. Markets
1996	4,83%	7,29%	12,41%	13,87%	33,78%	27,59%	30,93%	2,83%	11,89%
1997	4,42%	6,16%	18,31%	28,76%	19,76%	41,85%	52,36%	-14,90%	1,03%
1998	4,46%	10,94%	6,82%	-3,37%	-5,66%	17,21%	17,75%	-7,77%	-32,85%
1999	3,15%	-2,97%	10,67%	20,26%	39,01%	33,06%	42,14%	78,79%	90,86%
2000	4,32%	8,39%	10,49%	0,57%	17,38%	-2,46%	-5,84%	-18,79%	-26,37%
2001	4,74%	6,25%	4,75%	8,73%	6,93%	-16,83%	-8,77%	-19,61%	0,40%
2002	3,53%	8,49%	0,31%	-16,13%	-4,97%	-32,86%	-35,81%	-24,61%	-22,66%
2003	2,54%	3,77%	-4,92%	8,74%	1,47%	11,92%	6,09%	13,64%	25,87%
2004	2,18%	7,56%	2,09%	4,33%	5,10%	9,28%	1,40%	9,50%	13,54%
2005	2,20%	5,67%	7,93%	16,94%	24,11%	23,01%	21,23%	37,84%	50,45%
2006	3,02%	-0,28%	-5,11%	1,54%	-0,19%	16,65%	1,53%	1,12%	15,72%
2007	4,42%	0,97%	-0,86%	-7,09%	-0,53%	-0,73%	-5,46%	-5,47%	22,10%
2008	5,75%	9,97%	18,47%	-24,13%	-1,30%	-45,21%	-35,73%	-34,31%	-51,85%
2009	2,31%	4,32%	-1,28%	56,93%	14,66%	27,15%	22,35%	21,43%	69,06%
2010	1,11%	1,14%	14,10%	21,80%	17,28%	9,41%	23,71%	21,77%	24,04%
2011	1,59%	2,18%	13,58%	6,06%	5,15%	-12,18%	2,03%	-13,49%	-13,35%
2012	1,19%	11,42%	-0,66%	17,48%	10,50%	12,93%	10,72%	8,90%	12,92%
2013	0,23%	2,15%	-9,07%	2,17%	-10,93%	16,45%	21,51%	10,12%	-9,12%
2014	0,31%	13,50%	18,79%	6,00%	27,90%	13,68%	32,21%	20,40%	22,76%
2015	0,10%	1,71%	8,61%	6,74%	12,89%	5,59%	8,87%	12,00%	-7,39%
2016	-0,15%	3,13%	4,90%	18,53%	13,22%	3,51%	15,98%	7,89%	15,27%
2017	-0,32%	0,41%	-6,10%	-3,22%	-4,82%	10,95%	6,89%	9,83%	21,07%
2018	-0,32%	0,88%	4,05%	1,25%	-0,84%	-10,00%	-0,41%	-7,33%	-9,92%
2019	-0,31%	6,94%	8,06%	15,92%	14,77%	26,98%	34,01%	21,91%	21,17%

(in Euro)

- **Investors aim to maximise the return** (the final value) of their investments

Average Return (1/2)

Investors love financial markets with higher average returns:

$$\overline{R} = \frac{\left(R_1 + R_2 + \ldots.. + R_{T-1} + R_T\right)}{T}$$

$$\overline{R} = \frac{\sum_{t=1}^{T} R_t}{T}$$

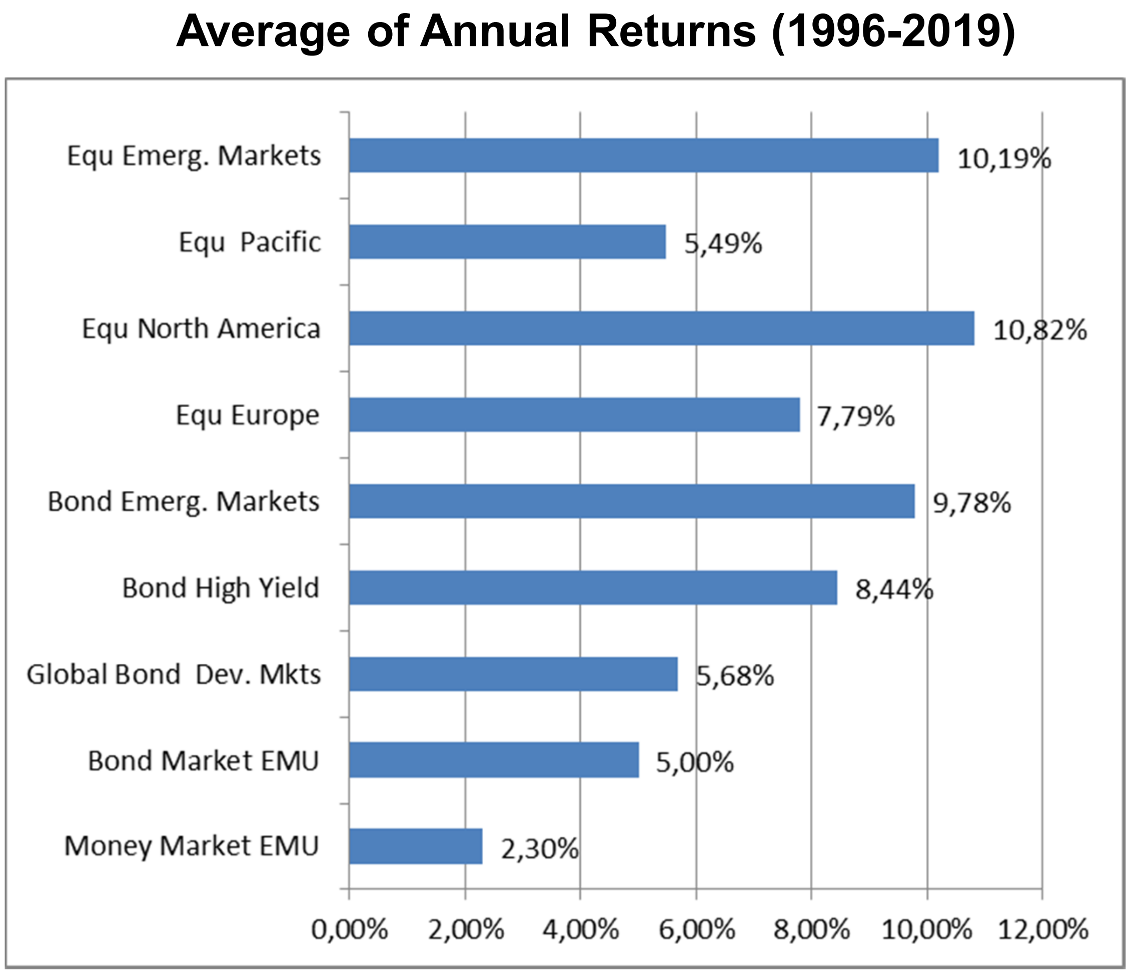
Excel:

=average(Time Series) EN

= media(Time Series) IT

1996-2019	Money Market EMU	Bond Market EMU	Global Bond Dev. Mkts	Bond High Yield	Bond Emerg. Markets	Equ Europe	Equ North America	Equ Pacific	Equ Emerg. Markets
Annual Average Return	2,30%	5,00%	5,68%	8,44%	9,78%	7,79%	10,82%	5,49%	10,19%

Average of Annual Returns (2/2)



Average Return of a Portfolio (1/3)

If we known:

- the Portfolio Weight of each market (w_i);

- the Average Returns of each market ($\overline{R}_i$) i

The estimation of the Portfolio Average Return is straightforward:

$$\overline{R}_{Port} = \sum_{i=1}^{k} w_i \times \overline{R}_i$$

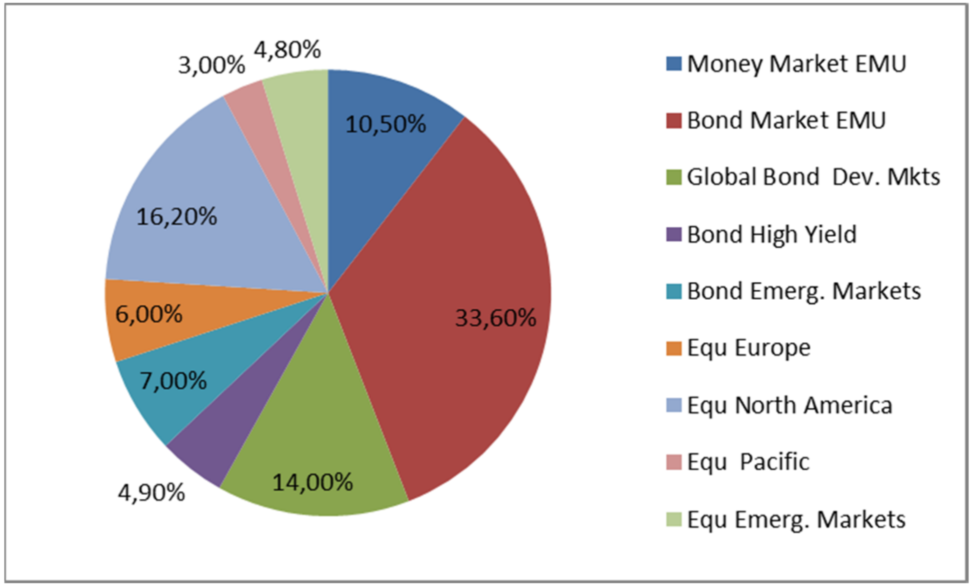
Excel:

=sumproduct(Weights, Average Returns) **EN**

=matr.somma.prodotto(Weights, Average Returns) **IT**

Average Return of a Portfolio (3/3)

	Money Market EMU	Bond Market EMU	Global Bond Dev. Mkts	Bond High Yield	Bond Emerg. Markets	Equ Europe	Equ North America	Equ Pacific	Equ Emerg. Markets
Average Annual Return	2,30%	5,00%	5,68%	8,44%	9,78%	7,79%	10,82%	5,49%	10,19%
Weights	10,50%	33,60%	14,00%	4,90%	7,00%	6,00%	16,20%	3,00%	4,80%



$$\overline{R}_{Port} = \sum_{i=1}^{k} w_i \times \overline{R}_i = 6.69\%$$

Investors want to maximise the average (or expected) return of the portfolio.

- **Investors are "risk adverse"**: given a targeted return they try to minimise the risk.

What is “risk”? (1/2)

The financial literature has formulated many mathematical & statistical indicators useful in order to “capture” risk.

Examples:

- Standard Deviation;
- Semi- Standard Deviation;
- Downside risk;
- Beta;
- Duration/Modified Duration;
- Rating
- Value at Risk (VaR);
- Shortfall probability;
- Tracking Error Volatility (TEV).

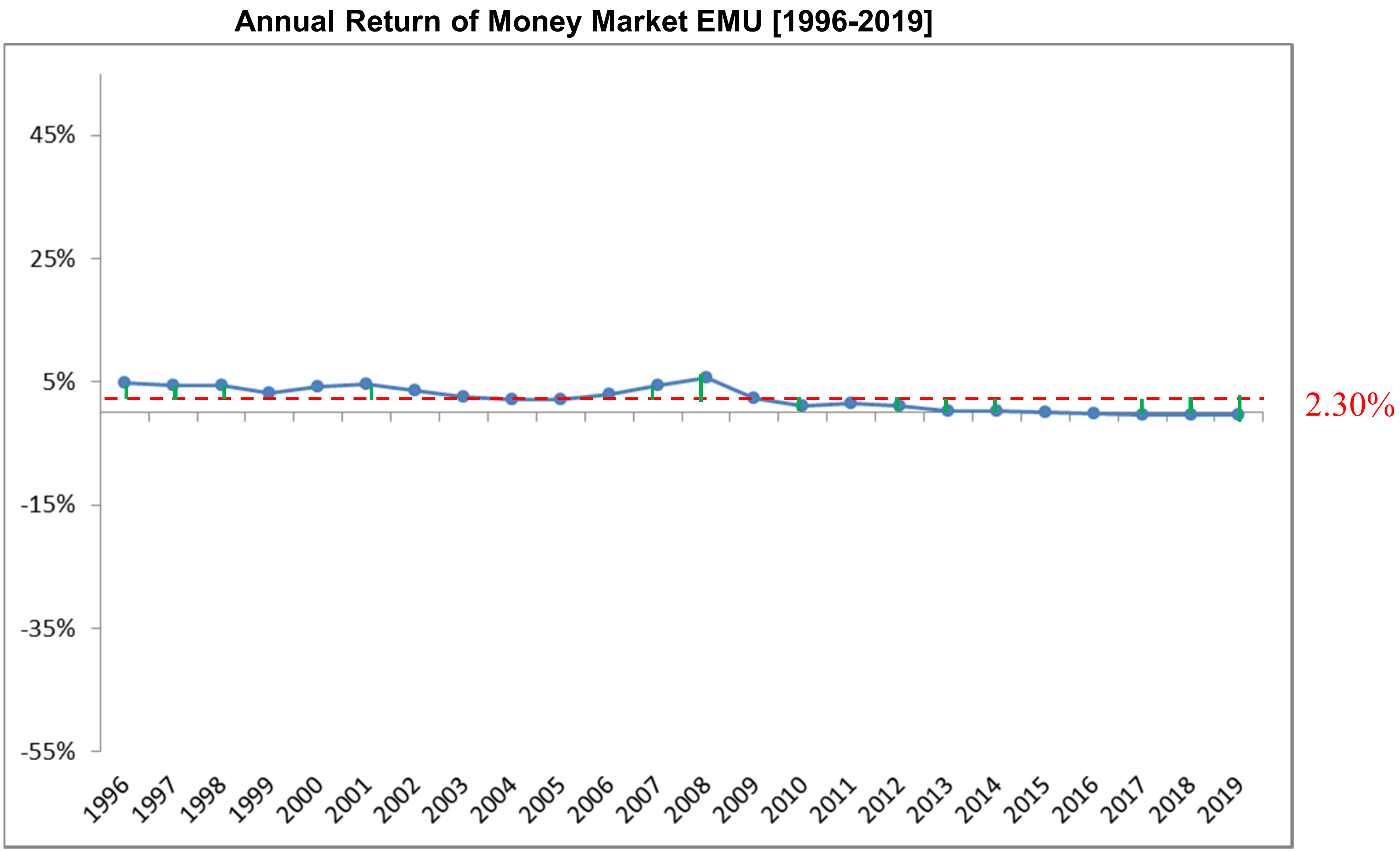
Many indicators…….maybe a variable difficult to measure.

What is “risk”? (2/2)

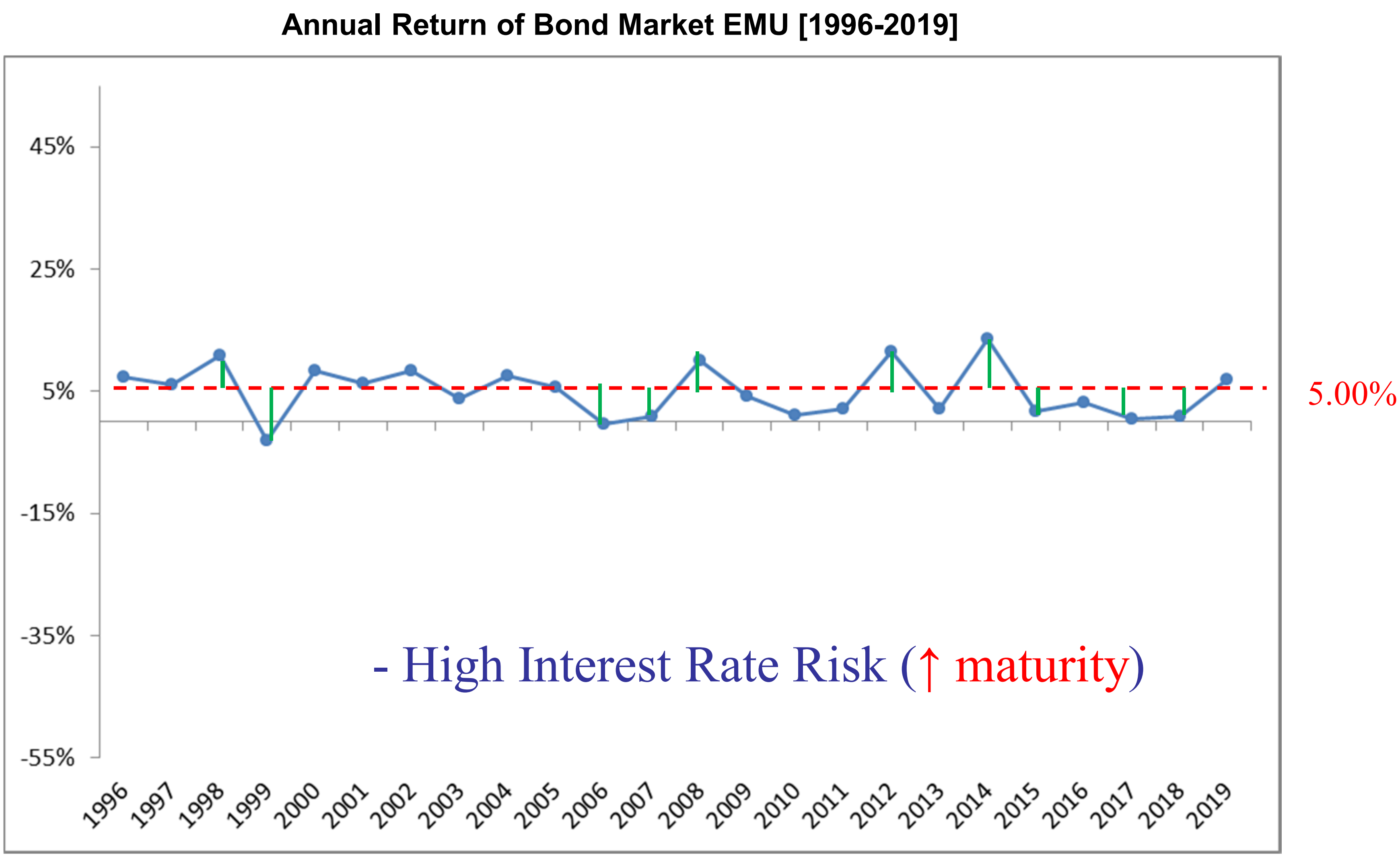
- Commonly in the financial environment risk is interpreted as the “uncertainty of returns”;
- So markets with volatile, unstable returns are considered risky.

Graphical evidences

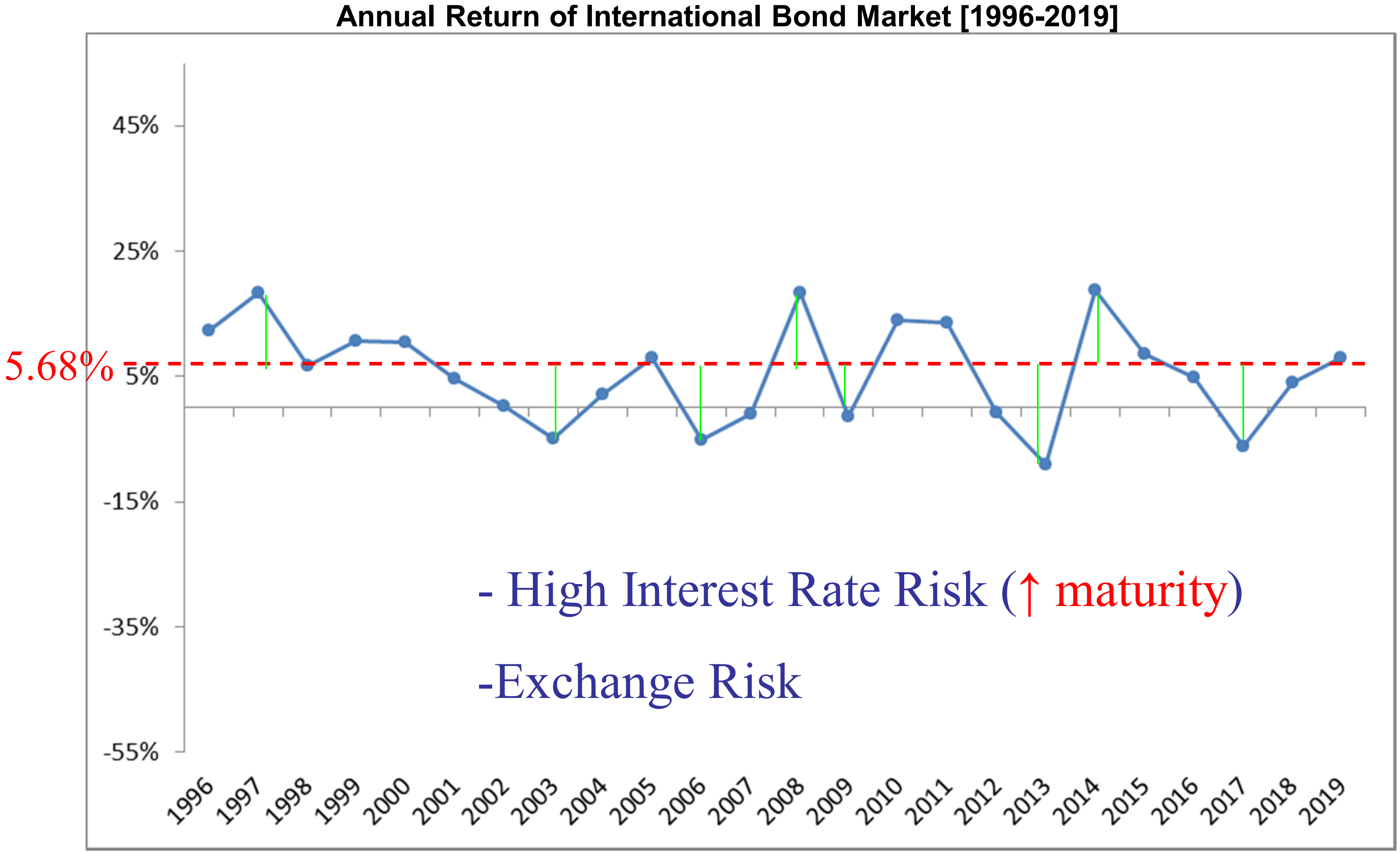
Very Low volatility: Money Market EMU



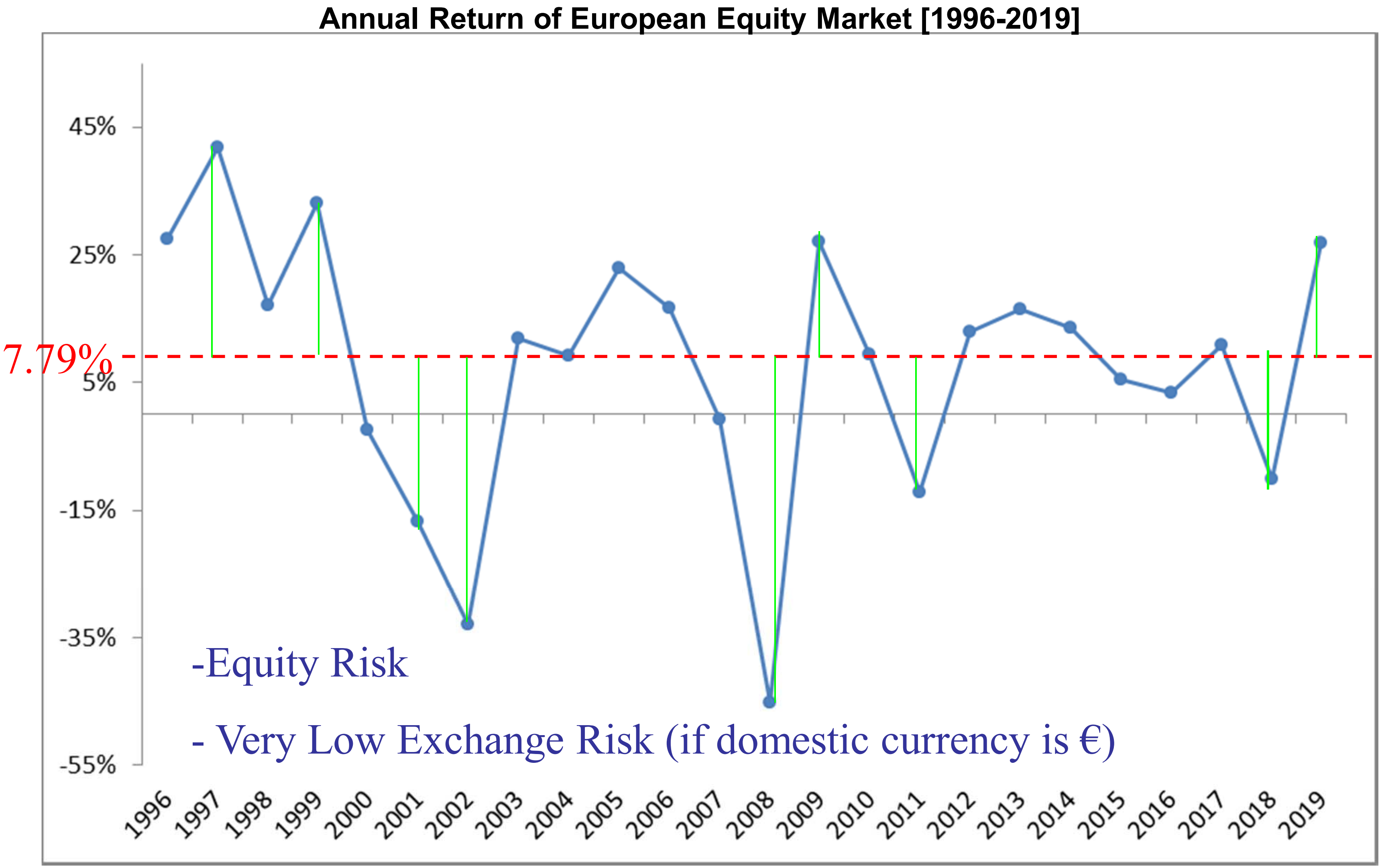
Low volatility: Bond Market EMU



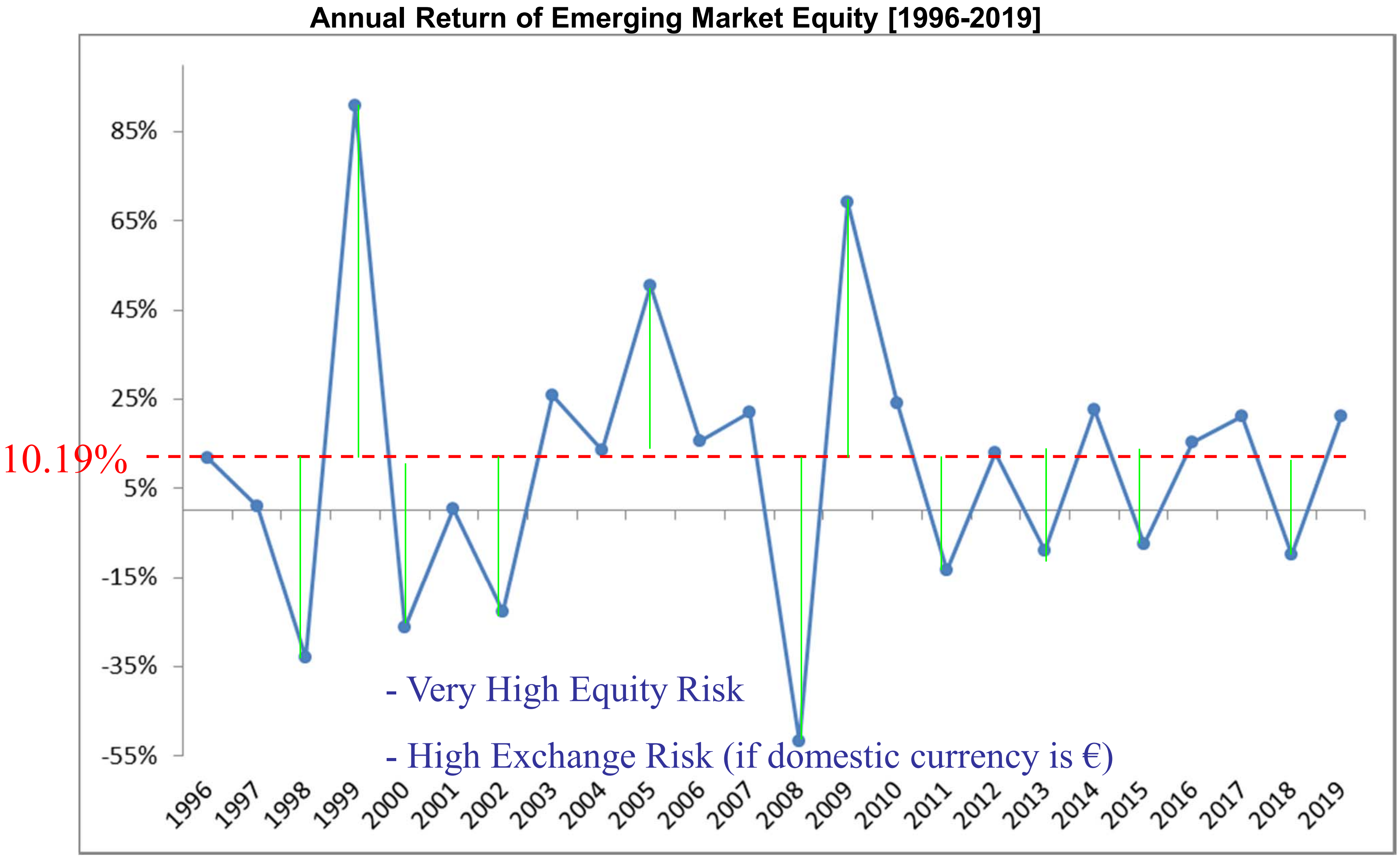
Average volatility: International Bond Market



High volatility: European Equity Market



Very High volatility: Emerg. Mkts Equity



Standard Deviation of Returns

- We need a statistical indicator able to synthesise the volatility.
- The most common parameter is the:

Standard deviation (σ)

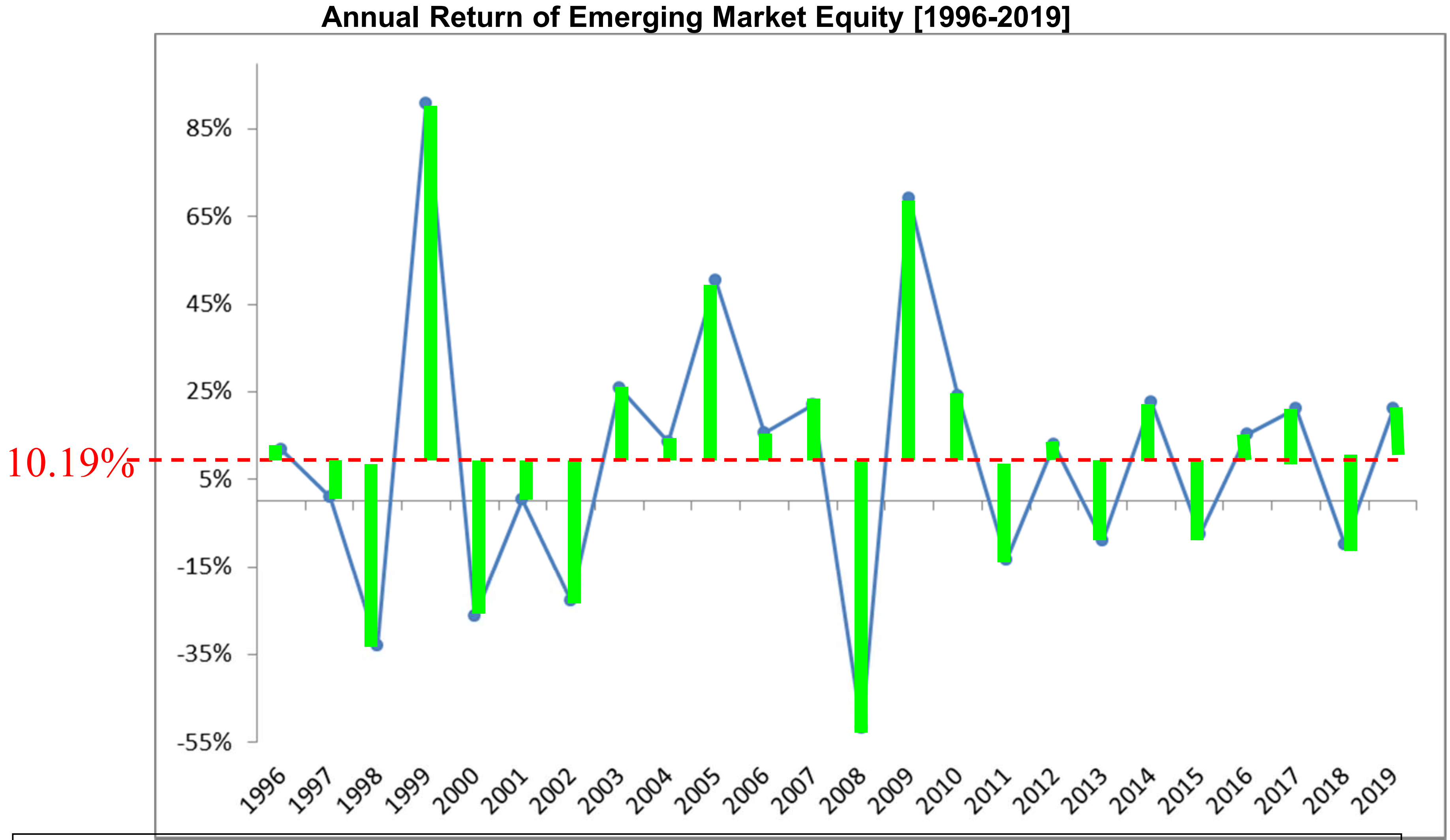
$$\sigma = \sqrt{\frac{\sum_{i=1}^{T}\left(R_i - \bar{R}\right)^2}{T-1}}$$

Excel:

=stdev(time series) **EN**

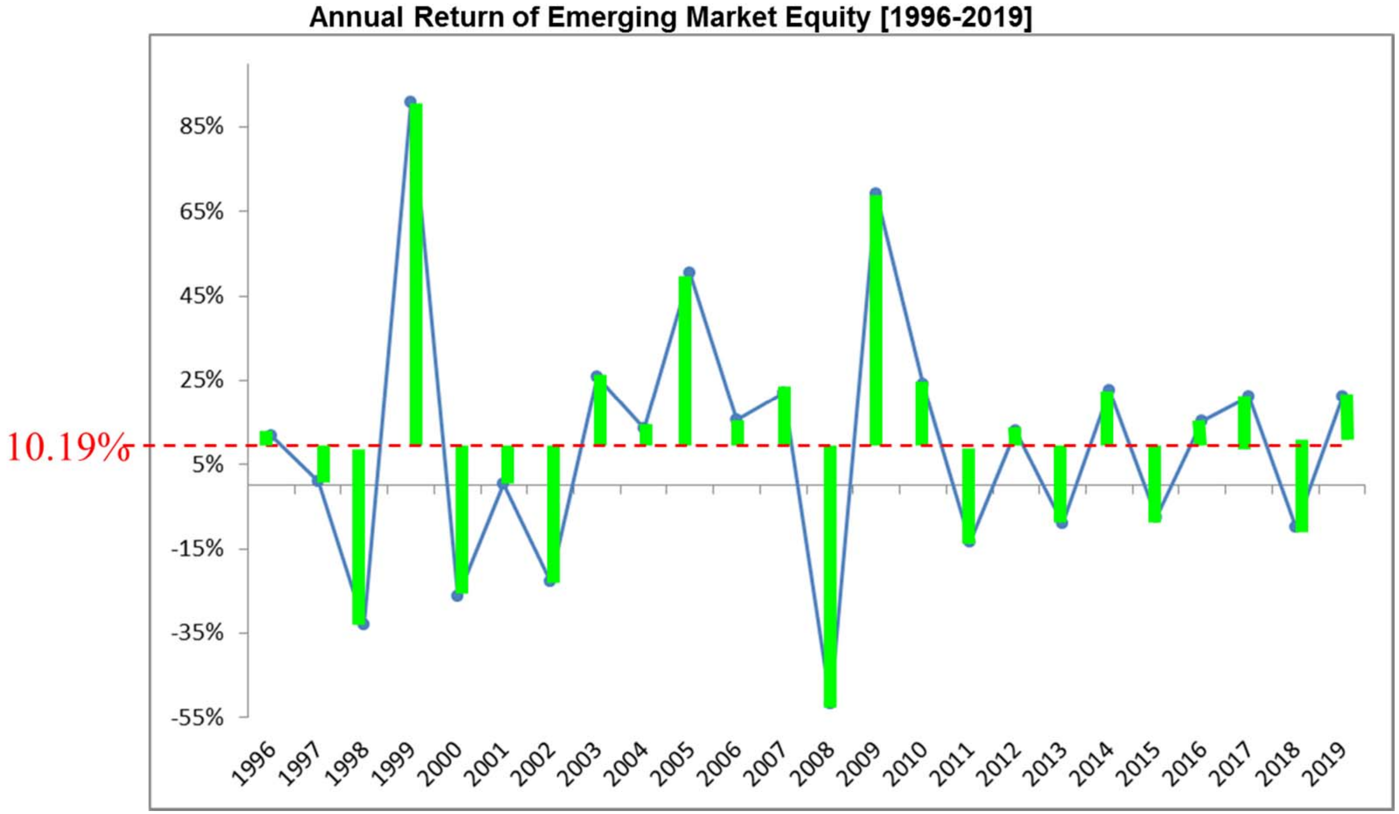
=dev.st(time series) **IT**

σ: an easy interpretation (1/2)



Standard deviation can be seen as the average of "*gaps*" between the average return and annual returns.

σ: an easy interpretation (2/2)

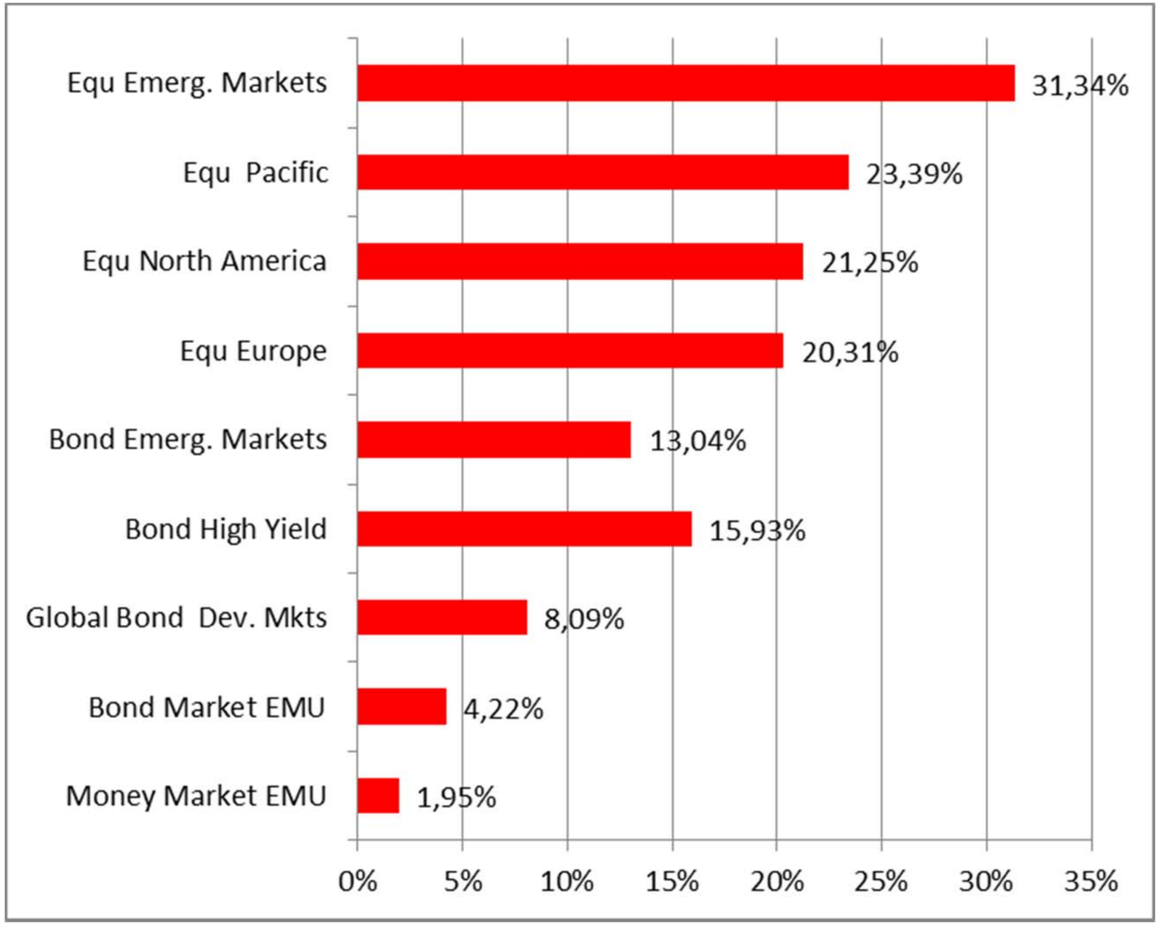


- The standard deviation of MSCI Em Mkts annual return is 31.34%;
- We can say that the annual return is likely to have an average deviation from the average return of 31.34% (average length of green bars is 31.34%).

Standard Deviations of Asset Classes

(1996-2019)	Money Market EMU	Bond Market EMU	Global Bond Dev. Mkts	Bond High Yield	Bond Emerg. Markets	Equ Europe	Equ North America	Equ Pacific	Equ Emerg. Markets
Standard Dev	1,95%	4,22%	8,09%	15,93%	13,04%	20,31%	21,25%	23,39%	31,34%

Standard Deviation of Annual Returns (1996-2019)



Standard deviation of a Portfolio: NOT a weighted average (1/2)

If we known:

- the Portfolio Weight of each market (w_i)

- the standard deviations of each market (σ_i)

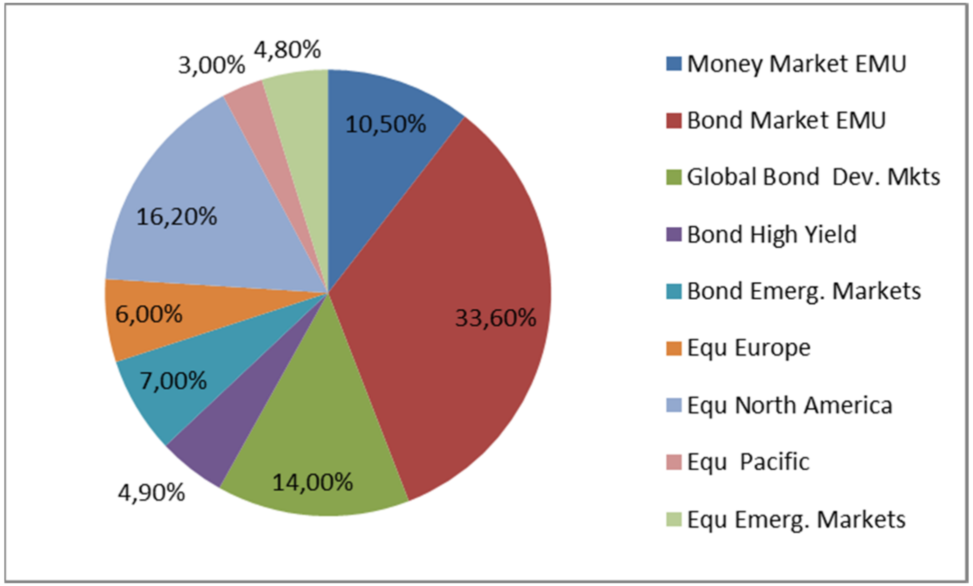
The estimation of the Portfolio standard deviation is NOT the following:

$$\cancel{\sigma_{Port} = \sum_{i=1}^{k} w_i \times \sigma_i}$$

That is, The portfolio standard deviation is NOT the weighted average of the standard deviation of the markets.

Standard deviation of a Portfolio: NOT a weighted average (2/2)

	Money Market EMU	Bond Market EMU	Global Bond Dev. Mkts	Bond High Yield	Bond Emerg. Markets	Equ Europe	Equ North America	Equ Pacific	Equ Emerg. Markets
Weights	10,50%	33,60%	14,00%	4,90%	7,00%	6,00%	16,20%	3,00%	4,80%
Standard Dev	1,95%	4,22%	8,09%	15,93%	13,04%	20,31%	21,25%	23,39%	31,34%



$$\cancel{\sigma_{Port} = \sum_{i=1}^{k} w_i \times \sigma_i = 11.31\%}$$

NOT a weighted average: 1st empirical evidence

Using time series of MSCI market indices, we measure the standard deviation of the following equity market sectors:

- MSCI Europe Pharmaceutical, σ_{Pharm} =12,54%;

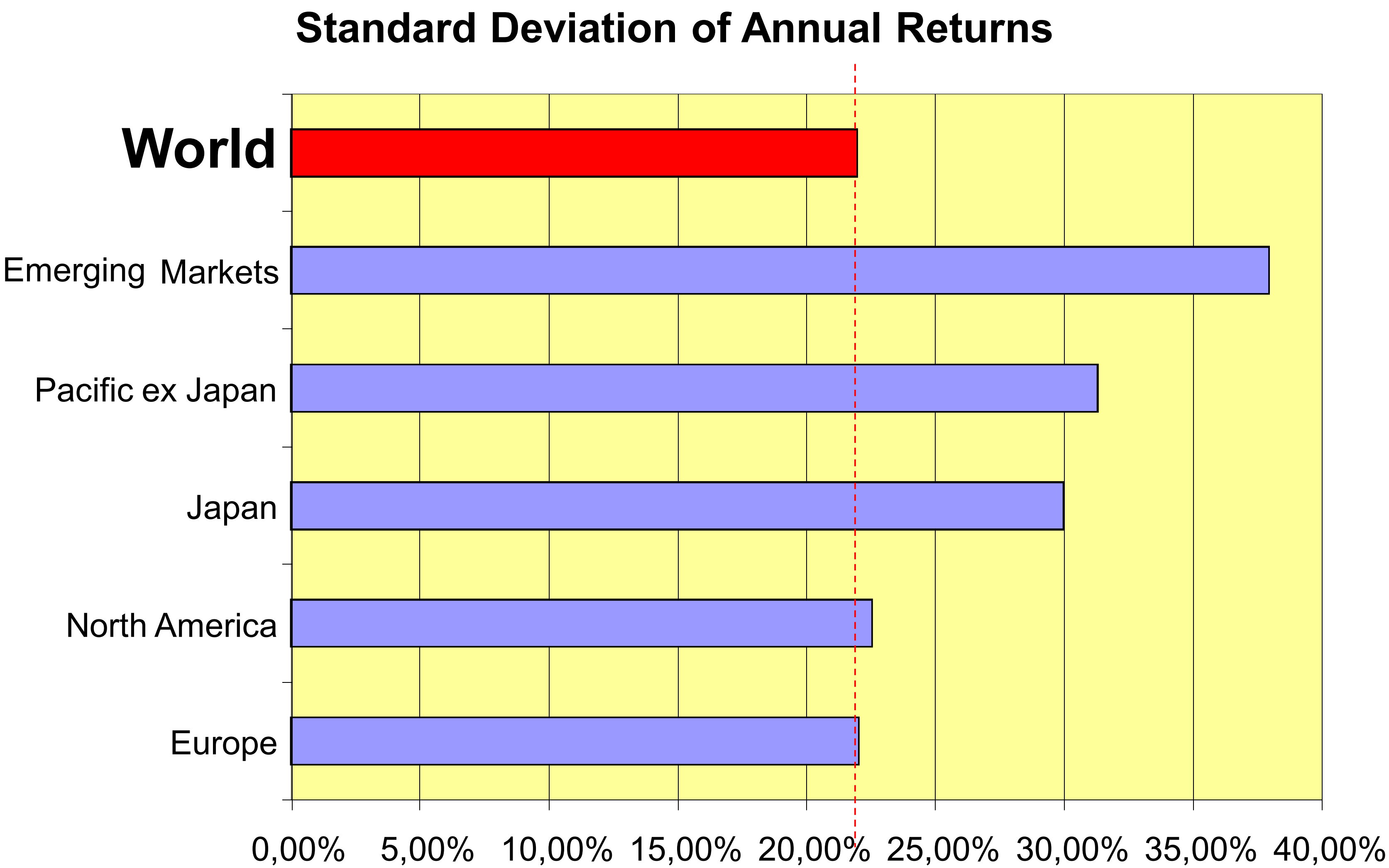
- MSCI Europe Biotechnology, σ_{Biotech}= 30,32%;

Which is the standard deviation of the MSCI Europe Pharma/Biotech?

$$\sigma_{Pharma/Biotech} = 12.44\%$$

It can't be the weighted average!

NOT a weighted average: 2nd empirical evidence



The world equity market had a standard deviation of returns that is lower than the standard deviation of all the regional markets. Again, risk can't be the weighted average.

The diversification effect

- Since 1952 is well known that it is possible to reduce risk avoiding concentration.
- Proverb: *"Don't put your eggs in the same basket"*

- Financial history shows that markets have the tendency not to move exactly in the same way:
  - year 1998: MSCI Europe (+17.21%) vs MSCI EM (-32.85%)
  - year 2011: MSCI North America (+2.03%) vs MSCI Pacific (-13.49%)

Thanks to diversification, the portfolio standard deviation is lower than the weighted average.

Overestimation

We need to "Capture" the diversification effect

- In order to measure the diversification effect (that is, the power of diversification in reducing risk) we must measure:

The Correlation (ρ)

$$\rho_{Port} = \frac{Cov(R_A; R_B)}{\sigma_A \times \sigma_B}$$

Correlation (ρ): characteristics (1/2)

- The correlation is calculated for a couple of markets;
- $-1 \leq \rho \leq +1$ $\rho \in [-1;+1]$
- If $\rho > 0$, *(in the majority of cases)* markets move in the same direction *(both gain or both lose)*
- If $\rho = +1$, markets *systematically* move in the same direction
- If $\rho < 0$, *(in the majority of cases)* markets move in opposite direction *(one gains, the other loses)*
- If $\rho = -1$, markets perfectly move in opposite direction *(they systematically move synchronised, but in opposite direction)*
- If $\rho = 0$, markets are independent (*no tendency to move in the same or in the opposite direction*) *(follows)*

Correlation (ρ): characteristics (2/2)

- *If ρ =+1, no* diversification
- *If ρ* <+1, yes diversification
- The lower the correlation, the higher the diversification (the risk reduction)

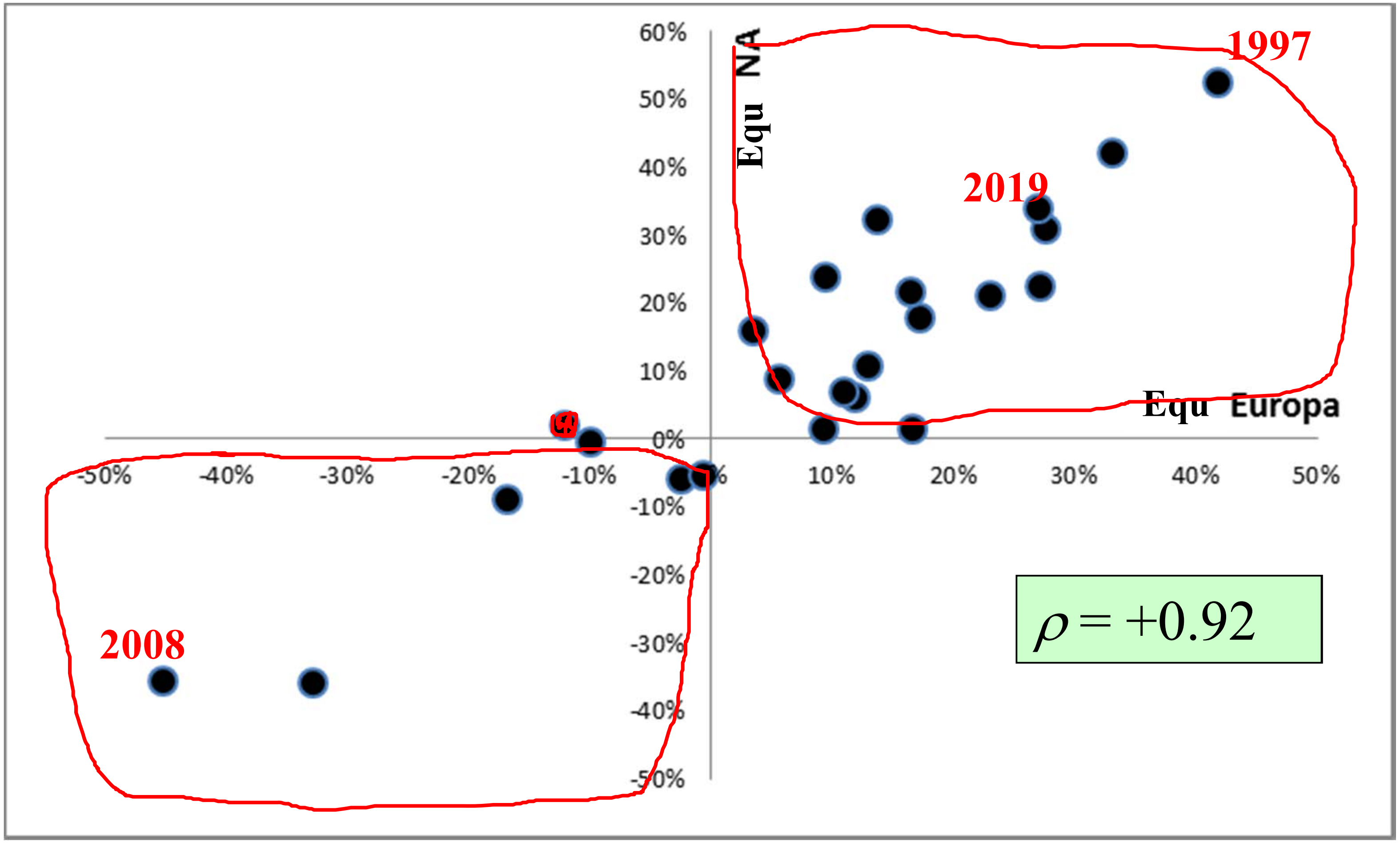
Excel:

=correl(time series mkt1, time series mkt2) **EN**

=correlazione(time series mkt1, time series mkt2) **IT**

Correlation: the scatter graph

Case 1: Positive correlation

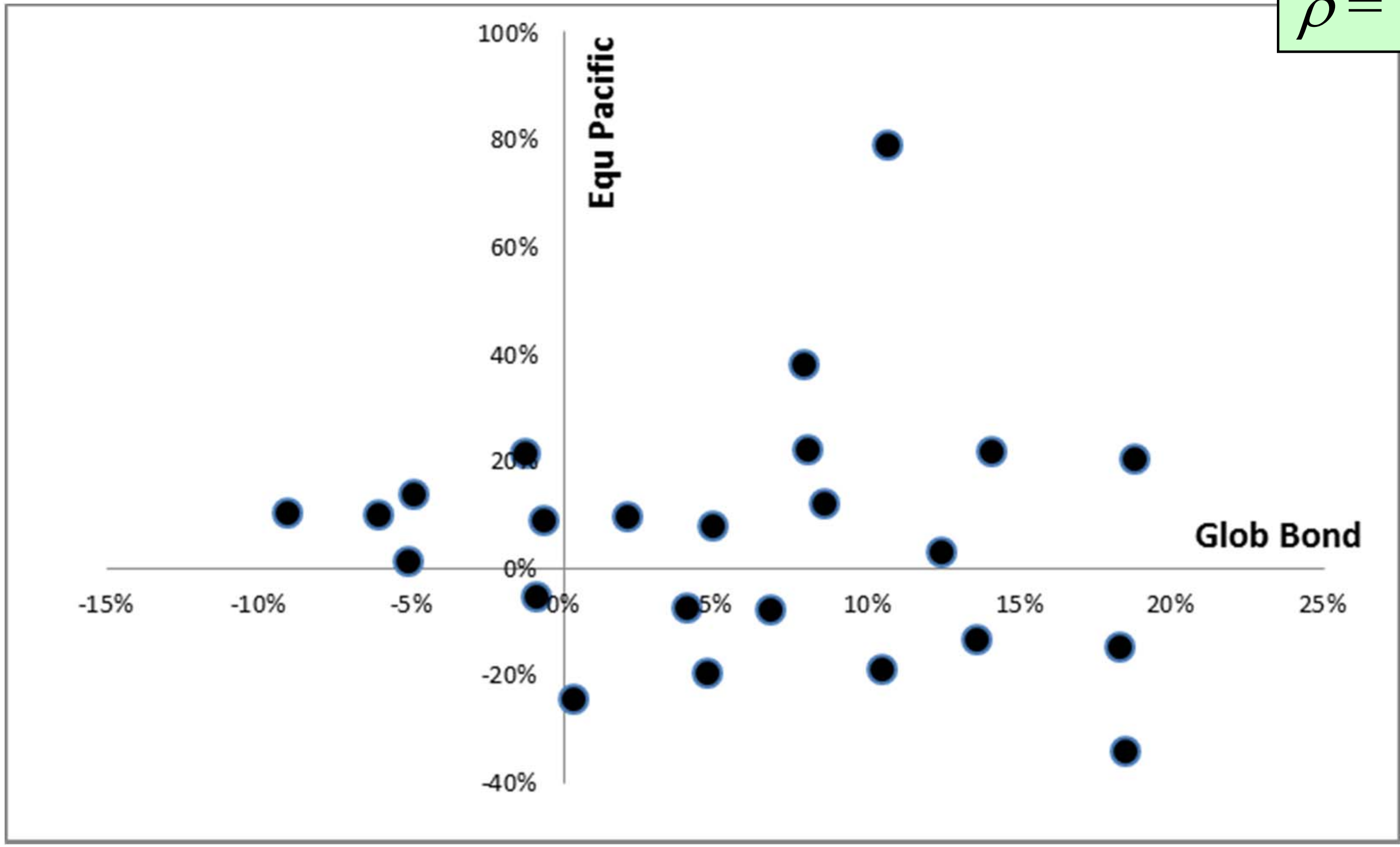


Strong tendency to move in the same direction (23 times on 24)

Correlation: the scatter graph

Case 2: Zero correlation

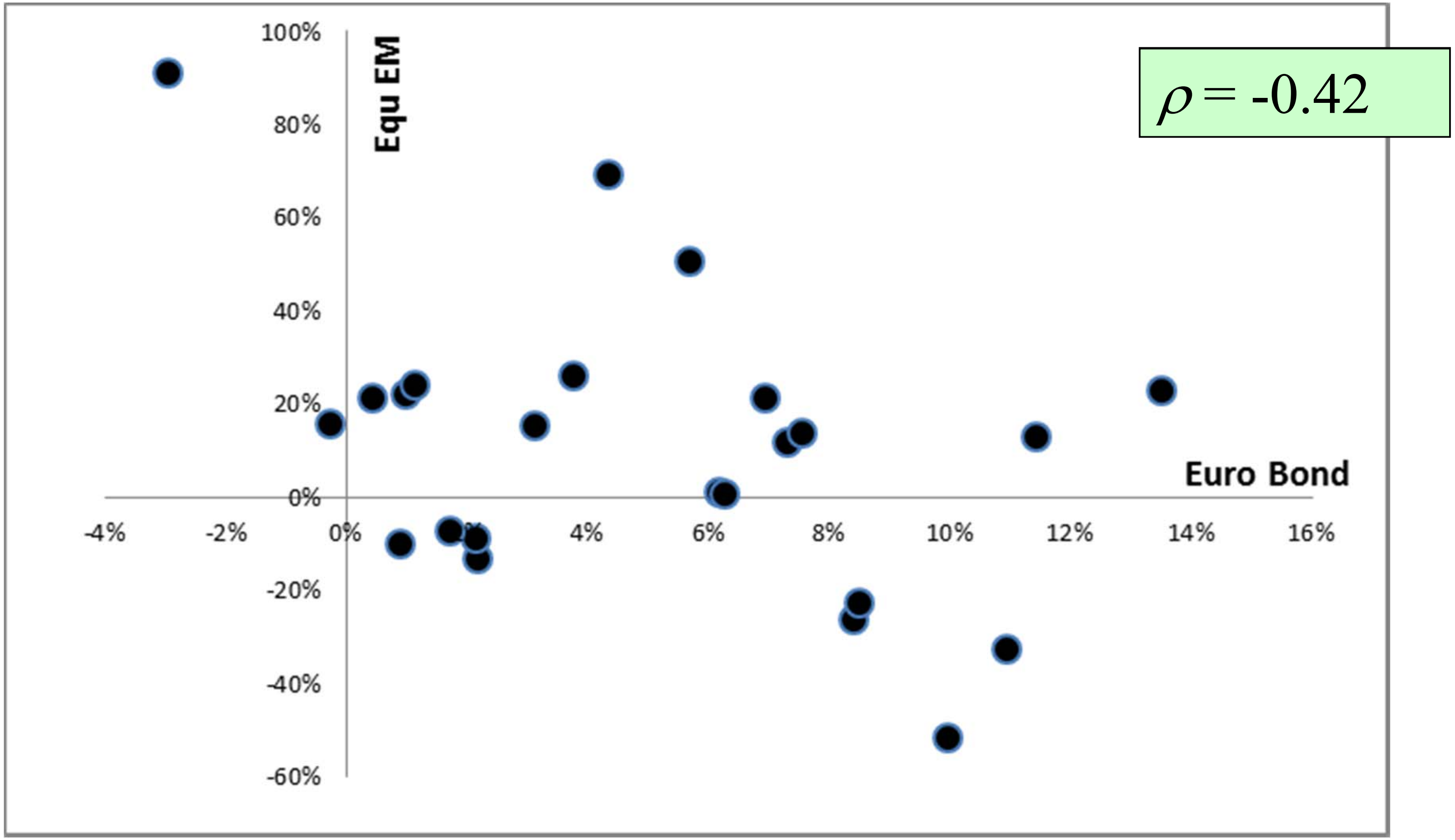
$\rho = -0.06$



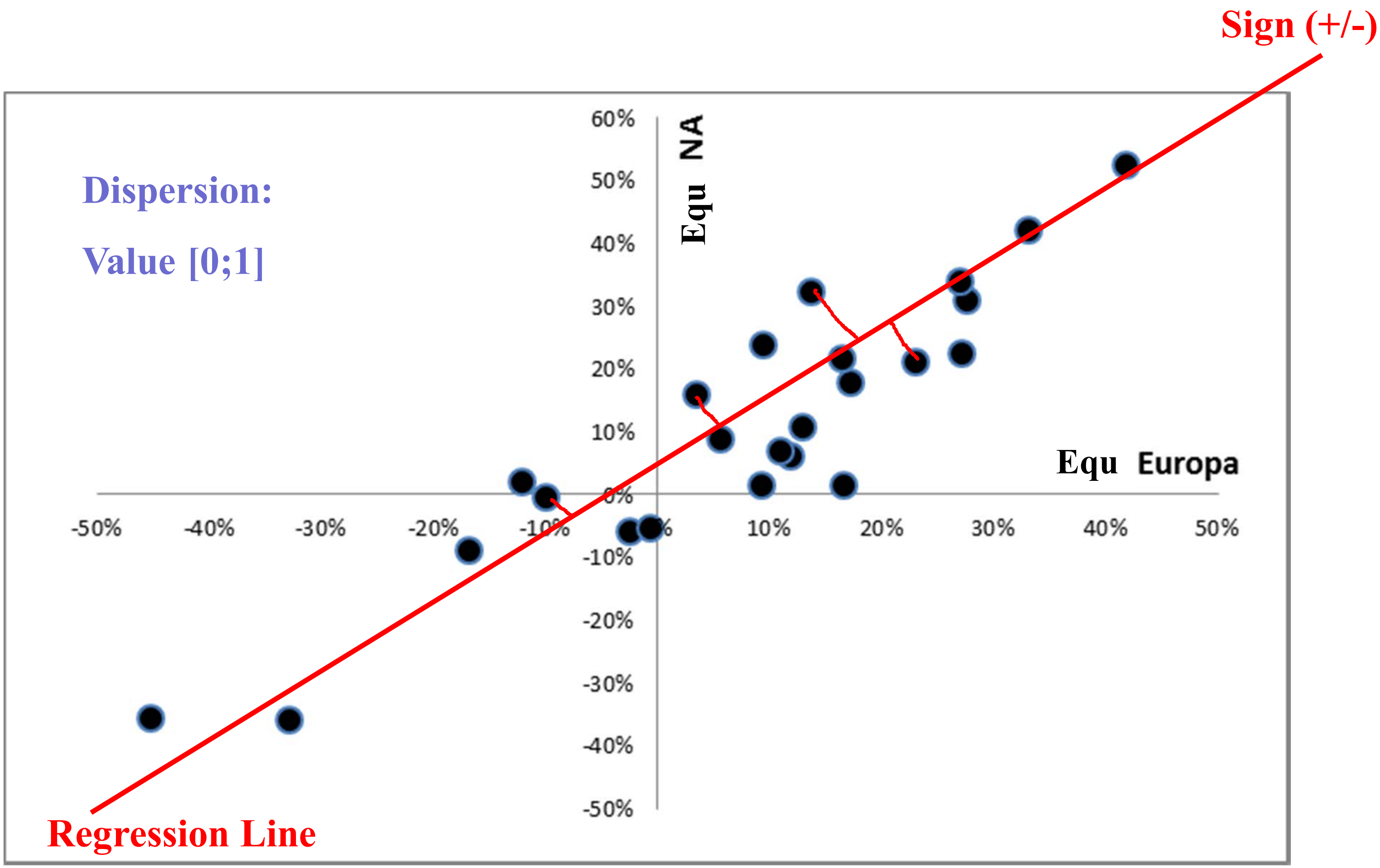
No strong tendency (15 times in the same direction - 9 times in the opposite direction)

Correlation: the scatter graph

Case 3: Negative correlation



Correlation: graphic interpretation



Correlation Matrix

➢ The Correlation Matrix shows the correlations between all the couples of markets:

$$Num^P = \frac{N \times (N-1)}{2} = \frac{9 \times 8}{2} = 36$$

(1996-2019)

Correlations	JPM Euro 3 months	JPM EMU All Mat	JPM Global	ML Global HY	JPM EMBI + Composite	MSCI Europe	MSCI North America	MSCI Pacific	MSCI Emerging Markets
JPM Euro 3 months	1								
JPM EMU All Mat	0,27	1							
JPM Global	0,25	0,34	1						
ML Global HY	-0,21	-0,16	0,05	1					
JPM EMBI + Composite	0,05	0,05	0,60	0,55	1				
MSCI Europe	-0,20	-0,17	-0,04	0,69	0,51	1			
MSCI North America	-0,28	-0,13	0,23	0,70	0,63	0,92	1		
MSCI Pacific	-0,40	-0,38	-0,06	0,52	0,58	0,66	0,64	1	
MSCI Emerging Markets	-0,23	-0,42	-0,16	0,69	0,57	0,64	0,57	0,87	1

In Excel: Data Analysis/Analysis Toolpack (EN) - Analisi Dati (IT)

The "*Gift*" of globalization

- As showed by the correlation matrix, globalization has strongly increased the correlation amoung equity markets.

	MSCI Europe	MSCI North America	MSCI Pacific	MSCI Emerging Markets
MSCI Europe	1			
MSCI North America	0,92	1		
MSCI Pacific	0,66	0,64	1	
MSCI Emerging Markets	0,64	0,57	0,87	1

- Today traditional risky assets are not able to produce big benefits of diversification.

This is the main reason why many institutional investors suggest not to limit the investment to the classical asset classes (bonds and listed stocks)……

They suggest to invest money also in "alternative investments":

- Hedge funds;
- Commodities;
- Private Equity;
- Real Estate.

Pay attention: They do not show negative correlation
And some are illiquid

Standard Deviation of a Portfolio (1/2)

If we known:

- the portfolio weight of each market (w_i)
- the standard deviations of each market (σ_i)
- the correlations between couples of markets ($\rho_{i,j}$)

We can estimate the standard deviation of a portfolio:

$$\sigma_{Port} = \sqrt{\sum_{i=1}^{k} \sum_{j=1}^{k} w_i \cdot w_j \cdot \sigma_i \cdot \sigma_j \cdot \rho_{i,j}}$$

In case of "two markets" portfolio:

$$\sigma_{port} = \sqrt{(w_1 \cdot \sigma_1)^2 + (w_2 \cdot \sigma_2)^2 + 2 \cdot w_1 \cdot w_2 \cdot \sigma_1 \cdot \sigma_2 \cdot \rho_{12}}$$

Standard deviation of a Portfolio: Numerical example

	Money Market EMU	Bond Market EMU	Global Bond Dev. Mkts	Bond High Yield	Bond Emerg. Markets	Equ Europe	Equ North America	Equ Pacific	Equ Emerg. Markets
Weights	10,50%	33,60%	14,00%	4,90%	7,00%	6,00%	16,20%	3,00%	4,80%
Standard Dev	1,95%	4,22%	8,09%	15,93%	13,04%	20,31%	21,25%	23,39%	31,34%

Correlations	JPM Euro 3 months	JPM EMU All Mat	JPM Global	ML Global HY	JPM EMBI + Composite	MSCI Europe	MSCI North America	MSCI Pacific	MSCI Emerging Markets
JPM Euro 3 months	1								
JPM EMU All Mat	0,27	1							
JPM Global	0,25	0,34	1						
ML Global HY	-0,21	-0,16	0,05	1					
JPM EMBI + Composite	0,05	0,05	0,60	0,55	1				
MSCI Europe	-0,20	-0,17	-0,04	0,69	0,51	1			
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MSCI Pacific	-0,40	-0,38	-0,06	0,52	0,58	0,66	0,64	1	
MSCI Emerging Markets	-0,23	-0,42	-0,16	0,69	0,57	0,64	0,57	0,87	1

$$\sigma_{Port} = \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{k} w_i \cdot w_j \cdot \sigma_i \cdot \sigma_j \cdot \rho_{i,j}} = 7.53\%$$

$$\cancel{\sigma_{Port} = \sum_{i=1}^{k} w_i \times \sigma_i = 11.31\%}$$

Standard deviation:
It is not the only measure of risk

If you continue your studies in “Finance”, you can learn that there are many other measures used to estimate the risk of financial markets.
Some of these parameters focus on risk measured as potential loss, instead of volatility.

Value at Risk (VaR)

In the following analysis we make the assumption that we are an Asset Management Committee involved in a Strategic Asset Allocation process.

Our objective is to identify a good model to build a SAA.

Focus on SAA

Strategic Asset Allocation: Naïve Portfolio Formation Rule

This first solution follows a qualitative approach

Naïve portfolios refuses mathematical solutions.

Naïve Portfolio Formation Rule: A primitive approach

Naïve strategies:

- are mathematics/statistics free;
- don't need optimization models;
- don't need numerical-quantitative estimations; estimations can be qualitative-judgemental (European Equity Market will beat the Japanese Equity Market).

Naïve strategies:

- are easy to put into practice;
- can generate good solution, never optimal ones;
- generate portfolios that are usually diversified and reasonable.

Naïve Portfolio Formation Rule: Example (7/7)

The final naïve portfolio:

- is diversified;
- has a reasonable composition.

……but at its best:

- it is a good solution….
- it is not the optimal one.

If you want more, you need

MODERN PORTFOLIO THEORY (MPT)

Focus on SAA

Strategic Asset Allocation: A Quantitative Approach

Quantitative Approach: The Markowitz Model

Harry Markowitz's *Portfolio selection* is the "father" of portfolio optimization……

…..and his model (even if it is 50 years old) is widely used in portfolio construction.

No doubt, there are other mathematical approach. But no one has the Markowitz's model aptitude to be:

- rigorous from a mathematical point of view;
- easy to be implemented.

Markowitz's gift in his Phd Thesis

- The first to **introduce risk** in portfolio construction.
- The first to guess that the **standard deviation** could be used as a risk measure.
- He was the first to talk about the **correlation** between investments and to quantify di **diversification benefits.** FATHER of DIVERSIF
- The first to show that a portfolio can be the output of an **optimization process**

Which was the reaction of the thesis commission (in 1954)?

Markowitz's gift in his Phd Thesis

Markowitz's "Portfolio Selection": A Fifty-Year Retrospective

Mark Rubinstein

The Journal of Finance, Vol. 57, No. 3. (Jun., 2002), pp. 1041-1045.

Near the end of his reign in 14 AD, the Roman emperor Augustus could boast that he had found Rome a city of brick and left it a city of marble. Markowitz can boast that he found the field of finance awash in the imprecision of English and left it with the scientific precision and insight made possible only by mathematics.

But which was the <u>reaction</u> of the thesis commission (in 1952)?

FOUNDATIONS OF PORTFOLIO THEORY

Nobel Lecture, December 7, 1990

by

HARRY M. MARKOWITZ

Finally, I would like to add a comment concerning portfolio theory as a part of the microeconomics of action under uncertainty. It has not always been considered so. For example, when I defended my dissertation as a student in the Economics Department of the University of Chicago, Professor Milton Friedman argued that portfolio theory was not Economics, and that they could not award me a Ph.D. degree in Economics for a dissertation which was not in Economics. I assume that he was only half serious, since they did award me the degree without long debate. As to the merits of his arguments, at this point I am quite willing to concede: at the time I defended my dissertation, portfolio theory was not part of Economics. But now it is.

The Markowitz Model: The hypothesises

Given a unique time horizon……

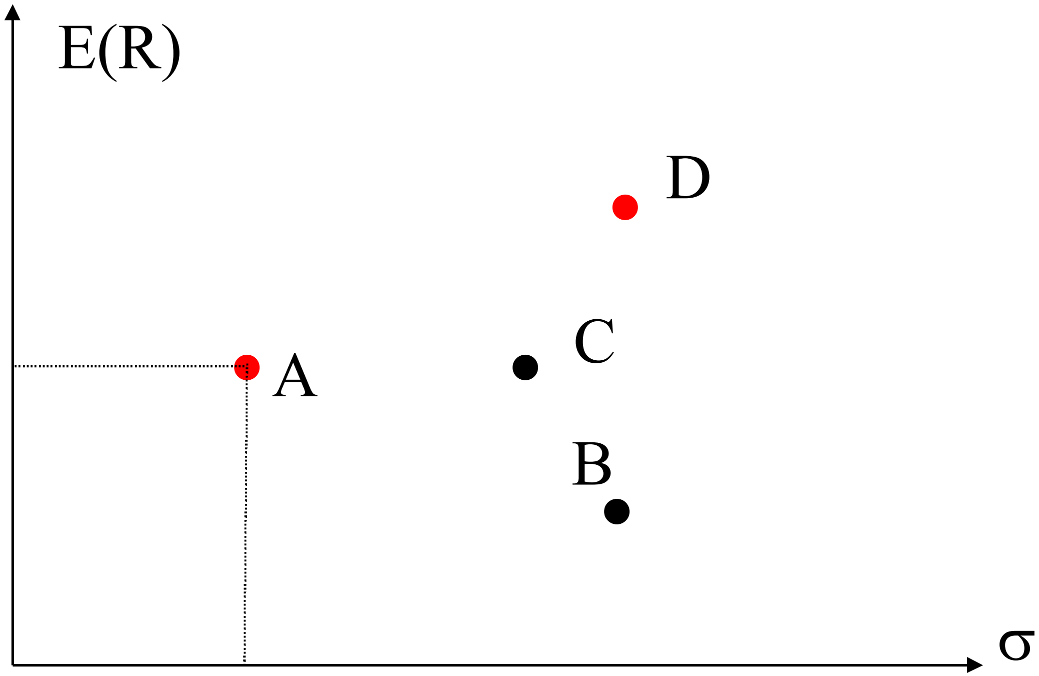
…..investors want to maximise the expected return (*"They love returns"*).

Investors are risk adverse (*"They hate risk"*)

The statistical parameter used to measure risk is the standard deviation.

The "Expected Return – Standard Deviation" Principle

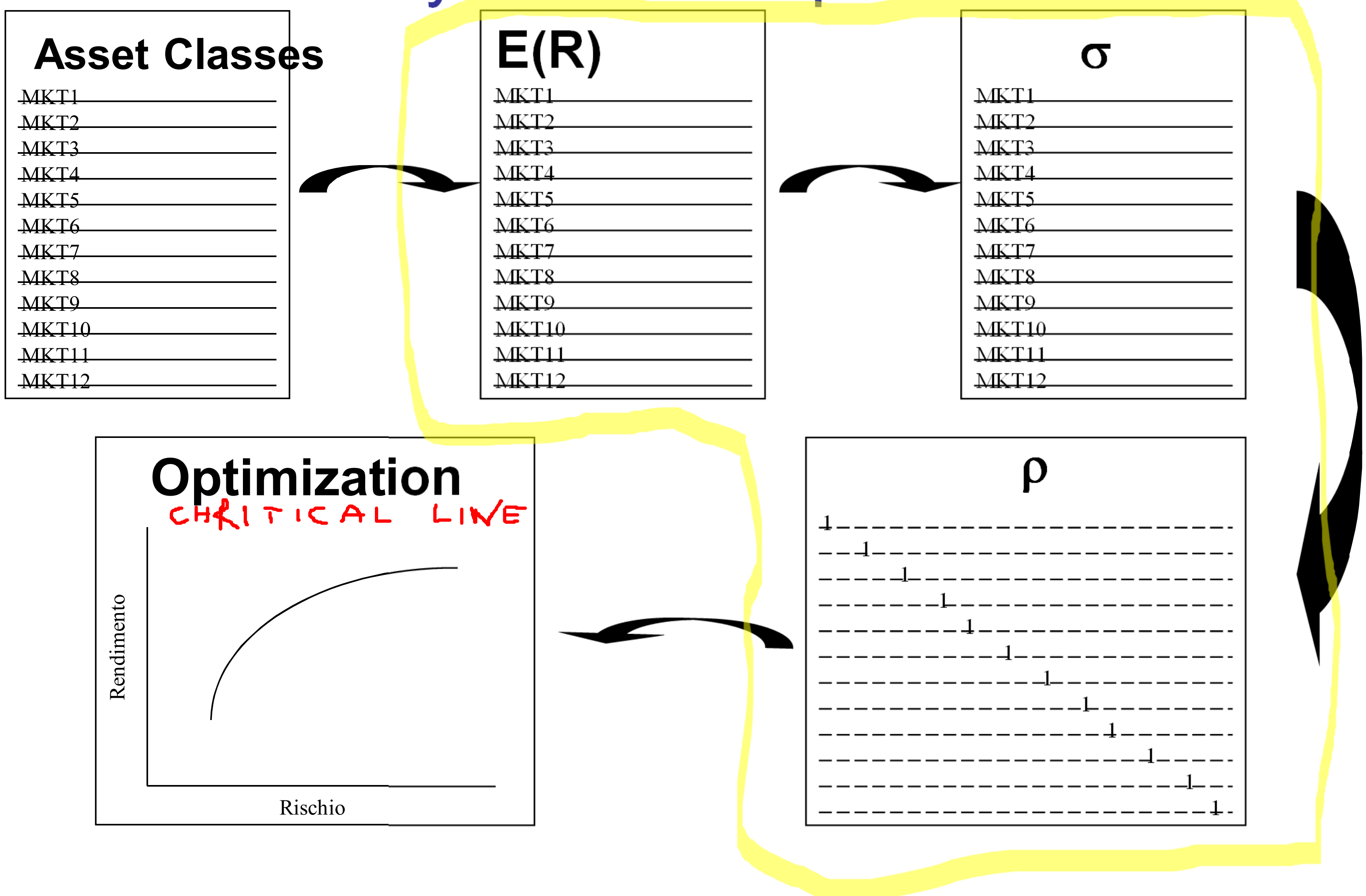
Risk is *bad variable*:
Therefore, investors are willing to increase risk only if higher risk produces higher return.



Solutions B & C are inefficient

Solutions D is efficient: it is an optimal solution for "*high risk tolerance*" investors

The Markowitz Model:
A very scheduled process



The Markowitz Model: A few remarks

1. 5 stages to be performed.
2. The process is time-expensive: you need to estimate many parameters.
3. For example: with 9 asset classes selected, it is necessary to estimate:
   - 9 expected return;
   - 9 standard deviation;
   - 36 correlation.
4. No way, if we want to use the Markowitz model, quantitative estimations are required.

Input estimation

THE JOURNAL OF FINANCE
Vol. VII, No. 1, March 1952
PRINTED IN U.S.A.

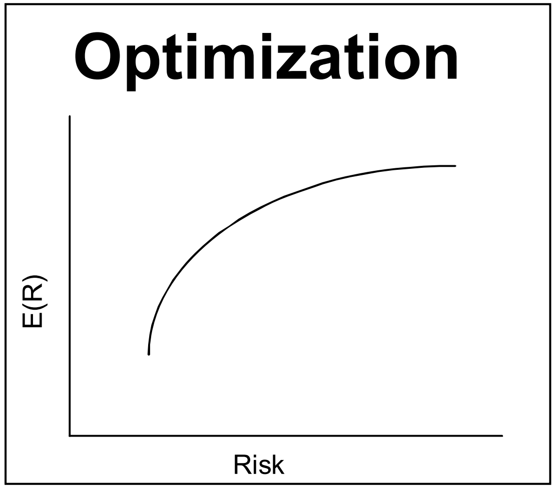
PORTFOLIO SELECTION*

HARRY MARKOWITZ

To use the E-V rule in the selection of securities we must have procedures for finding reasonable μ_i and σ_{ij}. These procedures, I believe, should combine statistical techniques and the judgment of practical men. My feeling is that the statistical computations should be used to arrive at a tentative set of μ_i and σ_{ij}. Judgment should then be used in increasing or decreasing some of these μ_i and σ_{ij} on the basis of factors or nuances not taken into account by the formal computations.

BLACK LITTERMAN

Final Stage: Optimization (1/2)



- If we have: Asset Classes, E(r), σ and ρ…
- We can optimize (Quadratic Programming).

Find the weights (w_i) able to:

Objective function → **MIN σ Portfoglio**

Constraints:

1st constraint: → Exp. Return = E(R)*

2nd constraint: → $w_1 + \ldots + \ldots w_i + \ldots\ldots w_n = 1$

3rd constraint: → $w_i \geq 0$

$$\min_{W} \quad \sigma_{Port}$$

Constraints :

$$\sum_{i=1}^{k} w_i E(R_i) = E(R^*)$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0 \quad \text{with } i = 1,\ldots,k$$

Final Stage: Optimization (2/2)

$$\min_{W} \sigma^2_{Port}$$

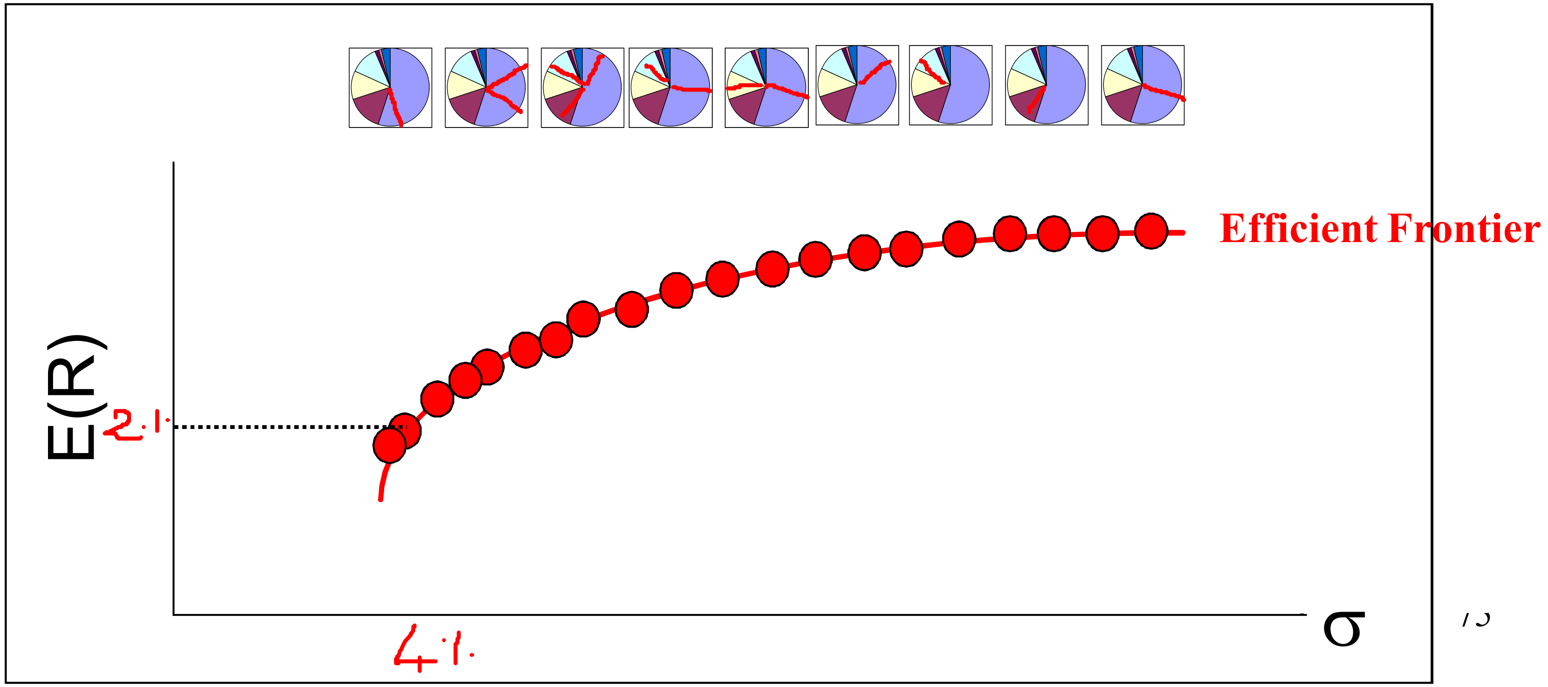
Constraints:

$$\sum_{i=1}^{k} w_i E(R_i) = E(R^*)$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0 \quad \text{with } i = 1, \ldots, k$$

- We run this optimization for a targeted expected return [E(R)*]……
- The optimization returns:
  - the portfolio composition……
  - that is efficient as, given the targeted E(R), it is able to minimise the standard deviation
- Running the optimization for different targeted E(R) we obtain a range of efficient portfolio



Markowitz Optimization: An application (1/6)

Asset Classes selected:

Benchmark selezionati

Nome
JPM Euro 3 mesi
JPM EMU Aggregate Tutte le Scadenze
MSCI Europa
MSCI Nord America
MSCI Giappone
MSCI Pacifico ex Giappone
MSCI Emerging Market Free

Markowitz Optimization: An application (2/6)

Expected Returns estimated:

	Rendimento atteso %
JPM EMU Aggregate Tutte le	3,2
JPM Euro 3 mesi	2,8
MSCI Giappone	4,5
MSCI Pacifico ex Giappone	6
MSCI Nord America	6
MSCI Europa	7
MSCI Emerging Market Free	8

Markowitz Optimization: An application (3/6)

Standard deviations estimated:

	Rischio atteso %
JPM EMU Aggregate Tutte le	1,8
JPM Euro 3 mesi	4,2
MSCI Giappone	22,8
MSCI Pacifico ex Giappone	23
MSCI Nord America	21
MSCI Europa	20
MSCI Emerging Market Free	29

Markowitz Optimization: An application (4/6)

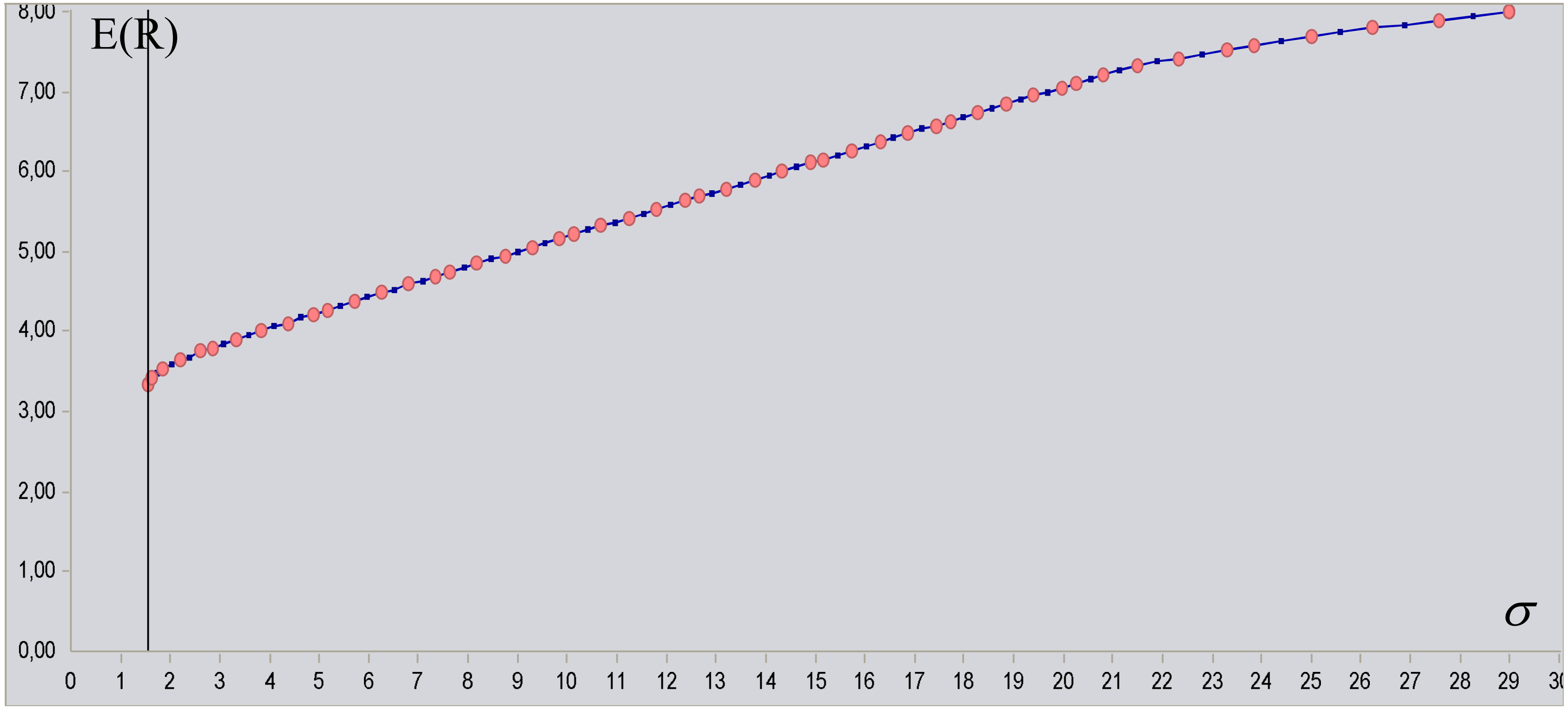
Correlations estimated:

Matrice delle correlazioni

	JPM EMU Aggregate Tutte le	JPM Euro 3 mesi	MSCI Giappone	MSCI Pacifico ex Giappone	MSCI Nord America	MSCI Europa	MSCI Emerging Market Free
JPM EMU Aggregate	1	Storico	Storico	Storico	Storico	Storico	Storico
JPM Euro 3 mesi	0,27	1	Storico	Storico	Storico	Storico	Storico
MSCI Giappone	-0,26	-0,27	1	Storico	Storico	Storico	Storico
MSCI Pacifico ex	-0,27	-0,15	0,61	1	Storico	Storico	Storico
MSCI Nord America	-0,35	-0,13	0,61	0,74	1	Storico	Storico
MSCI Europa	-0,4	-0,2	0,53	0,72	0,85	1	Storico
MSCI Emerging Market	-0,34	-0,18	0,62	0,87	0,76	0,76	1

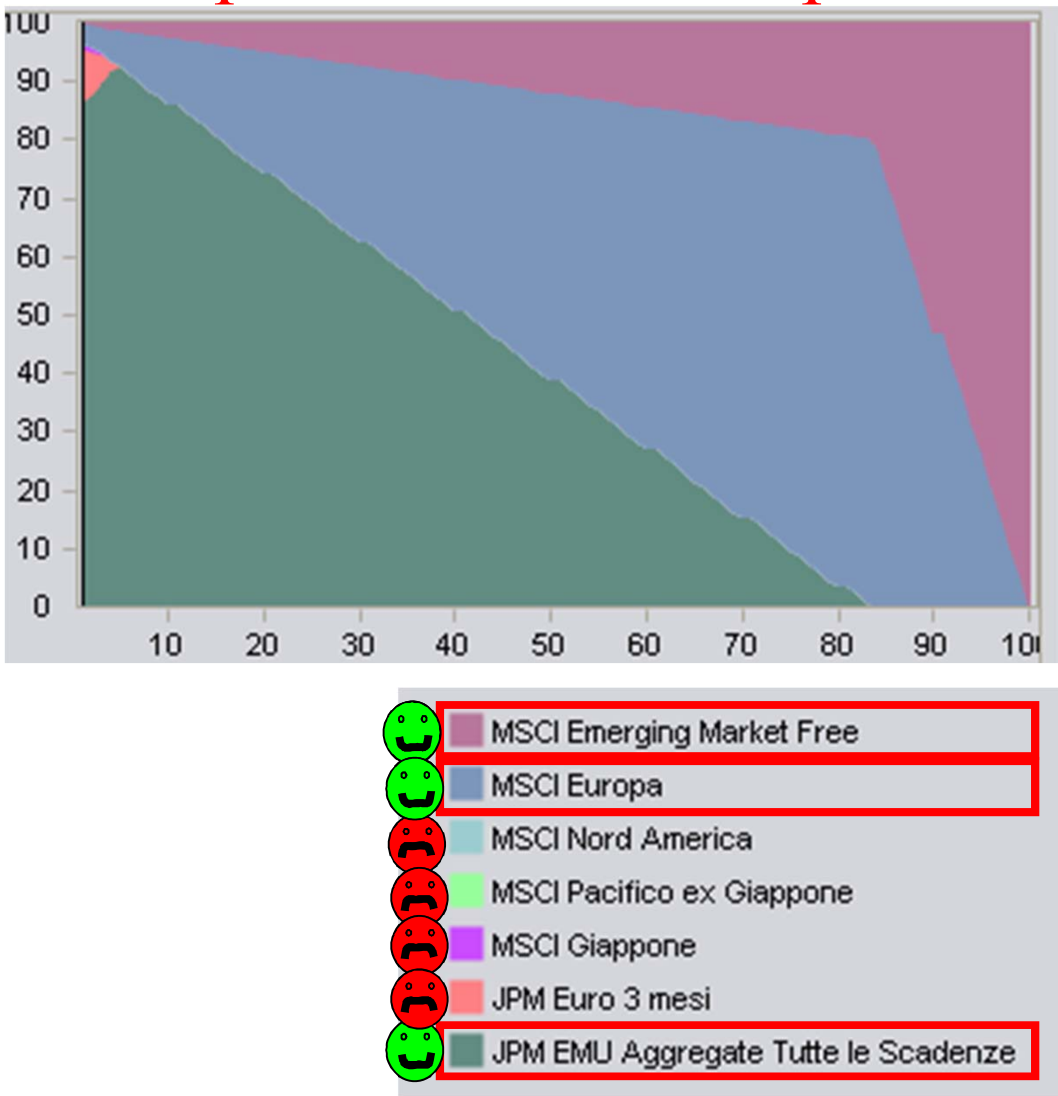
Markowitz Optimization: An application (5/6)

Output: Efficient Frontier



Markowitz Optimization: An application (6/6)

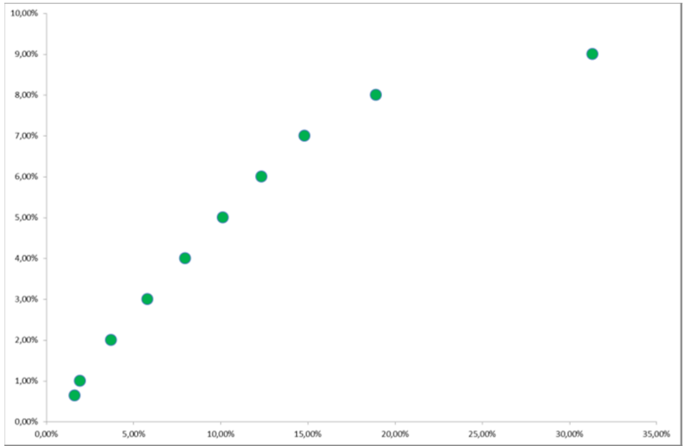
Output: Portfolio composition

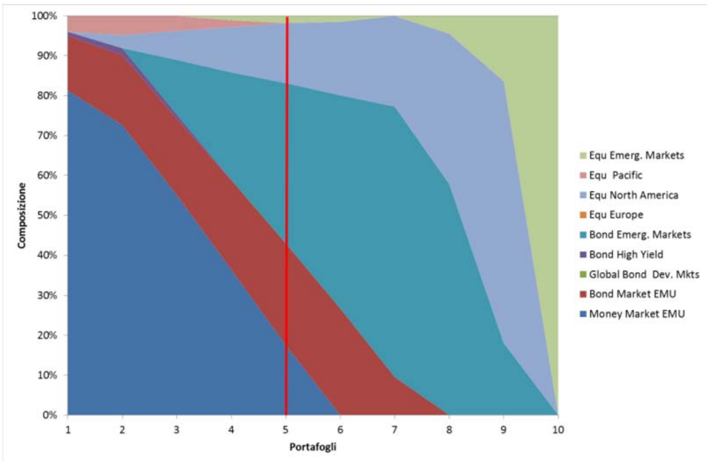


Markowitz Optimization: Excel

Asset Class	E(r)	σ	Weights
Money Market EMU	0,30%	1,95%	54,71%
Bond Market EMU	0,50%	4,22%	19,37%
Global Bond Dev. Mkts	1,60%	8,09%	0,00%
Bond High Yield	5,00%	15,93%	1,56%
Bond Emerg. Markets	6,00%	13,04%	13,54%
Equ Europe	6,50%	20,31%	0,00%
Equ North America	8,30%	21,25%	6,99%
Equ Pacific	7,00%	23,39%	3,81%
Equ Emerg. Markets	9,00%	31,34%	0,01%
PORTFOLIO	2,00%	3,73%	100,00%

2,00%
Targeted Return





It glitters but…….

Markowitz optimization seems to be the best solution.

Nevertheless financial literature has showed that this model has some problems:

1. Efficient portfolios are often unreasonable (Portfolios highly concentrated and/or big weights to "marginal markets").
2. Estimations are supposed to be perfect (Asset managers are clairvoyant! Estimation error doesn't exist). **Errors are ignored**

⇒ You can not fully trust a model that is not able to take into account errors in input estimation!

If you continue your studies in “Finance”, you can analyze how to improve the Markowitz model in order to force the model to:

- return reasonable and well diversified solutions, less affected by estimation errors

Beyond the Markowitz model:

The Capital Asset Pricing Model (CAPM)

The assumption of the CAPM are the same by Markowitz and more:

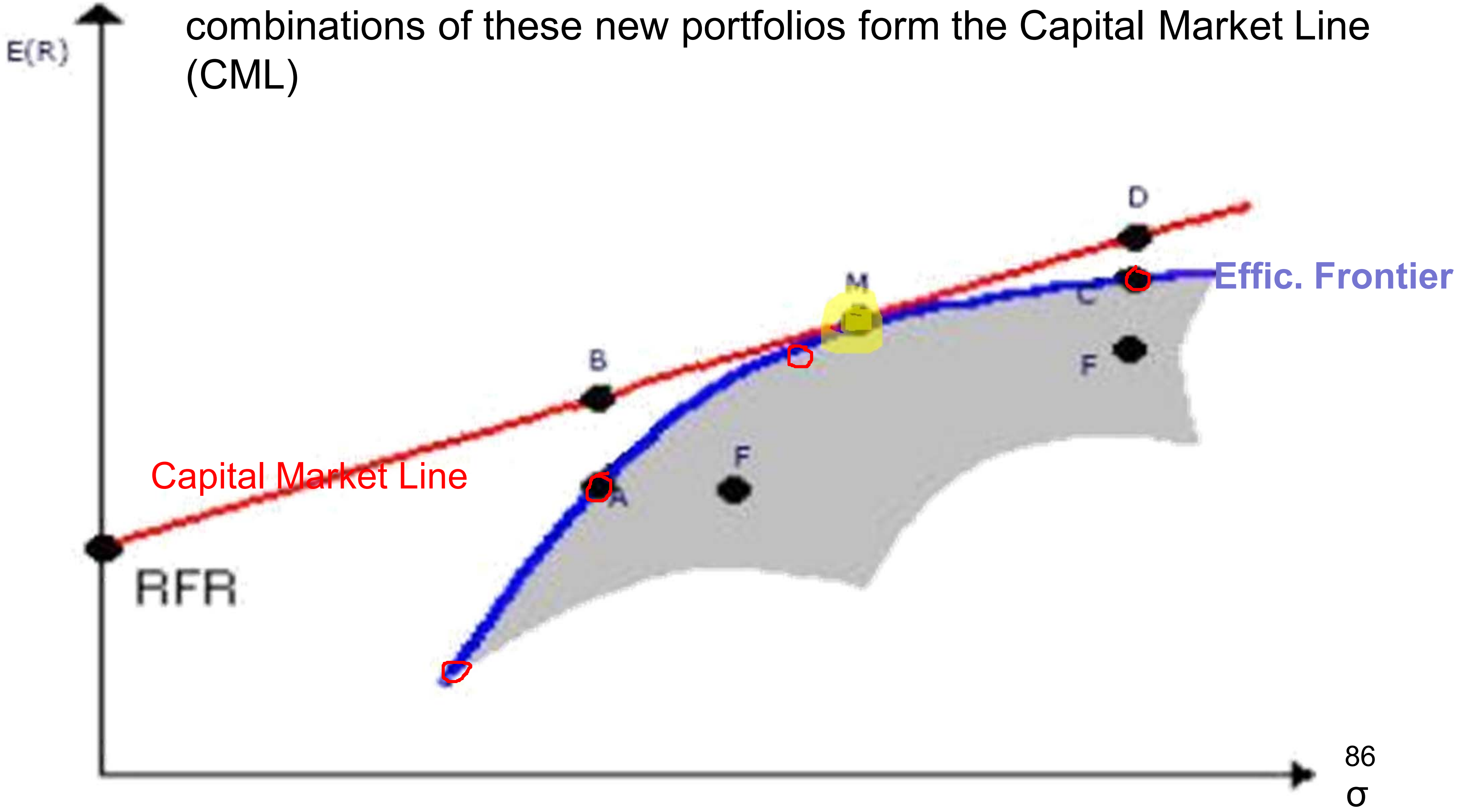
- Investors can freely borrow or invest at the Risk-free rate

- All the investors have same expectations

Unrealistic (Far from what really happens)

The Capital Asset Pricing Model (CAPM):
Optimal Portfolios

Optimal portfolios are the mix between the risk free asset and the efficient portfolio M (the tangency point). The [risk;expected return] combinations of these new portfolios form the Capital Market Line (CML)



The Capital Asset Pricing Model (CAPM):
Optimal Portfolios

If the investor can borrow or invest at the Risk-Free rate (RFR), there is a single efficient portfolio by Markowitz (**Market Portfolio**) that the investor should use to mix it with the **Risk-Free asset**

The Capital Asset Pricing Model (CAPM):

Further elements introduced by the CAPM

1) Formulas that simplify the estimation of the expected returns of the assets
2) and Beta…..

If you continue your studies in "Finance", you can analyze "in depth" CAPM and other models that represent the milestones of Modern Portfolio Theory.