Quantitative Methods – I

A.Y. 2020-21

Practice 1

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THEME #1

—

Types of Variables

1.6 The following table lists the number of billionaires in eight countries as of February 2011, as reported in The New York Times of July 27, 2011.

Country	Number of Billionaires
United States	413
China	115
Russia	101
India	55
Germany	52
Britain	32
Brazil	30
Japan	26

Variable

Element or member

Observation or measurement

Data set

Source: Forbes, International Monetary Fund.

Briefly explain the meaning of a member, a variable, a measurement, and a data set with reference to this table.

a. What is the variable for this data set?

b. How many observations are in this data set?

c. How many elements does this data set contain?

a. Number of Billionaires by Country

b. n=413+115+101+55+52+32+30+26=824

c. 8 *(United States, China, etc.)*

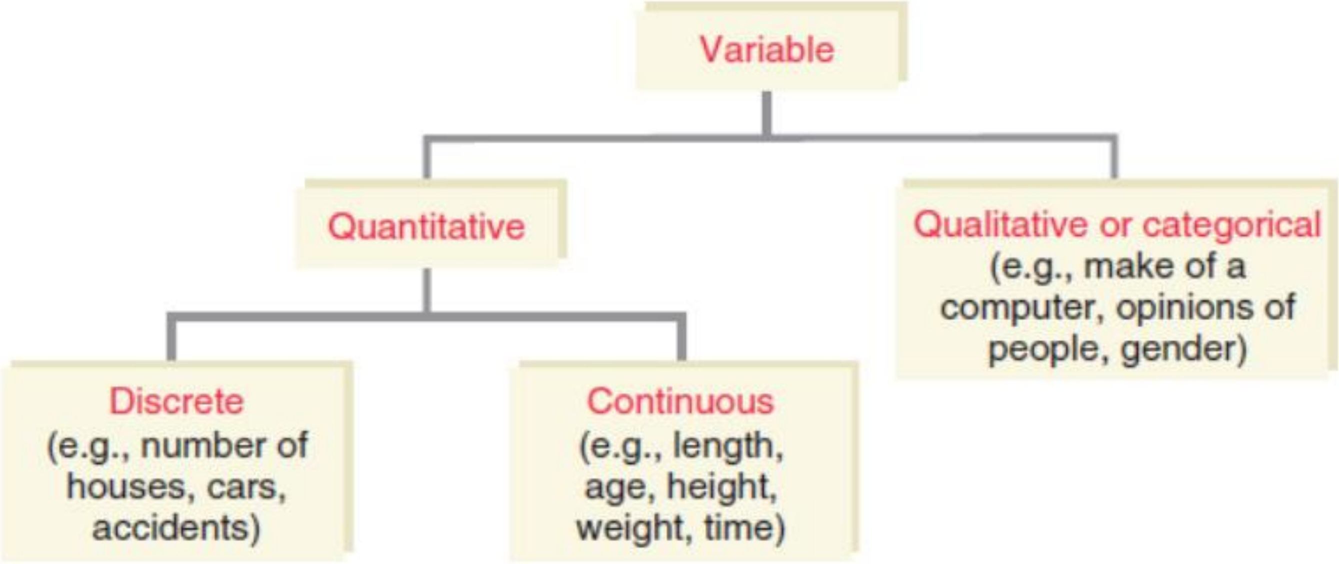


Figure 1.1 Types of variables.

A variable that can be measured numerically is called a **quantitative variable**.

A **discrete variable** can assume only certain values with no intermediate values.

A variable that can assume any numerical value is called a **continuous variable**.

Variables that cannot be measured numerically but can be divided into different categories are called **qualitative or categorical variables**.

1.9 Indicate which of the following variables are quantitative and which are qualitative.

Qualitative or categorical

Quantitative Discrete

- **a.** Number of children under 18 years of age in a family
- **b.** Colors of cars
- **c.** Age of people
- **d.** Time to commute from home to work

Quantitative Continuous

1.10 Indicate which of the following variables are quantitative and which are qualitative. Classify the quantitative variables as discrete or continuous.

- **a.** Women's favorite TV programs
- **b.** Salaries of football players
- **c.** Number of pets owned by families
- **d.** Favorite breed of dog for each of 20 persons

a. Qualitative (News, Series, Movies, etc.)

b. Quantitative Continuous (€ 25.365,30 per month)

c. Quantitative Discrete (1 cat, 3 dogs, etc.)

d. Qualitative (Husky, Fox Terrier, Labrador, etc.)

Stocks and Flows

- **Stock variables:** they can be measured only with reference to a specific time point
- **Flow variables:** they can be measured only with reference to a time interval

Ex. 1. - Indicate which of the following variables are stock and which are flow.

a. Residents in a municipality → Stock Variable
b. Employment status → Stock Variable
c. Sales of Iphone in 2020 → Flow Variable
d. Salary earned in 2020 → Flow Variable

THEME #2

Organizing and Graphing Data

A frequency distribution is a tabular way of summarizing the distribution of a character.

Collection of Raw Data: ex. Age of 50 students

21	19	24	25	29	34	26	27	37	33
18	20	19	22	19	19	25	22	25	23
25	19	31	19	23	18	23	19	23	26
22	28	21	20	22	22	21	20	19	21
25	23	18	37	27	23	21	25	21	24

From raw data to frequency distribution

Frequency Distribution

Age	Nr. Of Students
18	3
19	8
20	3
21	6
22	5
23	6
24	2
25	6
26	2
27	2
28	1
29	1
31	1
33	1
34	1
37	2
Total	50

The frequency distribution is used also for qualitative and quantitative variables.

Variable → Response; Number of Adults ← Frequency column

Response	Number of Adults
Very worried	162
Moderately worried	203
Not too worried	305
Not worried at all	325
Others	20
	Sum = 1015

Category → Not too worried; 305 ← Frequency

More generally, for quantitative variables is useful to subdivide the range of values that X can take into mutually exclusive and exhaustive intervals or classes

Variable → Weekly Earnings (dollars); Number of Employees f ← Frequency column

Weekly Earnings (dollars)	Number of Employees f
801 to 1000	4
1001 to 1200	11
1201 to 1400	39
1401 to 1600	24
1601 to 1800	16
1801 to 2000	6

Third class → 1201 to 1400; 39 ← Frequency of the third class

Lower limit of the sixth class → 1801

Upper limit of the sixth class → 2000

Data presented in the form of a frequency distribution are called grouped data.

Ex. 2 – Arrange the following data (n=20) into a frequency distribution table:

4 5 2 3 4 2 3 2 5 5 3 5 6 3 6 7 3 4 5

DATA VALUE	FREQUENCY
2	3
3	5
4	3
5	6
6	2
7	1

20

$$\text{Relative frequency of a category} = \frac{\text{Frequency of that category}}{\text{Sum of all frequencies}}$$

$$\text{Percentage} = (\text{Relative frequency}) \cdot 100\%$$

DATA VALUE	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
2	3	$\frac{3}{20}$ or 0.15 **15%**	0.15 **15%**
3	5	$\frac{5}{20}$ or 0.25 **25%**	0.15 + 0.25 = 0.40 **40%**
4	3	$\frac{3}{20}$ or 0.15 **15%**	0.40 + 0.15 = 0.55 **55%**
5	6	$\frac{6}{20}$ or 0.30 **30%**	0.55 + 0.30 = 0.85 **85%**
6	2	$\frac{2}{20}$ or 0.10 **10%**	0.85 + 0.10 = 0.95 **90%**
7	1	$\frac{1}{20}$ or 0.05 **5%**	0.95 + 0.05 = 1.00 **100%**

$$\begin{aligned}\text{Cumulative relative frequency} &= \frac{\text{Cumulative frequency of a class}}{\text{Total observations in the data set}}\\ \text{Cumulative percentage} &= (\text{Cumulative relative frequency}) \cdot 100\%\end{aligned}$$

A **cumulative frequency distribution** gives the total number of values that fall below the upper boundary of each class.

Ex. 3 – Calculate the relative frequency, the cumulative frequency and the cumulative relative frequency of the following table:

Class Interval	Class Frequency	Relative Frequency	Cumulative Frequency	Cumulative Relative Frequency
60–64	1	$\frac{1}{25} = 0.04$	1	0.04
65–69	1	$\frac{1}{25} = 0.04$	$1 + 1 = 2$	$0.04 + 0.04 = 0.08$
70–74	2	$\frac{2}{25} = 0.08$	$2 + 2 = 4$	$0.08 + 0.08 = 0.16$
75–79	6	$\frac{6}{25} = 0.24$	$4 + 6 = 10$	$0.16 + 0.24 = 0.4$
80–84	3	$\frac{3}{25} = 0.12$	$10 + 3 = 13$	$0.4 + 0.12 = 0.52$
85–89	5	$\frac{5}{25} = 0.2$	$13 + 5 = 18$	$0.52 + 0.2 = 0.72$
90–94	5	$\frac{5}{25} = 0.2$	$18 + 5 = 23$	$0.72 + 0.2 = 0.92$
95–99	2	$\frac{2}{25} = 0.08$	$23 + 2 = 25$	$0.92 + 0.08 = 1$

25

5. Forty-eight randomly selected car owners were asked about their typical monthly expense on gas. The following data show the responses of these 48 car owners.

$210	160	430	255	176	135	221	359	380	405	391	477
333	209	267	121	357	87	167	95	347	487	302	545
351	256	492	277	245	367	159	187	253	287	456	64
76	166	304	444	193	479	188	148	53	327	234	110

a. Construct a frequency distribution table. Use the classes 50–149, 150–249, 250–349, 350–449, and 450–549.

b. Calculate the relative frequency and percentage for each class.

Monthly Expense on Gas (in dollars)	Frequency	Relative Frequency	Percentage
50 to 149	9	9/48 = 0.188	18.8
150 to 249	13	13/48 = 0.271	27.1
250 to 349	11	11/48 = 0.229	22.9
350 to 449	9	9/48 = 0.188	18.8
450 to 549	6	6/48 = 0.125	12.5

THEME #4

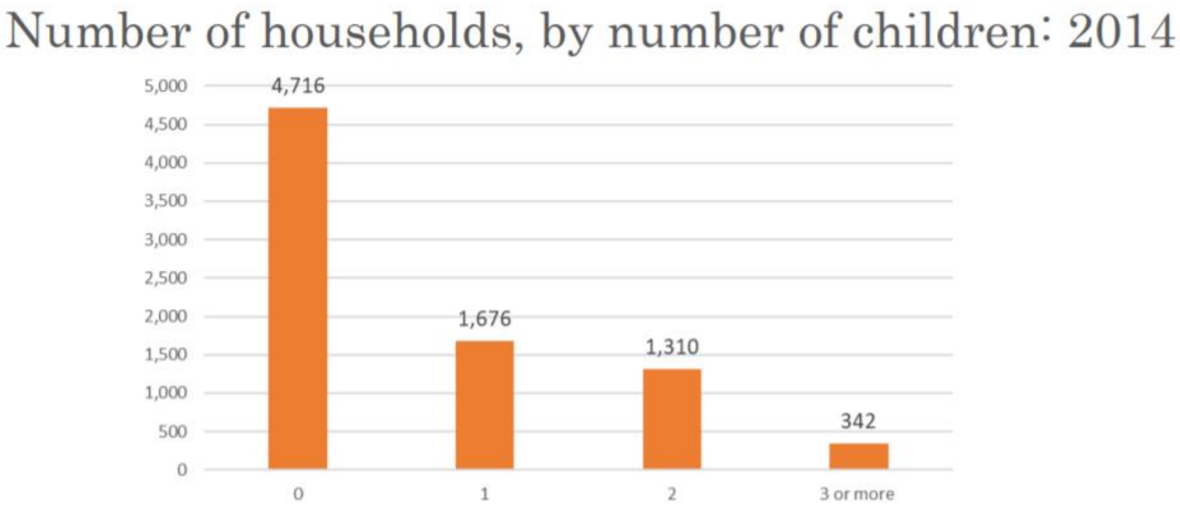
Graphical representations of frequency distributions

Qualitative variables

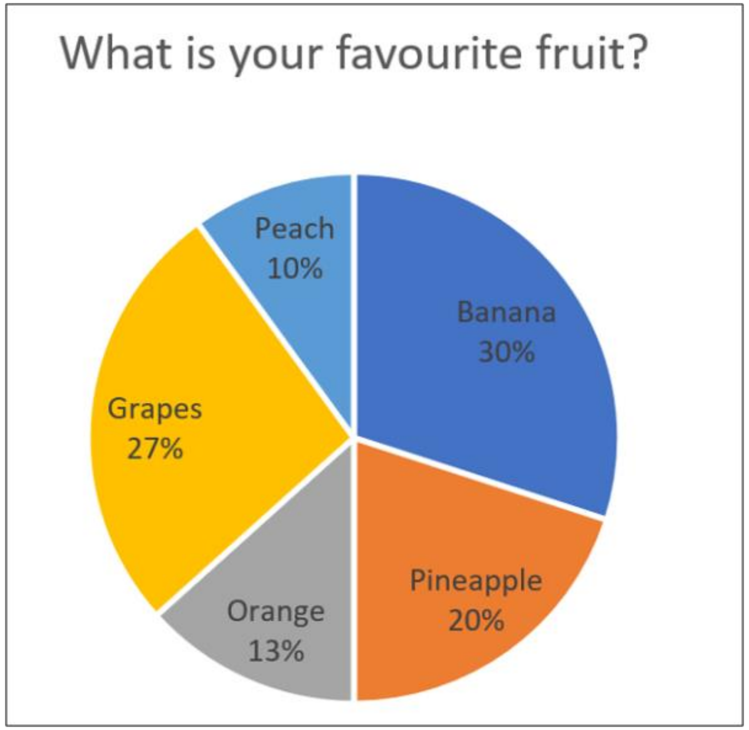
Bar Chart

Each bar's height represents the frequencies (absolute, relative, percentage) of each category/value.

Appropriate for: Qualitative, Quantitative Discrete



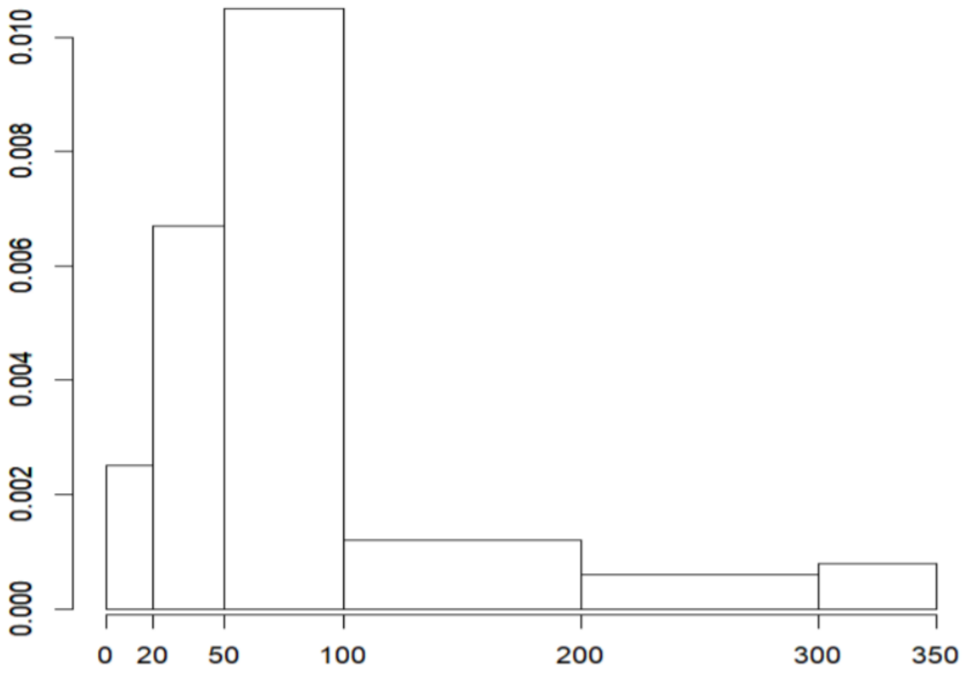
Pie Chart



Continuous variables

Histogram

A bar chart in which each contiguous bar represents a class:
1. the width is proportional to the class width
2. the area is proportional to the relative frequency, rf_i
3. the height is given by the density, h_i



Appropriate for: Quantitative Continuous (in class)

Steps:

1) Compute the relative frequency of each class, $rf_i = \frac{f_i}{n}$
2) Compute the width of each class, $W_i = upper\ limit - lower\ limit$
3) Derive the density, as $h_i = \frac{rf_i}{W_i}$

Ex. 4 - Forty-eight randomly selected car owner were asked about their typical monthly expence

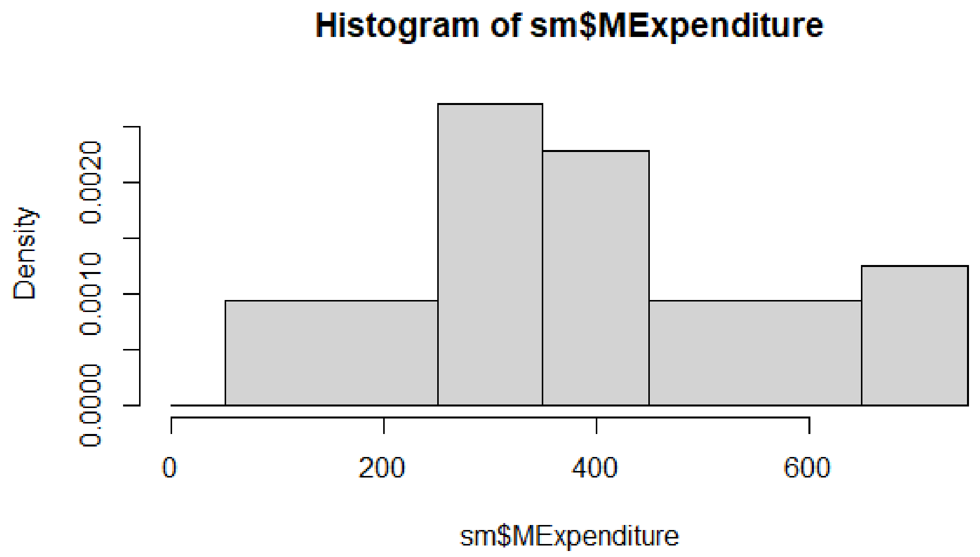
$ 52,00	$ 125,00	$ 360,00	$ 280,00	$ 128,00	$ 220,00	$ 201,00	$ 355,00	$ 400,00	$ 500,00	$ 510,00	$ 550,00
$ 254,00	$ 145,00	$ 290,00	$ 652,00	$ 720,00	$ 240,00	$ 198,00	$ 175,00	$ 278,00	$ 401,00	$ 288,00	$ 610,00
$ 700,00	$ 264,00	$ 401,00	$ 295,00	$ 300,00	$ 554,00	$ 548,00	$ 740,00	$ 628,00	$ 330,00	$ 410,00	$ 298,00
$ 699,00	$ 674,00	$ 351,00	$ 612,00	$ 360,00	$ 629,00	$ 320,00	$ 375,00	$ 330,00	$ 337,00	$ 440,00	$ 444,00

a. Construcy a frequency distribution table using the classes 50 ⊢ 250, 250 ⊢ 350, 350 ⊢ 450, 450 ⊢ 650, and 650 ⊢ 750.
b. Calculat the relative frequency and percentage for each class.
c. Contruct a histogram.

$$w_i = \text{upper limit of the class } - \text{lower limit of the class}$$

$$h_i = \frac{rf_i}{w_i}$$

$Classes$	f_i	$rf_i = f_i/n$	w_i	h_i
50 ⊢ 250	9	9/48= 0,19	200	0,19/200= 0,0009
250 ⊢ 350	13	13/48= 0,27	100	0,27/100= 0,0027
350 ⊢ 450	11	11/48= 0,23	100	0,23/100= 0,0023
450 ⊢ 650	9	9/48= 0,19	200	0,19/200= 0,0009
650 ⊢ 750	6	6/48= 0,13	100	0,13/100= 0,0013
Total	48	1,00		



THEME #4

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Measures of position

Measures of position

Mode
Value/category/class with the highest frequency

Median
Value of the observation(s) in the middle of the ranked data, where the middle position is $\frac{n+1}{2}$

Quartiles
Three values that divide the **ranked** data into four equal parts

Percentiles
Values that divide the **ranked** data into 100 equal parts

Arithmetic mean/average
Sum of all values divided by number of observations

POPULATION

$$\mu = \frac{\sum x}{N}$$

SAMPLE

$$\bar{x} = \frac{\sum x}{n}$$

Geometric mean
The *n*th root of the product of all observations

$$\left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}} = \sqrt[n]{a_1 a_2 \cdots a_n}.$$

Harmonic mean
The reciprocal of the arithmetic mean

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

Ex. 5

Calculating the mode for ungrouped data.

The following data give the speeds (in miles per hour) of eight cars that were stopped on I-95 for speeding violations.

77	82	74	81	79	84	74	78

Find the mode.

Solution In this data set, 74 occurs twice, and each of the remaining values occurs only once. Because 74 occurs with the highest frequency, it is the mode. Therefore,

Mode = **74 mile per hour**

A small company has 12 employees. Their commuting times (rounded to the nearest minute) from home to work are 23, 36, 14, 23, 47, 32, 8, 14, 26, 31, 18, and 28, respectively. Find the mode for these data.

Solution In the given data on the commuting times of these 12 employees, each of the values 14 and 23 occurs twice, and each of the remaining values occurs only once. Therefore, this data set has two modes: 14 and 23 minutes.

The ages of 10 randomly selected students from a class are 21, 19, 27, 22, 29, 19, 25, 21, 22, and 30 years, respectively. Find the mode.

Solution This data set has three modes: **19**, **21**, and **22**. Each of these three values occurs with a (highest) frequency of 2.

Ex. 7

Calculating the median for ungrouped data: even number of data values.

The following data give the cell phone minutes used last month by 12 randomly selected persons.

230	2053	160	397	510	380	263	3864	184	201	326	721

Find the median for these data.

Solution To calculate the median, we perform the following two steps.

Step 1: In the first step, we rank the given data in increasing order as follows:

160	184	201	230	263	326	380	397	510	721	2053	3864

Step 2: In the second step, we find the value that divides the ranked data set in two equal parts. This value will be the median.
The value that divides 12 data values in two equal parts falls between the 6th and the 7th values and the median will be given as follows:

160	184	201	230	263	326	380	397	510	721	2053	3864
					↑						
					Median = 353						

$$\text{Median} = \text{Average of the two middle values} = \frac{326 + 380}{2} = \mathbf{353\ \ minutes}$$

Thus, the median cell phone minutes used last month by these 12 persons was 353. We can state that half of these 12 persons used less than 353 cell phone minutes and the other half used more than 353 cell phone minutes last month. Note that this data set has two outliers, 2053 and 3864 minutes, but these outliers do not affect the value of the median.

$$\frac{n+1}{2} = \frac{12+1}{2} = 6{,}5 \quad (\text{between } 6th \text{ and } 7th \text{ values})$$

Ex. 8

Calculating the population mean for ungrouped data.

The following are the ages (in years) of all eight employees of a small company:

53	32	61	27	39	44	49	57

Find the mean age of these employees.

Solution Because the given data set includes *all* eight employees of the company, it represents the population. Hence, $N = 8$. We have

$$\sum x = 53 + 32 + 61 + 27 + 39 + 44 + 49 + 57 = 362$$

The population mean is

$$\mu = \frac{\sum x}{N} = \frac{362}{8} = \mathbf{45.25\ \ years}$$

Thus, the mean age of all eight employees of this company is 45.25 years, or 45 years and 3 months.

3.82 The following data give the 2014 profits (in millions of dollars) of the top 10 companies listed in the 2014 *Fortune 500*. Find the mean and median for these data. Do these data have a mode?

Company	2014 Profits (mil. of dollars)
Wal-Mart Stores	16022
Exxon Mobil	3258
Chevron	21423
Berkshire Hathaway	19476
Apple	37037
Phillips 66	3726
General Motors	5346
Ford Motor	7155
General Electric	13057
Valero Energy	2720

Step 1. Rank data

Company	2014 Profits (mil. of dollars)
Valero Energy	2720
Exxon Mobil	3258
Phillips 66	3726
General Motors	5346
Ford Motor	7155
General Electric	13057
Wal-Mart Stores	16022
Berkshire Hathaway	19476
Chevron	21423
Apple	37037
Total	129220

Step 2. Median

$$\frac{n+1}{2} = \frac{11}{2} = 5.5 \text{ (between 5th and 6th values)}$$

$$\text{Median}=\frac{7155+13057}{2} = 10106$$

Step 3. Mean

$$\mu = \frac{\sum x}{N} = \frac{129220}{10} = 12922.0$$

Step 4. No Mode

$$\text{Class midpoint or mark} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

Class	Midpoint (m or c)
$5 \mid - 10$	$\frac{10+5}{2} = 7{,}5$
$10 \mid - 25$	$\frac{25+10}{2} = 17{,}5$

For population data:

Arithmetic Mean

$$\mu = \sum \frac{x_i\, f_i}{N} \qquad \mu = \sum \frac{m_i\, f_i}{N} \quad \text{(classes)}$$

For sample data:

Arithmetic Mean

$$\bar{x} = \sum \frac{x_i\, f_i}{n} \qquad \bar{x} = \sum \frac{m_i\, f_i}{n} \quad \text{(classes)}$$

3.53 For 50 airplanes that arrived late at an airport during a week, the time by which they were late was observed. In the following table, x denotes the time (in minutes) by which an airplane was late, and f denotes the number of airplanes.

Find the mean.

x	f
0 ├ 20	14
20 ├ 40	18
40 ├ 60	9
60 ├ 80	5
80 ├ 100	4

x	f	m	mf
0 ├ 20	14	10	140
20 ├ 40	18	30	540
40 ├ 60	9	50	450
60 ├ 80	5	70	350
80 ├ 100	4	90	360
	50		1840

$$\mu = \frac{\sum x}{N} = \frac{1840}{50} = 36.8$$

THEME #5

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Measures of dispersion

Range

It is obtained by taking the difference between the largest and the smallest values in a data set.

Range=Largest value−Smallest value

Interquartile Range

The difference between the third and the first quartiles

IQR = Q3 − Q1

Variance and Standard Deviation

The variance is the squared deviation of a variable from its mean.

The standard deviation is obtained by taking the positive square root of the variance.

$$\sigma^2 = \frac{\sum (x-\mu)^2}{N} \quad \text{and} \quad s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$

$$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}} \quad \text{and} \quad s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$$

Coefficient of Variation

The coefficient of variation, denoted by CV, expresses standard deviation as a percentage of the mean.

For population data : $\mathrm{CV} = \frac{\sigma}{\mu} \times 100\%$

For sample data : $\mathrm{CV} = \frac{s}{\bar{x}} \times 100\%$

Ex. 9

Calculate range, median and interquartile range of following values:

14 3 5 4 6 8 12 5 3 7 18

n=11

(n+1)/2=6

Rank the data

3
3
4
5
5
6
7
8
12
14
18

n=6

(n+1)/2=3.5

Q1= 4,5

Median= 6

Q3= 10

Calculate

Min= 3
Max= 18

Range= 18 - 3 = 15

IR= 10 - 4,5 = 5,5

3.83 The following data represent the differences (in seconds) between each winner's time of Belmont Stakes horse racing for the years 1999–2011 and the best time of 1973.

3.80 7.20 2.80 5.71 4.26 3.50 4.75 3.81 4.74 5.65 3.54 7.57 6.88

a. Compute the range, variance, and standard deviation for these data.

Rank the data

2.80
3.50
3.54
3.80
3.81
4.26
4.74
4.75
5.65
5.71
6.88
7.20
7.57

Calculate:

Min= 2.80
Max= 7.57

Range= 7.57 – 2.80 = 4.77

Calculate the mean

$$\mu = \frac{\sum x}{N} = \frac{64.21}{13} = 4.94$$

3.83 The following data represent the differences (in seconds) between each winner's time of Belmont Stakes horse racing for the years 1999–2011 and the best time of 1973.

3.80 7.20 2.80 5.71 4.26 3.50 4.75 3.81 4.74 5.65 3.54 7.57 6.88

a. Compute the range, variance, and standard deviation for these data.

$$\mu = \frac{\sum x}{N} = \frac{64.21}{13} = 4.94$$

x	x-μ	$(x-\mu)^2$
2.80	-2.14	4.58
3.50	-1.44	2.07
3.54	-1.40	1.96
3.80	-1.14	1.30
3.81	-1.13	1.28
4.26	-0.68	0.46
4.74	-0.20	0.04
4.75	-0.19	0.04
5.65	0.71	0.51
5.71	0.77	0.59
6.88	1.94	3.77
7.20	2.26	5.11
7.57	2.63	6.92
		28.61

Calculate the variance and the standard deviation

$$\sigma^2 = \sum \frac{(x-\mu)^2}{N} = \frac{28.61}{13} = 2.20$$

$$\sigma = \sqrt{2.20} = 1.48$$

Exercise 10

Let the following unitary distribution of the character X be given:

2 4 2 2 4 2 0 4 0 2 4 16

Calculate the variance and the standard deviation.

x_i	n_i	$x_i n_i$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^2 n_i$
0	2	0	12.25	24.5
2	5	10	2.25	11.25
4	4	16	0.25	1
16	1	16	156.25	156.25
	12	42		193

Calculate the mean

$$\bar{x} = \sum \frac{x_i\, n_i}{n}$$

Calculate the variance and the standard deviation

$$s^2 = \frac{1}{n} \sum_{i=1}^{k} n_i (x_i - \bar{x})^2$$

$$s^2 = \frac{1}{n} \sum_{i=1}^{k} (x_i - \bar{x})^2 n_i = \frac{193}{12} = 16.08\bar{3}.$$

$$s = \sqrt{s^2} \simeq \sqrt{16.083} = 4.01.$$

Box-Whiskers Plot (or Box-Plot)

Comprehensive graphical representation of a distribution.

Indeed, it provides info on:

-position: median, Q1, and Q3 (lines)
-dispersion: interquartile range (box)
-shape of the distribution (whiskers)
-extreme values: outliers

bp.1 An employee of a computer store recorded the number of sales he made each month. In the past 12 months, he sold the following numbers of computers:

51, 20, 25, 39, 7, 44, 92, 41, 22, 6, 42, 18.

Make the box and whisker plots.

First, put the data in ascending order. Then find the median.
N=12
6, 7, 18, 20, 22, 25, 39, 41, 42, 44, 51, 92

Median position = (N+1)/2 = (12 + 1) / 2 = 6.5th value
Median = (sixth + seventh observations) / 2 = (25 + 39) / 2 = 32

There are six numbers below the median, namely: 6, 7, 18, 20, 22, 25.
Q1 position = the median of these six items = (6 + 1) / 2 = 3.5th value
Q1 = (third + fourth observations) / 2 = (18 + 20) / 2 = 19

There are six numbers above the median, namely: 39, 41, 42, 44, 51, 92.
Q3 position = the median of these six items = (6 + 1) / 2 = 3.5th value
Q3 = (third + fourth observations) / 2 = (42+44) / 2 = 43

bp.1 An employee of a computer store recorded the number of sales he made each month. In the past 12 months, he sold the following numbers of computers:

51, 20, 25, 39, 7, 44, 92, 41, 22, 6, 42, 18.

Make the box and whisker plots.

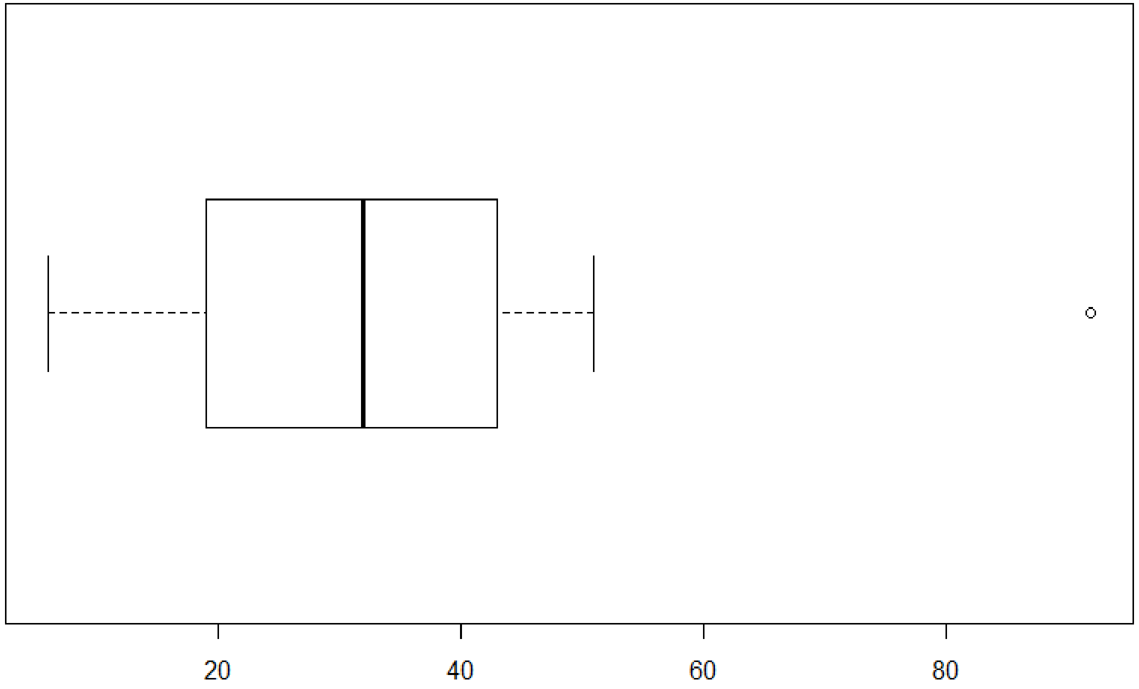
Median = 32
Q1 = 19
Q3 = 43

IQR= Q3 – Q1 = 43 - 19 = 24

Whiskers:
Upper = Q3 + 1.5 IQR = 43 + 1.5 · 24 = 43 + 36 = 79

Lower = Q1 – 1.5 IQR = 19 – 36 = -17
(smaller than the minimum value)

1 Upper outlier (92)



Ex. 6

Calculating the median for ungrouped data: odd number of data values.

Table 3.2 lists the 2014 compensations of female CEOs of 11 American companies (*USA TODAY*, May 1, 2015). (The compensation of Carol Meyrowitz of TJX is for the fiscal year ending in January 2015.)

Table 3.2 Compensations of 11 Female CEOs

Company & CEO	2014 Compensation (millions of dollars)
General Dynamics, Phebe Novakovic	19.3
GM, Mary Barra	16.2
Hewlett-Packard, Meg Whitman	19.6
IBM, Virginia Rometty	19.3
Lockheed Martin, Marillyn Hewson	33.7
Mondelez, Irene Rosenfeld	21.0
PepsiCo, Indra Nooyi	22.5
Sempra, Debra Reed	16.9
TJX, Carol Meyrowitz	28.7
Yahoo, Marissa Mayer	42.1
Xerox, Ursula Burns	22.2

Find the median for these data.

Solution To calculate the median of this data set, we perform the following two steps.

Step 1: The first step is to rank the given data. We rank the given data in increasing order as follows:

16.2	16.9	19.3	19.3	19.6	21.0	22.2	22.5	28.7	33.7	42.1

Step 2: The second step is to find the value that divides this ranked data set in two equal parts. Here there are 11 data values. The sixth value divides these 11 values in two equal parts. Hence, the sixth value gives the median as shown below.

16.2	16.9	19.3	19.3	19.6	21.0	22.2	22.5	28.7	33.7	42.1
					↑					
					Median					

$$\frac{(n+1)}{2} = \frac{12}{2} = 6$$

3.16 The following data give the annual salaries (in thousand dollars) of 20 randomly selected health care workers.

50	71	57	39	45	64	38	53	35	62
74	40	67	44	77	61	58	55	64	59

a. Calculate the mean, median, and mode for these data.

n=20

Rank data

35 38 39 40 44 45 50 53 55 57 58 59 61 62 64 64 67 71 74 77

Mean

$$\bar{x} = \frac{\Sigma x}{n} = \frac{35+38+39+40+44+45+50+53+55+57+58+59+61+62+64+64+67+71+74+77}{20} = \frac{1113}{20} = 55{,}65$$

Median

$$\frac{n+1}{2} = \frac{21}{2} = 10{,}5$$

35	38	39	40	44	45	50	53	55	57	58	59	61	62	64	64	67	71	74	77
1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°

$$Median = \frac{57+58}{2} = 57{,}5$$

Mode

35 38 39 40 44 45 50 53 55 57 58 59 61 62 64 64 67 71 74 77

$Mode =$ 64

2 times