Consumer's choices under uncertainty

Adapted from Luca Panaccione's
Lectures on Microeconomics

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Overview

1. Describing Risky Outcome – Basic Tools
   - *Lotteries and Probabilities*
   - *Expected Values*

2. Evaluating Risky Outcomes
   - *Risk Preferences and the Expected Utility Function*

Tools for Describing Risky Outcomes

A **lottery** is defined by a list of uncertain outcomes and the probabilities with which these outcomes occur.

<u>Examples:</u> Investment, Roulette, Football Game.

Tools for Describing Risky Outcomes

Definition: The **probability** of an outcome of a lottery is the likelihood that this outcome occurs.

Example: The probability is often estimated using the historical frequencies of the outcome.

Probability Distribution

Definition: A **probability distribution** is a list of probabilities, one for each outcome of the lottery.

It depicts all possible payoffs in the lottery and their associated probabilities.

Probability Distribution

Properties of probability distributions:

- The probability of each outcome is a number between 0 and 1
- The sum of the probabilities of all possible outcomes equals 1.

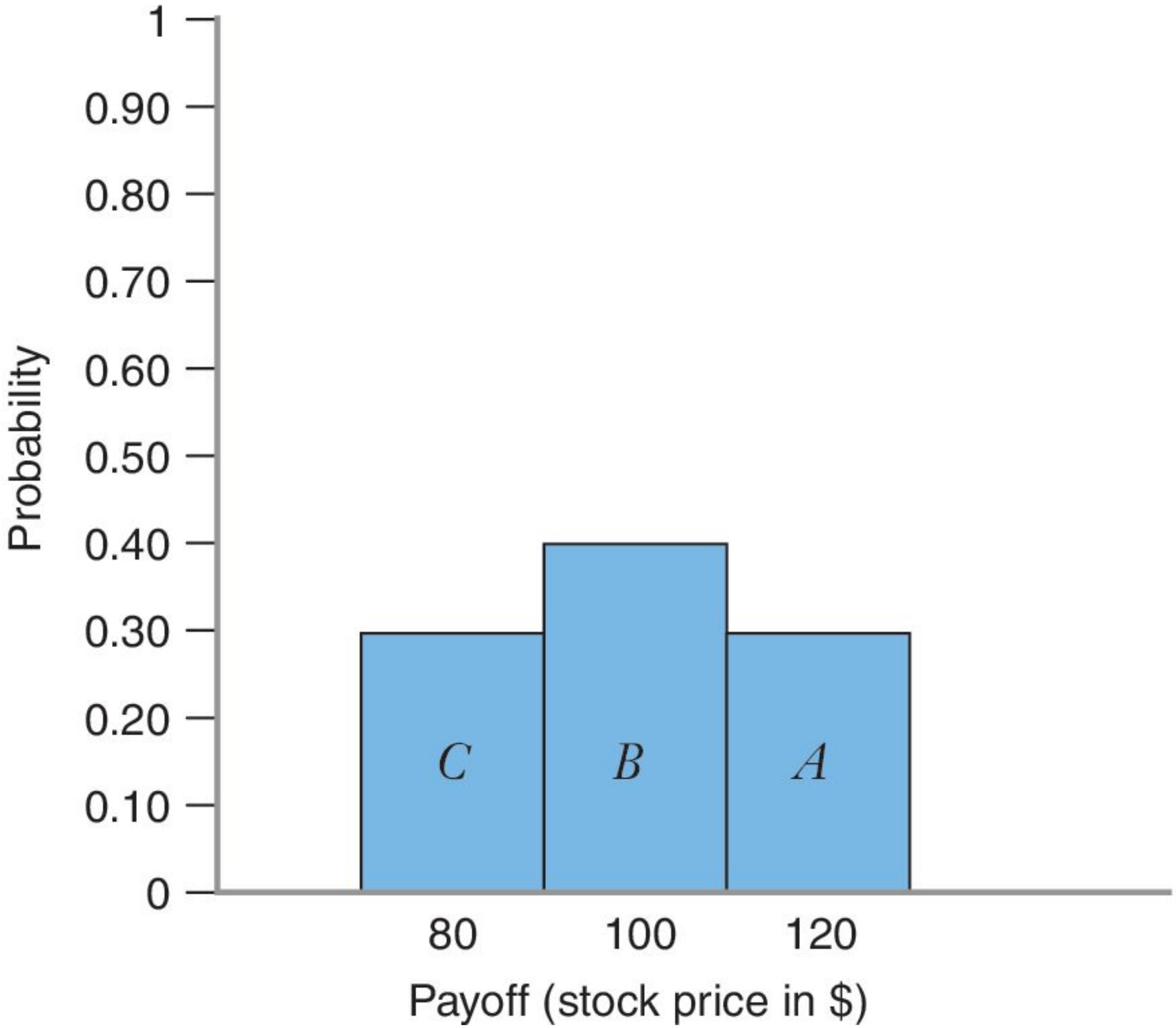
An example of a lottery

Example: Consider the lottery L over the finite set of outcomes {A,B,C}.

This lottery associates a probability of:

1) 0.3 to the outcome A,
2) 0.4 to the outcome B,
3) 0.3 to the outcome C.

Probability Distribution



Expected Value

<u>*Definition:*</u> The **expected value** of a lottery is a measure of the average outcome that the lottery will generate.

$$EV = Pr(A)\,A + Pr(B)\,B + Pr(C)\,C$$

where:

- $Pr(x)$ is the probability of outcome $x = A, B, C$.
- A,B, and C are outcomes.

Expected Value

For the lottery K which pays 120euros with probability 1/4, 100euros with probability 2/4 and 80euros with probability 1/4, the expected value is:

$$EV = (1/4 \times 120) + (2/4 \times 100) + (1/4 \times 80) = 100 \text{ euros}$$

Notice that the expected value may be one of the outcomes of the lottery.

Variance & Standard Deviation

Definition: The **variance** of a lottery is the sum of the probability-weighted squared deviations between outcomes and the expected value of the lottery. It is a measure of the lottery's riskiness.

$$Var = (A - EV)^2 Pr(A) + (B - EV)^2 Pr(B) + (C - EV)^2 Pr(C)$$

Definition: The **standard deviation** of a lottery is the square root of the variance. It is an alternative measure of risk

Variance & Standard Deviation

For the three-outcome lottery, the variance is obtained a follows.

The squared deviation of outcome A is: $(120 - 100)^2 = 400$

The squared deviation of outcome B is: $(100 - 100)^2 = 0$

The squared deviation of outcome C is: $(80 - 100)^2 = 400$

The variance is:

$$(400 \text{ X } 0.3) + (0 \text{ X } 0.4) + (400 \text{ X } 0.3) = 240$$

Evaluating Risky Outcomes

Suppose that an individual facing risky alternatives attempts at maximizing her ***expected utility****, i.e., the probability-weighted average of the utility from each possible outcome they face.*

For a lottery with two outcomes, this corresponds to:

$$\text{EU} = Pr(A) \times U(A) + Pr(B) \times U(B)$$

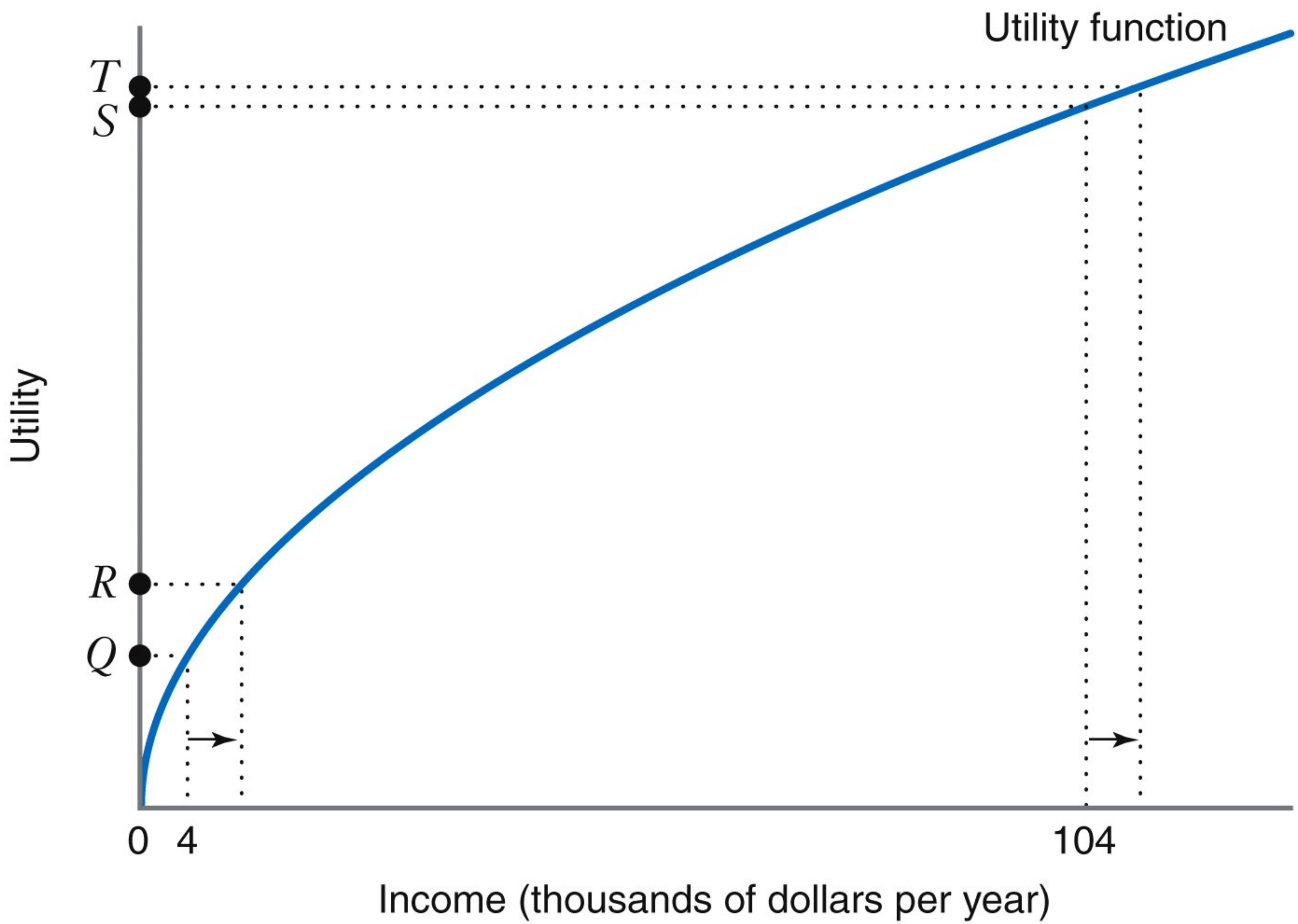
Evaluating Risky Outcomes

Which assumptions on $U(\cdot)$?

Suppose that

- $U(\cdot)$ is increasing (positive marginal utility)
- $U(\cdot)$ is increasing at a decreasing rate (diminishing marginal utility)

Evaluating Risky Outcomes



Evaluating Risky Outcomes

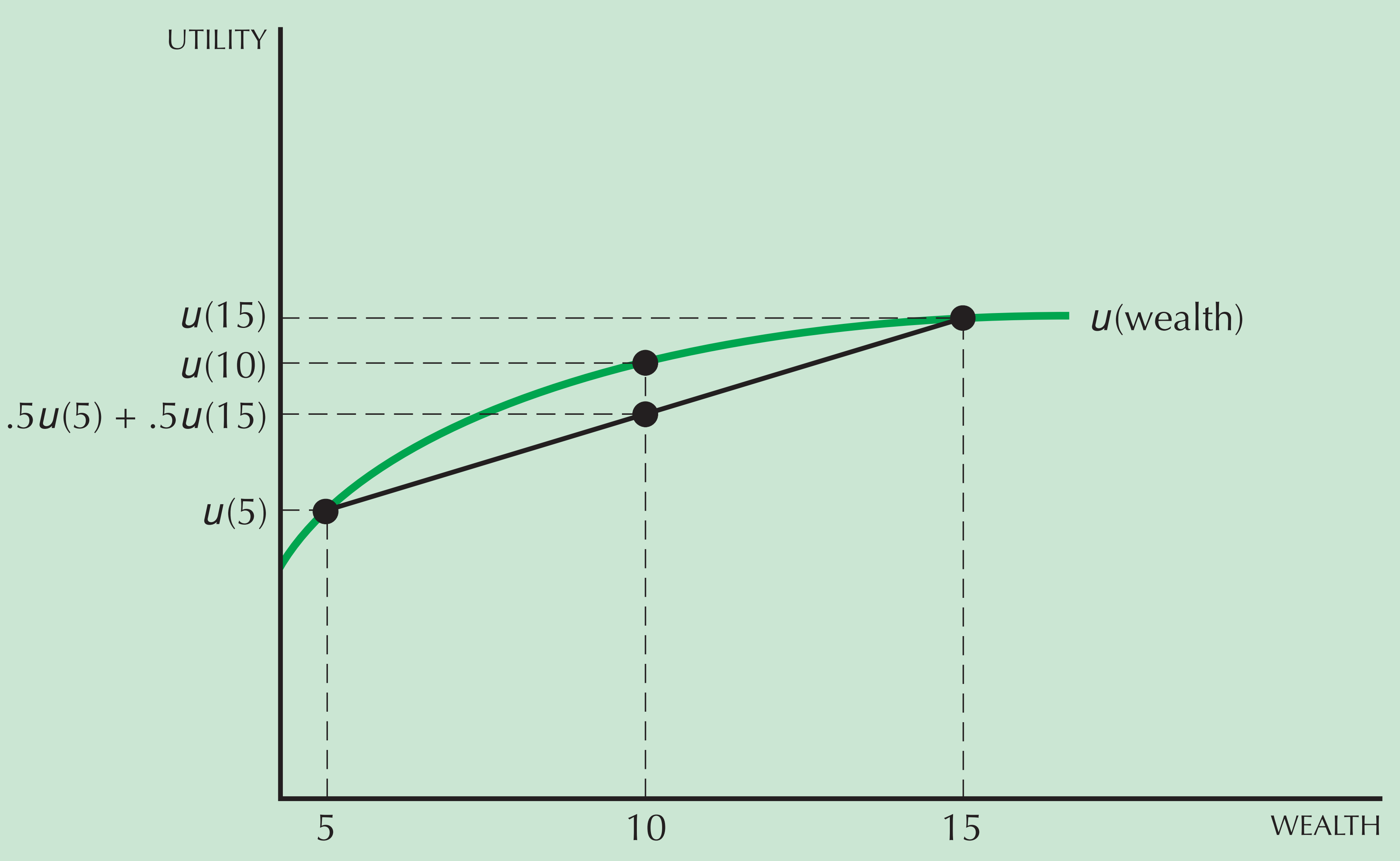
In this case, utility of the expected value is higher than expected utility:

$$U\left(Pr(A) \times A + Pr(B) \times B\right) > Pr(A) \times U(A) + Pr(B) \times U(B)$$

Evaluating Risky Outcomes

- Example: lottery with equally likely outcomes $A = 5$ and $B = 15$
- $EV = 0.5 \times 5 + 0.5 \times 15 = 10$

Evaluating Risky Outcomes



Evaluating Risky Outcomes

- The preferences of this decision maker are such that he prefers the utility of 10, i.e of a safe lottery which yields 10 for sure, to the expected utility he may get by participating in the lottery that has an expected value equal to 10.

- He is a **risk-averse** decision maker.

Evaluating Risky Outcomes

- Along the black line,

$$U = U(5) + \left(\frac{U(15) - U(5)}{15 - 5}\right)(\text{wealth} - 5)$$

- When wealth $= 10$ (which is EV of the lottery),

$$U = U(5) + \left(\frac{U(15) - U(5)}{15 - 5}\right)(10 - 5) = 0.5U(5) + 0.5U(15)$$

Evaluating Risky Outcomes

- Consider two lotteries:
  - lottery 1: two outcomes A, B with probability $Pr(A), Pr(B)$
  - lottery 2: sure outcome equal to $Pr(A) \times A + Pr(B) \times B$
- A decision maker who has utility function as above will prefer lottery 2

Evaluating Risky Outcomes

Example: Work for a large, established firm (choice 1) or a start-up (choice 2)?

- Choice 1: sure salary of 54,000

- Choice 2: sure salary of 4,000, plus benefit of 100.000 in case of success (0.5 probability)

- These choices correspond to lotteries:

  - choice 1 is a sure-thing lottery

  - choice 2 is a lottery with two equally likely outcomes

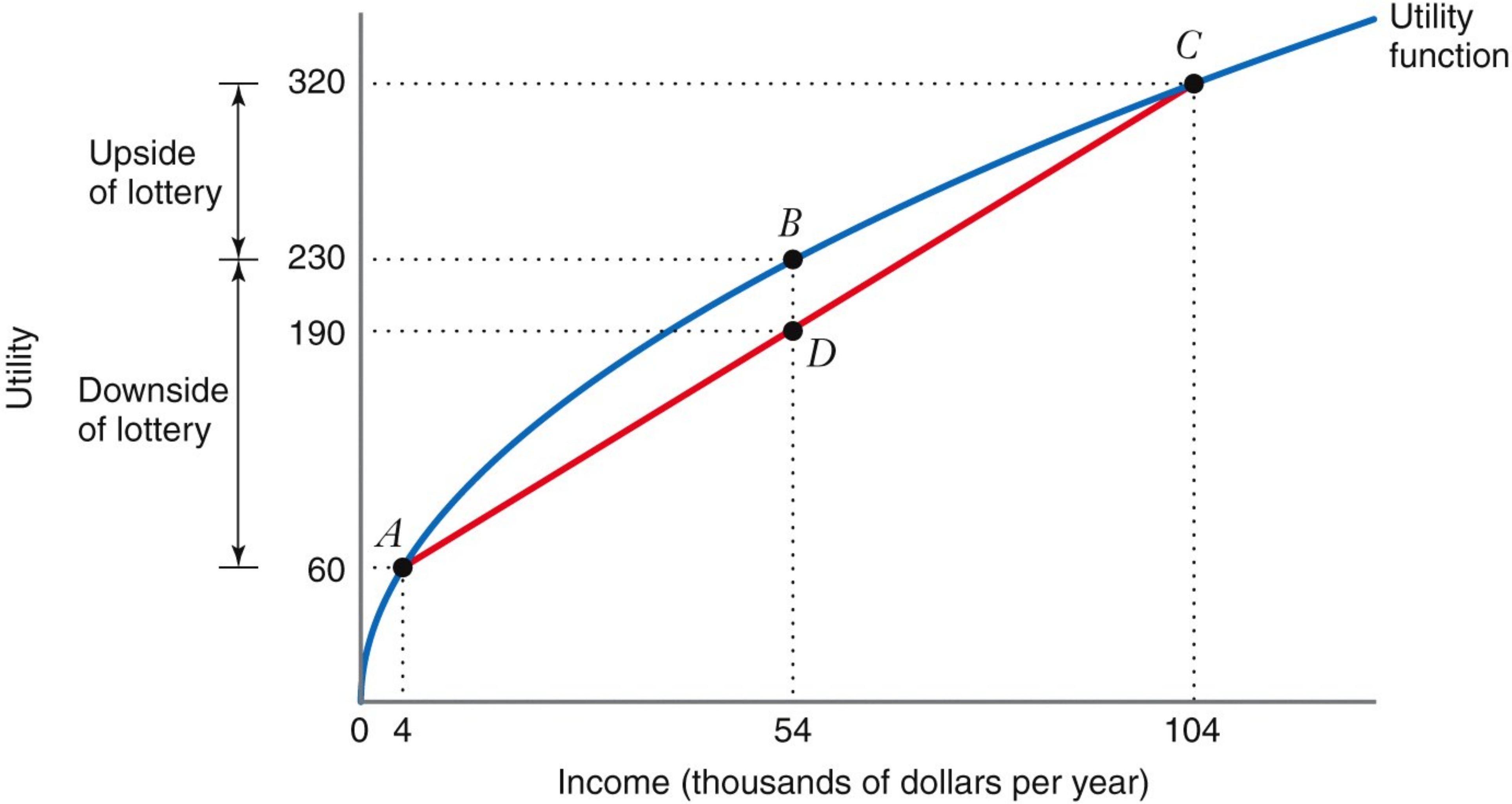
Evaluating Risky Outcomes

EV(choice 1) = 54.000

EV(choice 2) = 0.5 X 4.000 + 0.5 X 104.000 = 54.000

Evaluating Risky Outcomes

$$D \;\rightarrow\; 0.5\times U(4) + 0.5\times U(104) = 0.5\times 60 + 0.5\times 320 = 190$$



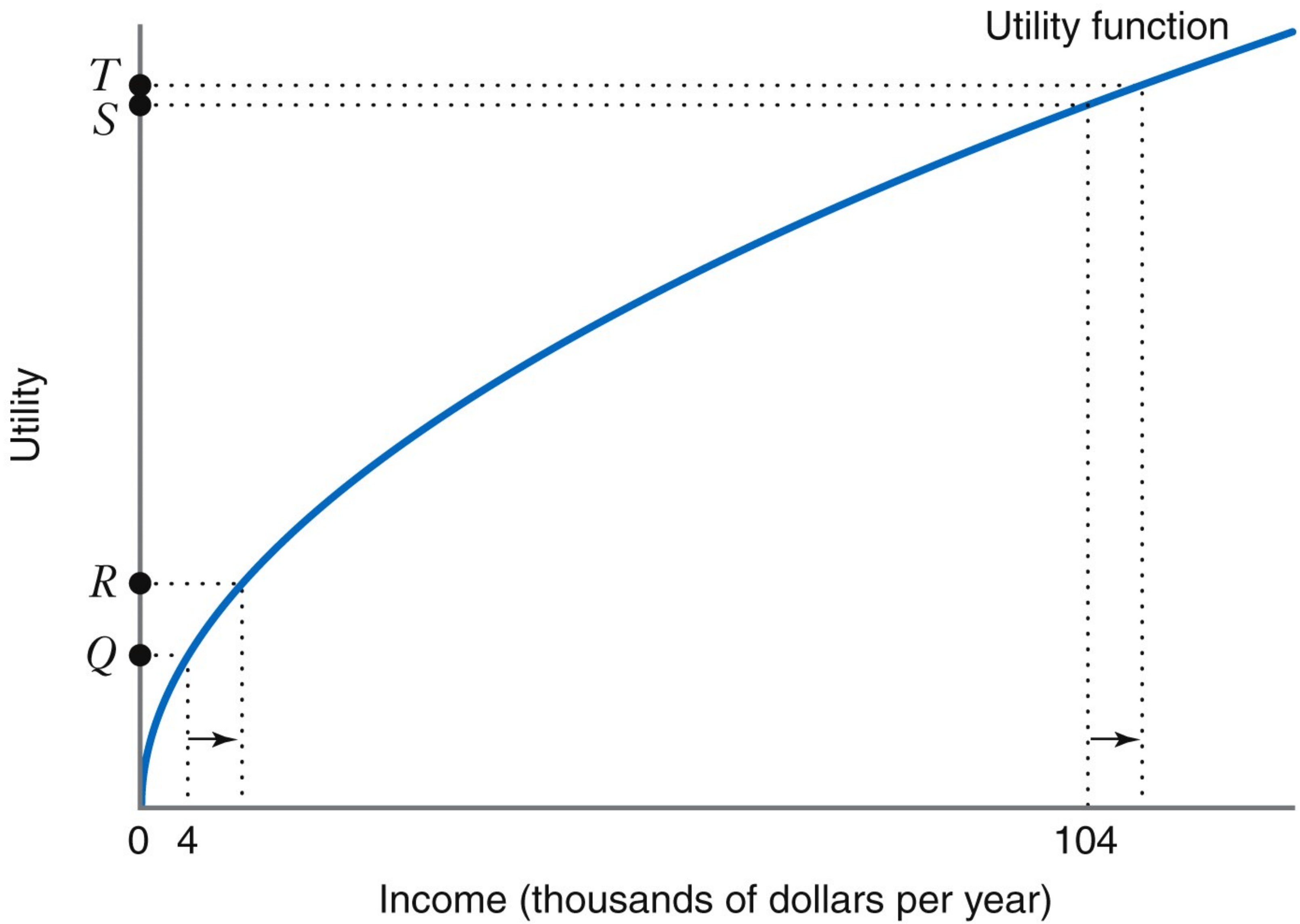
Risk Preferences

- An individual who prefers a sure thing to a lottery with the same expected value is risk averse

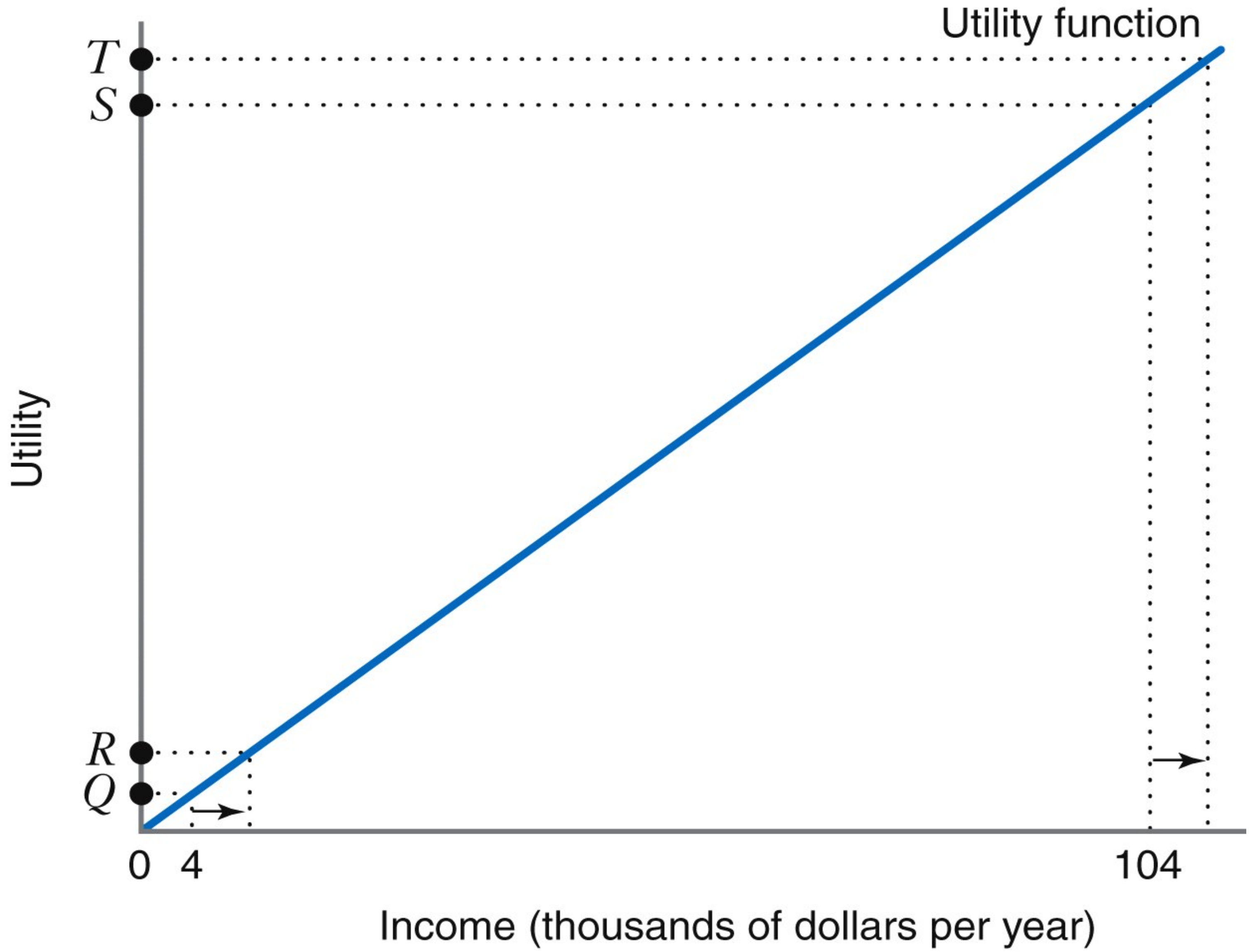
- An individual who is indifferent about a sure thing or a lottery with the same expected value is risk neutral

- An individual who prefers a lottery to a sure thing that equals the expected value of the lottery is risk loving (or risk preferring)

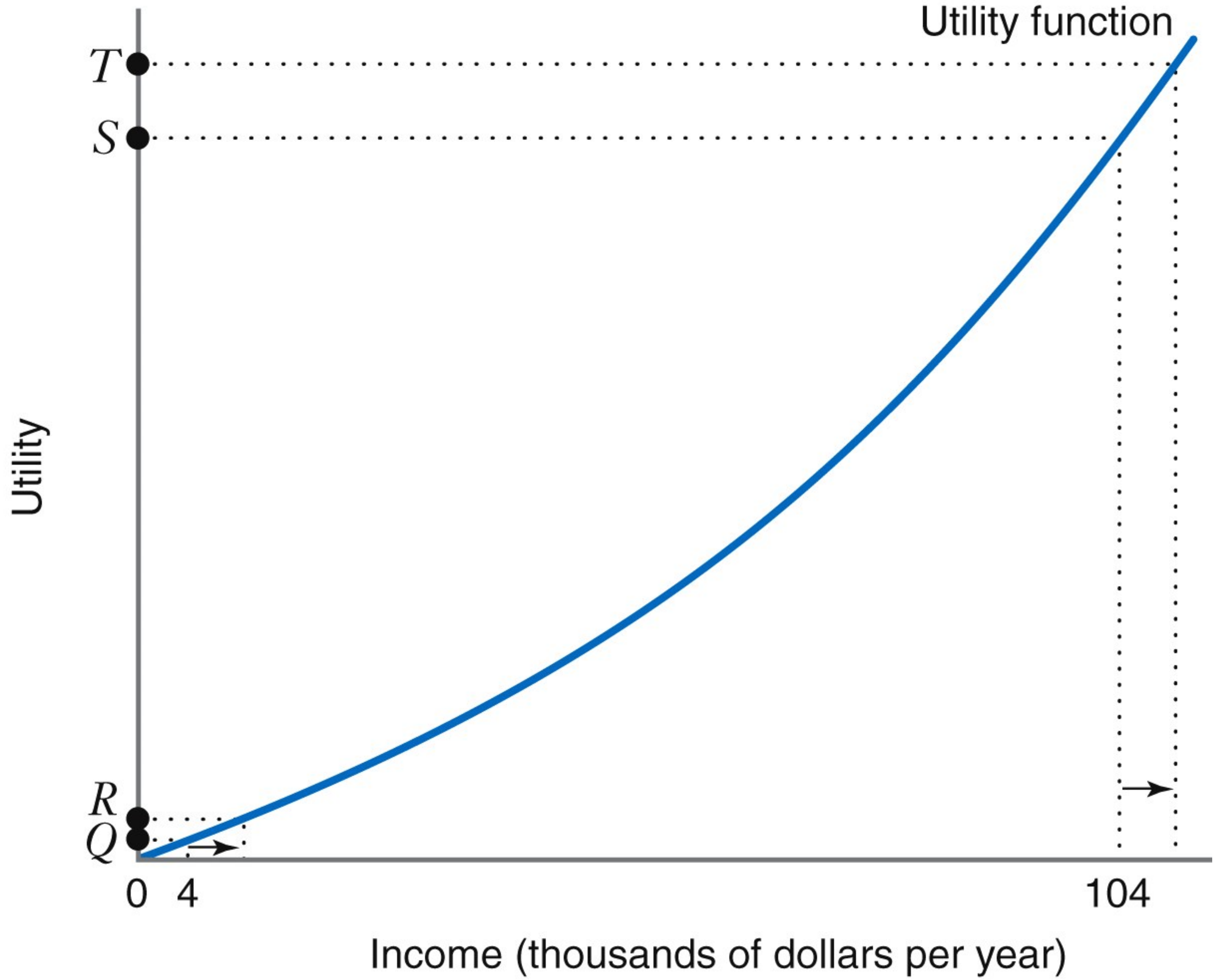
Risk Averse Decision Maker



Risk Neutral Decision Maker



Risk Loving Decision Maker

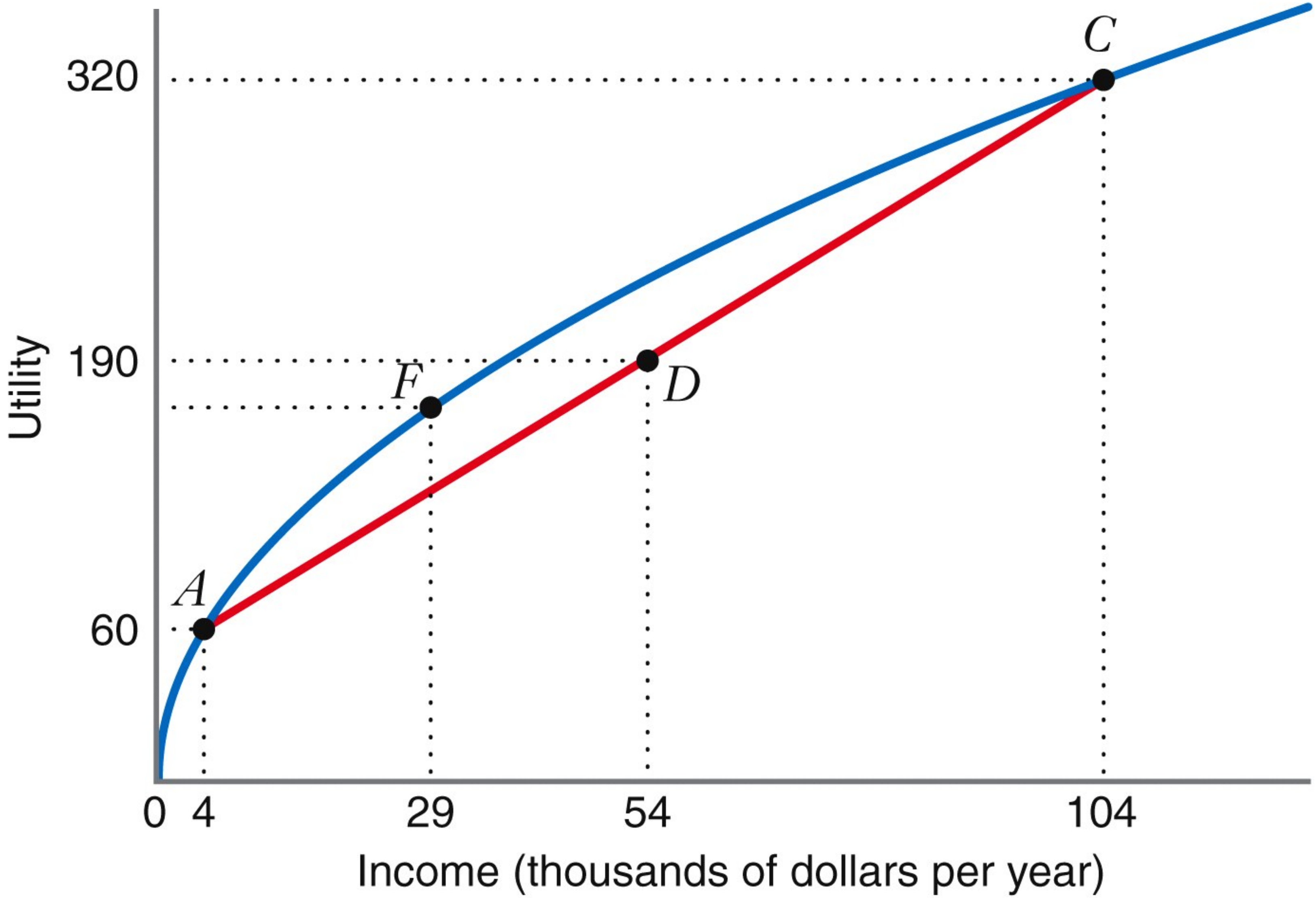


Bearing or Avoiding Risk?

- Will a risk averse decision maker always avoid risk (when possible)?
- Consider the previous example about job offers and assume expected salary from start-up is larger than sure salary from established firm
- Is it possible that a risk averse decision maker accept the uncertainty associated to the first job offer?

Bearing or Avoiding Risk?

In this case, sure salary is 29.000



Risk Premium

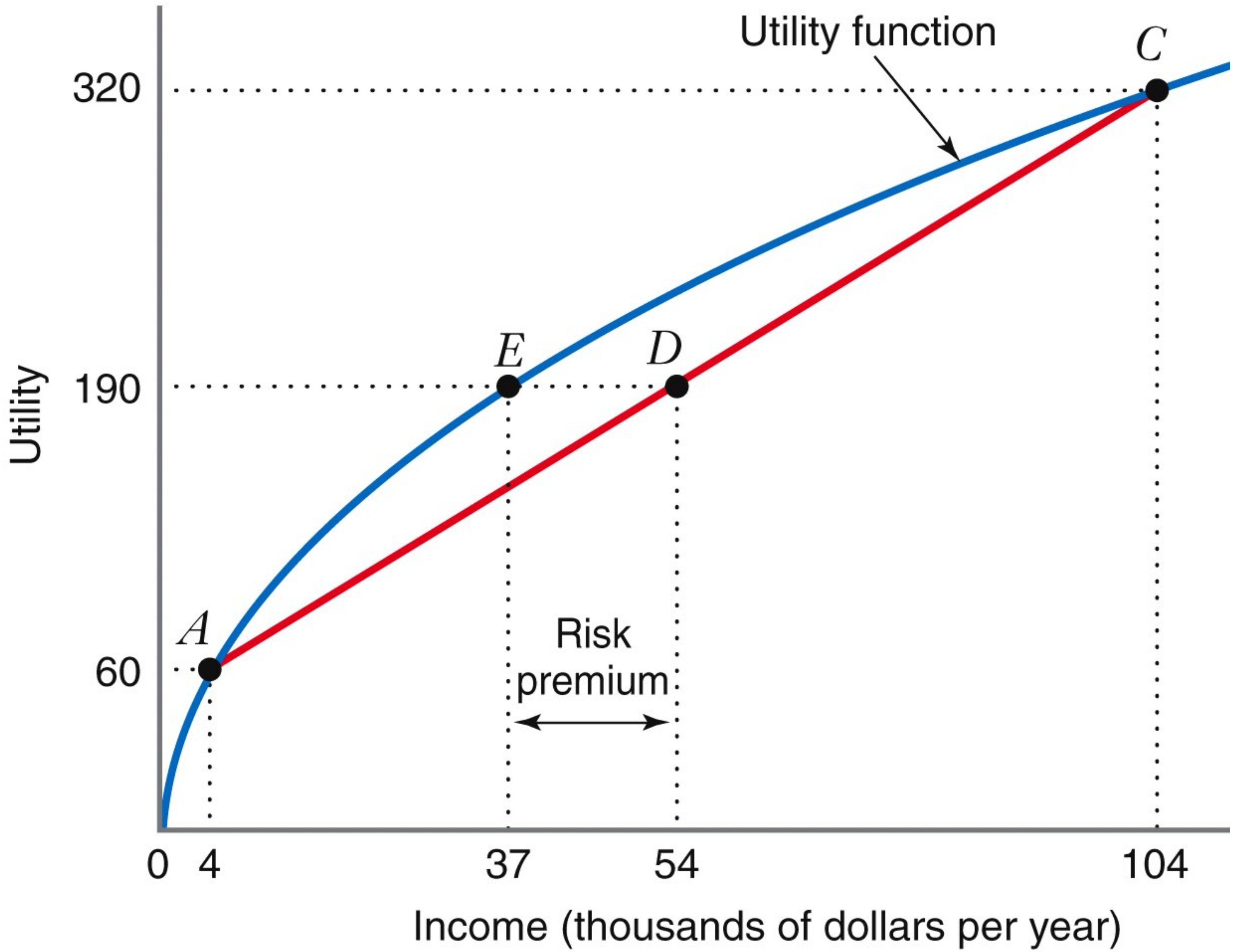
- The **risk premium** (RP) of a lottery is the necessary difference between the expected value of a lottery and the sure thing so that the decision maker is *indifferent* between the lottery and the sure thing:

$$pU(I_1) + (1-p)U(I_2) = U(pI_1 + (1-p)I_2 - RP)$$

- This is equivalent to:

$$pU(I_1) + (1-p)U(I_2) = U(EV - RP)$$

Risk Premium



The Demand for Insurance

- Consider the following scenario: a decision maker
  - has disposable income equal to 50.000
  - may incur in a loss equal to 10.000 with probability 0.05
  - may purchase an insurance policy which rembourses 10.000 is case of accident for a price (*insurance premium*) of 500
- In this case, the insurance policy:
  - provides full coverage
  - is *fairly priced*: the premium equals the expected value of the loss

The Demand for Insurance

- In this case, the decision maker faces two lotteries (in thousands):

(L1) *no insurance*: 50 with probability 0.95 and 40 with probability 0.05

(L2) *insurance*: 49.5 with probability 0.95 and 49.5 with probability 0.05

- Expected value of lottery (L1) is equal to:

$$0.95 \times 50 + 0.05 \times 40 = 47.5 + 2 = 49.5$$

- A risk-averse decision maker prefers (L2), hence will buy the fairly priced insurance policy