

MATHEMATICS II  
Thursday March 8 2018  
Second Exercise Class

1. Find local maxima/minima of the following functions.

a.

$$F(x) = \int_1^x \frac{\ln(t)}{1+t} dt$$

.

b.

$$F(x) = \int_{-1}^x \ln(t^3 + 2) dt$$

.

c.

$$F(x) = \int_3^x \frac{\ln(t^2 - 1)}{\ln(t^2 - 4)} dt$$

.

d.

$$F(x) = \int_0^x \frac{e^t - e^{-t}}{e^t + e^{-t}} dt$$

.

2. Compute

a.

$$\lim_{x \rightarrow +\infty} \frac{\int_0^x e^{t^2} dt}{e^{x^2}}$$

.

b.

$$\lim_{x \rightarrow +\infty} \frac{\int_0^x t^2 \sin(t) dt}{\arctan(x)}$$

.

3. Decide if the function

$$F(x) = \int_{-1}^x \sqrt{1+t^3} dt$$

admits an inverse function  $F^{-1}(x)$ .

4. Given the matrices

$$A = \begin{pmatrix} 1 & 4 & -1 \\ 0 & 2 & -2 \\ 3 & -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 2 \\ -3 & 0 \\ -2 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 0 \\ -2 & 0 & -1 \\ 0 & -3 & 1 \end{pmatrix},$$

- calculate  $2A + 3C$ ,
- verify that  $(A - B) \cdot C = A \cdot C - B \cdot C$ ,
- compute  $C^2$  and  $C \cdot C^T$ .

5. Given the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix},$$

- recognize that  $(A \cdot B)^T = B^T \cdot A^T$ .
- determine  $A^{-1}$ ,  $B^{-1}$ ,
- determine  $A^{-1} \cdot B^{-1}$ , and compare it to  $(B \cdot A)^{-1}$ .

6. Recognize if it is possible to realize the product between the indicated matrices, and when possible calculate it:

- $A = \begin{pmatrix} -1 & 5 & -2 & 0 \\ 3 & 2 & 0 & -6 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & -2 & -1 \\ 1 & 0 & 7 \\ 0 & -3 & 1 \\ 2 & -4 & -1 \end{pmatrix}$
- $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & -5 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & -4 & 5 \\ 2 & 1 & 0 \\ -6 & 0 & 3 \end{pmatrix}$
- $A = \begin{pmatrix} 1 & 6 & 2 \\ 7 & 3 & -1 \end{pmatrix}$  and  $B = \begin{pmatrix} 4 & -6 \\ 1 & 3 \end{pmatrix}$
- $A = \begin{pmatrix} 3 & 7 \\ -2 & -1 \end{pmatrix}$  and  $B = \begin{pmatrix} 4 & 0 & 9 \\ 0 & 6 & 3 \\ 2 & 1 & -8 \end{pmatrix}$  and

7. Given the matrix

$$A = \begin{pmatrix} -3 & 4 & 0 & 1 \\ -1 & 1 & 2 & 6 \\ 0 & -4 & 7 & 5 \\ 1 & -1 & 2 & 3 \end{pmatrix}$$

write the minors  $M_{13}, M_{22}, M_{32}, M_{44}$ .

8. Calculate the determinant of the following matrices.

- $C = \begin{pmatrix} 2 & 1 & -1 \\ -3 & 0 & 3 \\ 0 & 1 & -2 \end{pmatrix}$
- $D = \begin{pmatrix} -5 & -1 & 3 \\ 5 & 2 & -4 \\ 0 & 0 & 1 \end{pmatrix}$
- $E = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -2 & 4 \\ 7 & -1 & -3 \end{pmatrix}$
- $G = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -2 & 0 \\ 4 & 0 & 0 \end{pmatrix}$
- $H = \begin{pmatrix} 1 & 3 & 1 & 5 \\ 3 & 4 & 5 & 6 \\ 2 & 1 & -2 & 0 \\ 4 & 2 & 1 & 3 \end{pmatrix}$

9. For which values of  $\alpha$  is it  $A^{-1} = A^T$ ? (such a matrix  $A$  is called *orthogonal*)

$$\begin{pmatrix} \frac{1}{2} & \alpha \\ -\alpha & \frac{1}{2} \end{pmatrix}$$