MATHEMATICS II
Thursday March 8 2018
Second Exercise Class

1. Find local maxima/minima of the following functions.

   a.

$$F(x) = \int_1^x \frac{\ln(t)}{1+t} dt$$

.

SOLUTION: $x = 0$ min. b.

$$F(x) = \int_{-1}^x \ln(t^3+2) dt$$

.

SOLUTION: $x = -\sqrt[3]{2}$ min.

c.

$$F(x) = \int_3^x \frac{\ln(t^2-1)}{\ln(t^2-4)} dt$$

.

SOLUTION: $x = 3$ min.

d.

$$F(x) = \int_0^x \frac{e^t - e^{-t}}{e^t + e^{-t}} dt$$

.

SOLUTION: $x = 0$ min.

2. Compute

   a.

$$\lim_{x\to+\infty} \frac{\int_0^x e^{t^2}\,dt}{e^{x^2}}$$

.

   SOLUTION: 0.

   b.

$$\lim_{x\to+\infty} \frac{\int_0^x t^2 \sin(t)dt}{\arctan(x)}$$

.

   SOLUTION: $\nexists$.

3. Decide if the function

$$F(x) = \int_{-1}^{x} \sqrt{1+t^3}dt$$

   admits an inverse function $F^{-1}(x)$.

   SOLUTION: yes.

4. Given the matrices

$$A = \begin{pmatrix} 1 & 4 & -1 \\ 0 & 2 & -2 \\ 3 & -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 2 \\ -3 & 0 \\ -2 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 0 \\ -2 & 0 & -1 \\ 0 & -3 & 1 \end{pmatrix},$$

   - calculate $2A+3C$,
     SOLUTION:
     $\begin{pmatrix} 5 & 14 & -2 \\ -6 & 4 & -7 \\ 6 & -11 & 3 \end{pmatrix}$,
   - verify that $(A-C)\cdot B = A\cdot B - C\cdot B$,
     SOLUTION:
     $(A-C)\cdot B = \begin{pmatrix} -4 & -3 \\ -4 & 1 \\ -4 & 3 \end{pmatrix}, A\cdot B = \begin{pmatrix} -10 & -1 \\ -2 & -6 \\ 3 & 6 \end{pmatrix}, C\cdot B = \begin{pmatrix} -6 & 2 \\ 2 & -7 \\ 7 & 3 \end{pmatrix}$

- compute C^2 and $C \cdot C^T$.
  SOLUTION:
  $C \cdot C = \begin{pmatrix} -3 & 2 & 2 \\ -2 & -1 & -1 \\ 6 & -3 & 4 \end{pmatrix}, \quad C \cdot C^T = \begin{pmatrix} 5 & -2 & -6 \\ -2 & 5 & -1 \\ -6 & -1 & 10 \end{pmatrix}$

5. Given the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix},$$

- recognize that $(A \cdot B)^T = B^T \cdot A^T$.
  SOLUTION:
  $(A \cdot B)^T = B^T \cdot A^T = \begin{pmatrix} 5 & -1 \\ 4 & -2 \end{pmatrix},$
- determine A^{-1}, B^{-1},
  SOLUTION:
  $A^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} \frac{1}{3} & 0 \\ -\frac{1}{6} & \frac{1}{2} \end{pmatrix}$
- determine $A^{-1} \cdot B^{-1}$, and compare it to $(B \cdot A)^{-1}$.
  SOLUTION:
  $A^{-1} \cdot B^{-1} = (B \cdot A)^{-1} = \begin{pmatrix} 0 & 1 \\ \frac{1}{6} & -\frac{1}{2} \end{pmatrix}$

6. Recognize if it is possible to realize the product between the indicated matrices, and when possible calculate it:

- $A = \begin{pmatrix} -1 & 5 & -2 & 0 \\ 3 & 2 & 0 & -6 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -2 & -1 \\ 1 & 0 & 7 \\ 0 & -3 & 1 \\ 2 & -4 & -1 \end{pmatrix}$
  SOLUTION:
  $A \cdot B = \begin{pmatrix} 5 & 8 & 34 \\ -10 & 18 & 17 \end{pmatrix}$
- $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & -5 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -4 & 5 \\ 2 & 1 & 0 \\ -6 & 0 & 3 \end{pmatrix}$
  SOLUTION:
  $A \cdot B = \begin{pmatrix} 2 & -11 & 15 \\ -22 & -5 & 6 \end{pmatrix}$
- $A = \begin{pmatrix} 1 & 6 & 2 \\ 7 & 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -6 \\ 1 & 3 \end{pmatrix}$
  SOLUTION: $\nexists A \cdot B$

- $A = \begin{pmatrix} 3 & 7 \\ -2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 0 & 9 \\ 0 & 6 & 3 \\ 2 & 1 & -8 \end{pmatrix}$

  SOLUTION: $\nexists A \cdot B$

7. Given the matrix

$$A = \begin{pmatrix} -3 & 4 & 0 & 1 \\ -1 & 1 & 2 & 6 \\ 0 & -4 & 7 & 5 \\ 1 & -1 & 2 & 3 \end{pmatrix}$$

write the minors M_{13},M_{22}, M_{32}, M_{44}.

SOLUTION:

$M_{13} = \begin{pmatrix} -1 & 1 & 6 \\ 0 & -4 & 5 \\ 1 & -1 & 3 \end{pmatrix}$, $M_{22} = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 7 & 5 \\ 1 & 2 & 3 \end{pmatrix}$, $M_{32} = \begin{pmatrix} 3 & 0 & 1 \\ -1 & 2 & 6 \\ 1 & 2 & 3 \end{pmatrix}$, $M_{44} = \begin{pmatrix} 3 & 4 & 0 \\ -1 & 1 & 2 \\ 0 & -4 & 7 \end{pmatrix}$

8. Calculate the determinant of the following matrices.

- $C = \begin{pmatrix} 2 & 1 & -1 \\ -3 & 0 & 3 \\ 0 & 1 & -2 \end{pmatrix}$

  SOLUTION: -9
- $D = \begin{pmatrix} -5 & -1 & 3 \\ 5 & 2 & -4 \\ 0 & 0 & 1 \end{pmatrix}$

  SOLUTION: -5
- $E = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -2 & 4 \\ 7 & -1 & -3 \end{pmatrix}$

  SOLUTION: 14
- $G = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -2 & 0 \\ 4 & 0 & 0 \end{pmatrix}$

  SOLUTION: 8
- $H = \begin{pmatrix} 1 & 3 & 1 & 5 \\ 3 & 4 & 5 & 6 \\ 2 & 1 & -2 & 0 \\ 4 & 2 & 1 & 3 \end{pmatrix}$

  SOLUTION: 65

9. For which values of α is it $A^{-1} = A^T$? (such a matrix A is called *orthogonal*)

$$\begin{pmatrix} \frac{1}{2} & \alpha \\ -\alpha & \frac{1}{2} \end{pmatrix}$$

SOLUTION: $\alpha = \pm\frac{\sqrt{3}}{2}$