

MATHEMATICS II
Thursday March 22 2018
Fourth Exercise Class

1. Determine the rank of the following sets of vectors.

a) $\vec{v}_1 = (1, 2, 0), \vec{v}_2 = (2, 0, 1), \vec{v}_3 = (1, 1, 0), \vec{v}_4 = (1, -1, 2).$

b) $\vec{v}_1 = (2, 0, 1, 0), \vec{v}_2 = (-1, 1, 0, 1), \vec{v}_3 = (2, -2, 0, 0), \vec{v}_4 = (0, 1, 3, 0).$

c) $\vec{v}_1 = (0, 0, 1, 1, 0), \vec{v}_2 = (1, 1, 0, 0, 1), \vec{v}_3 = (1, 0, 1, 0, 1).$

2. Determine the rank of the following set of vectors on the varying parameter α .

$\vec{v}_1 = (\alpha, 1, 1), \vec{v}_2 = (1, \alpha, 1), \vec{v}_3 = (0, 1, \alpha).$

3. Determine the rank of the following matrices.

a)

$$\begin{pmatrix} 1 & -1 & 1 \\ -1 & -3 & 1 \\ 4 & 4 & 0 \end{pmatrix}$$

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b)

$$\begin{pmatrix} 2 & 3 & -1 & 1 \\ 1 & -2 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ -1 & -3 & 3 & 1 \end{pmatrix}$$

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4. Determine the rank of the following matrices on the varying parameter α .

a)

$$\begin{pmatrix} 0 & 1 - \alpha & 2 \\ 1 + \alpha & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

b)

$$\begin{pmatrix} 0 & \alpha & 1 \\ \alpha + 2 & 0 & -1 \\ 1 & 2 & 0 \end{pmatrix}$$

5. Study the following systems of equations. For each of them determine the solutions if they exist.

a)

$$\begin{cases} x + y - z = 1 \\ 2x - y - 2z = -1 \\ -x + y - z = 2 \end{cases}$$

b)

$$\begin{cases} 2x - 5y + 2z = 1 \\ 3x - 2y + z = 4 \\ x + 3y - z = -3 \end{cases}$$

c)

$$\begin{cases} 2x - 3z = 4 \\ -y + 4w = -3 \\ x - 2y = -2 \\ 2w + z = 2 \end{cases}$$

d)

$$\begin{cases} 2x - 2y = 0 \\ x + 2z = 1 \\ 3x - w = 2 \\ w + z = -1 \end{cases}$$

6. Study the following systems of equations on the varying parameter λ and μ .

a)

$$\begin{cases} x - y - \lambda x = 0 \\ -x + y - \lambda y = 0 \end{cases}$$

b)

$$\begin{cases} y + z = 1 \\ 2x + 2z = \lambda \\ x + z = 1 \end{cases}$$

c)

$$\begin{cases} y + z = 1 \\ 2x + 2z = \lambda \\ x + y + z = 1 \end{cases}$$

d)

$$\begin{cases} y + z = 1 \\ 2x + 2z = \lambda \\ x + \gamma y + z = 1 \end{cases}$$

e)

$$\begin{cases} x + y + z = \lambda x \\ 2x - y + z = \lambda y \\ -x + 2y + 2z = \lambda z \end{cases}$$