

MATHEMATICS II
Thursday, March 29 2018
Fifth Exercise Class

1. Given the set of vectors $F = \{(1, 3, 0), (1, -1, 2), (0, 0, 1)\}$, verify that F is a base for \mathbb{R}^3 . Moreover, given the vector v whose coordinates in F are $(2, -1, 0)$, write its coordinates in the natural base.
2. Determine the value for the parameter h such that the following matrix A admits eigenvalue $\lambda = 1$. For this value of h determine the eigenvalues of A :

$$D = \begin{pmatrix} h & 1 & 0 \\ 1-h & 0 & 2 \\ 1 & 1 & h \end{pmatrix}$$

3. Calculate the eigenvalues and the eigenvectors of the following matrices

$$\begin{aligned} A &= \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} & B &= \begin{pmatrix} 4 & 1 & 1 \\ -3 & 0 & -1 \\ -1 & -1 & 2 \end{pmatrix} \\ C &= \begin{pmatrix} 2 & -3 & 0 \\ -1 & 0 & 0 \\ -1 & 1 & 1 \end{pmatrix} & D &= \begin{pmatrix} 4 & 6 & 0 \\ -3 & -5 & 0 \\ -3 & -6 & -5 \end{pmatrix} \end{aligned}$$

4. Determine the algebraic and the geometric multiplicity of the eigenvalues of the following matrices

$$\begin{aligned} A &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} & B &= \begin{pmatrix} 2 & 0 & 0 \\ -3 & -1 & 3 \\ 0 & 0 & 2 \end{pmatrix} \\ C &= \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix} & D &= \begin{pmatrix} -7 & -3 & -16 \\ -2 & 0 & -2 \\ 6 & 3 & 15 \end{pmatrix} \end{aligned}$$

5. Verify that the following matrices are similar and determine the invertible matrix M such that $B = M \cdot A \cdot M^{-1}$

$$A = \begin{pmatrix} 4 & -2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

6. Determine whether the following matrices are diagonalizable. If yes, diagonalize it.

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -2 \end{pmatrix} \qquad D = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 4 & 1 \end{pmatrix}$$