

MATHEMATICS 2

Sixth Practice

Exercise 1: Identify the domain of the following functions

$$f(x, y) = \log(1 - x^2 - y^2)$$

$$\text{sol} : x^2 + y^2 < 1$$

$$f(x, y) = \sqrt{x^2 + 4y^2 - 4}$$

$$\text{sol} : x^2 + 4y^2 - 4 \geq 0$$

$$f(x, y) = e^{\frac{3}{x-2y}}$$

$$\text{sol} : y \neq \frac{x}{2}$$

$$f(x, y) = \frac{3x + 2y}{4x^2 - y^2}$$

$$\text{sol} : y \neq \pm 2x$$

$$f(x, y) = e^{\frac{y}{\sqrt{x^2 - 4}}}$$

$$\text{sol} : x < -2 \vee x > 2$$

$$f(x, y) = \frac{\sin xy^2}{\sqrt{1 - 4x^2 - 9y^2}}$$

$$\text{sol} : 4x^2 + 9y^2 < 1$$

Exercise 2: Write the level curves of the following functions:

$$f(x, y) = \log(1 - x^2 - y^2)$$

$$\text{sol} : x^2 + y^2 = 1 - e^c$$

$$f(x, y) = \sqrt{x^2 + 4y^2 - 4}$$

$$\text{sol} : x^2 + 4y^2 = 4 + c^2$$

$$f(x, y) = e^{\frac{3}{x-2y}}$$

$$\text{sol} : y = x/2 - 3/\log c$$

$$f(x, y) = \frac{1}{4x^2 + y^2}$$

$$\text{sol} : 4x^2 + y^2 = 1/c$$

Exercise 3: Compute the partial derivatives and write the gradient of the following functions:

$$f(x, y) = 3x^2y^3 - 2x^2y^4$$

$$\text{sol} : f_x = 6xy^3 - 4xy^4;$$

$$f_y = 9x^2y^2 - 8x^2y^3$$

$$f(x, y) = e^{x^2y}$$

$$\text{sol} : f_x = e^{x^2y}2xy$$

$$f_y = e^{x^2y}x^2$$

$$f(x, y) = \log(1 - x^2 - y^2)$$

$$\text{sol} : f_x = \frac{-2x}{x^2 + y^2 - 1}$$

$$f_y = \frac{-2y}{x^2 + y^2 - 1}$$

$$f(x, y) = \arctan(2x^3y^2)$$

$$\text{sol} : f_x = \frac{6x^2y^2}{1 + 4x^6y^4}$$

$$f_y = \frac{4x^3y^2}{1 + 4x^6y^4}$$

$$f(x, y) = x^2 \sin(2x - 3y)$$

$$\text{sol} : f_x = 2x \sin(2x - 3y) + 2x^2 \cos(2x - 3y)$$

$$f_y = -3x^2 \cos(2x - 3y)$$

Exercise 4: Find the stationary points of the following functions and establish if they are local min, local max or saddle points:

$$f(x, y) = x^2 + xy + y^2 - 2x - y$$

sol : (1, 0) local min

$$f(x, y) = (x - 1)^2 - y^2$$

sol : (1, 0) saddle point

$$f(x, y) = e^{(x - y)(x^2 - 2y^2)}$$

sol : (0, 0) saddle point

(-4, -2) local max

$$f(x, y) = x^2 + y^2 + xy + x$$

sol : (-2/3, 1/3) local min

$$f(x, y) = x^3 + y^3 - 3xy$$

sol : (0, 0) saddle point

(1, 1) local min