MATHEMATICS 2

Exam

15/01/2019, A.Y. 2017/2018

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1) *(8 p.ts)* Solve the following integral

$$\int x^4 \ln(x)dx$$

By integration by parts

$$\int x^4 \ln(x)dx = \frac{x^5}{5} - \frac{1}{5}\int x^5 \cdot \frac{1}{x}dx = \frac{x^5}{5} - \frac{1}{5}\int x^4 dx = \frac{x^5}{5} - \frac{1}{25}x^5 + c$$

2) *(10 p.ts)* Find eigenvalues and eigenvectors of the following matrix and determine if it is diagonalizable. If so, identify the invertible matrix T that transforms A into a diagonal matrix, and show how T realizes this transformation.

$$A = \begin{pmatrix} 0 & 4 & 6 \\ 0 & -3 & -5 \\ -5 & -3 & -6 \end{pmatrix}$$

Let us look for Real eigenvalues

$$|A - \lambda| = \begin{vmatrix} -\lambda & 4 & 6 \\ 0 & -3-\lambda & -5 \\ -5 & -3 & -6-\lambda \end{vmatrix} = -\lambda^3 - 9\lambda^2 - 33\lambda + 10 = -(\lambda^3 + 9\lambda^2 + 33\lambda - 10) = 0$$

Let us try to apply the remainder Theorem to evaluate the presence of rational zeroes of the characteristic polynomial. Therefore all the possible rational zeroes must be searched among the divisors of the given term -10: hence $\{\pm 1; \pm 2; \pm 5; \pm 10\}$. We test these possibilities by substituting them in the characteristic polynomial $P(\lambda) = -(\lambda^3 + 9\lambda^2 + 33\lambda - 10)$:

- $P(1) = -33$
- $P(-1) = 35$
- $P(2) = -100$
- $P(-2) = +48$
- $P(-5) = 75$
- $P(5) = -505$
- $P(10) = -2220$
- $P(-10) = 440$

none of the previous cases cancel $P(\lambda)$, hence, as $P(\lambda)$ is a third degree polynomial with a strictly increasing graph we may deduce that it has a single real zero that is an irrational number and the remaining zeroes are complex numbers; hence the given matrix is not diagonalizable in $\mathbb{R}$.

3) *(10 p.ts)* Given the function

$$f(x,y,z) = 4x^2 + 2y^2 + z^2 - xy + yz - 5x - 9y + z$$

, find max and/or min and say if they are local or global.

Let us search for critical points by cancelling the first partial derivatives

$$\begin{cases} f_x & = 8x - y - 5 = 0 \\ f_y & = 4y - x + z - 9 = 0 \\ f_z & = 2z + y + 1 = 0 \end{cases}$$

the previous system is solved by $x = 1$, $y = 3$, $z = -2$. The Hessian matrix is

$$H(x,y,z) = \begin{pmatrix} 8 & -1 & 0 \\ -1 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix},$$

notice that $H(x,y,z)$ doesn't depend on variables, hence eventual max/min will be global. The second order conditions are

1. $|f_{xx}| = 8 > 0$;
2. $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 8 & -1 \\ -1 & 4 \end{vmatrix} = 32 - 1 = 31 > 0$;
3. $|H(x,y,z)| = \begin{vmatrix} 8 & -1 & 0 \\ -1 & 4 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 54 > 0$

The positivity of the previous Principal Minors implies that the point $(1,3,-2)$ is a global minimum.