

Mathematics II

Second Practice

1. Let A, B and C be the following matrices:

$$A := \begin{pmatrix} 2 & 3 \\ 4 & 0 \\ 1 & -1 \end{pmatrix}, \quad B := \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix}, \quad C := \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 1 & 4 \end{pmatrix}$$

Compute (if possible):

$$\begin{array}{lll} a) & 2A + C & b) \quad A + B + C \quad c) \quad A^T \cdot C \\ d) & (A^T \cdot C)^{-1} & e) \quad C^T \cdot A + B \quad f) \quad A \cdot C^T + B \\ g) & \det(A^T \cdot C) + B & h) \quad B + B^{-1} \quad i) \quad B^{-1} \cdot (A \cdot C^T) \end{array}$$

2. Compute $A \cdot B$ when such an operation is possible:

$$a) \quad A := \begin{pmatrix} 3 & 3 \\ -1 & 1 \\ 1 & 0 \end{pmatrix}, \quad B := \begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix}$$

$$b) \quad A := \begin{pmatrix} 3 & -2 & -4 \\ -1 & 1 & 2 \\ -1 & 1 & -1 \end{pmatrix}, \quad B := \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 2 & -2 & -3 & 1 \end{pmatrix}$$

$$c) \quad A := (1 \ 0 \ 0), \quad B := \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 5 \end{pmatrix}$$

$$d) \quad A := \begin{pmatrix} 1 & 1 & -\frac{1}{3} & \frac{1}{6} \\ 1 & -1 & \frac{4}{3} & -\frac{1}{6} \end{pmatrix}, \quad B := \begin{pmatrix} 1 & 2 & 0 \\ 1 & -1 & 1 \\ 1 & 3 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

3. Calculate the following determinants:

$$a) \begin{vmatrix} 2 & 1 & 2 \\ 0 & 3 & -1 \\ 4 & 1 & 1 \end{vmatrix}, \quad b) \begin{vmatrix} -1 & 5 & 3 \\ 4 & 0 & 0 \\ 2 & 7 & 8 \end{vmatrix}, \quad c) \begin{vmatrix} 3 & 1 & 1 \\ 2 & 5 & 5 \\ 8 & 9 & 9 \end{vmatrix}$$

$$d) \begin{vmatrix} 1 & 1 & -2 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & -1 & 1 & 0 \\ 3 & 1 & 2 & 5 \end{vmatrix}, \quad e) \begin{vmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & -2 & 1 & 4 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & -1 & 1 & -1 & 0 \\ 3 & 1 & 2 & 1 & 5 \end{vmatrix}$$

4. Given in \mathbb{R}^3 the vectors $\vec{v} = (1, 0, -1)$, $\vec{w} = (2, 2, -1)$ and $\vec{z} = (-2, 1, 0)$, calculate:

- a) $\vec{v} + 2\vec{w} - 3\vec{z}$;
- b) $(\vec{v} \cdot \vec{w}) \vec{z} - \vec{w}$;
- c) $(\vec{v} \times \vec{w}) \cdot \vec{z}$;
- d) the length of each vector.

5. Given in \mathbb{R}^3 the vectors $\vec{v} = (1, 0, -1)$, $\vec{w} = (2, 2, -1)$ and $\vec{z} = (0, -2, -1)$, calculate the values of the parameters α, β, γ , such that: $\alpha\vec{v} + \beta\vec{w} + \gamma\vec{z} = \vec{0}$.

6. Given the vectors $\vec{v} = (k, 1, -1)$ and $\vec{w} = (2, 2k, -1)$ determine values for k such that their inner product is zero.

7. For each of the following vectors find the corresponding versor

- a) $\vec{v} = (1, \alpha)$;
- b) $\vec{v} = (-\alpha, \alpha)$;
- c) $\vec{v} = (1, \alpha, 2\alpha)$.

8. Determine the parametric equation and the Cartesian equation of the line on the space

- a) passing through the points $A(1, 0, 2)$ and $B(1, 3, 0)$;
- b) passing through the point $P(8, 0, 1)$ and parallel to the vector $\vec{v} = (0, 10, 0)$;
- c) passing through the point $P(-1, 0, 4)$ and parallel to the line of parametric equations $x = 2t, y = -t, z = 3t$.

9. Determine the reciprocal position of the lines r and s of Cartesian equations

$$\begin{cases} x + y + z = 0 \\ y = 2 \end{cases} \quad ; \quad \begin{cases} x + 2y + z = 2 \\ y = 3 \end{cases}$$

10. Establish if the vectors $\vec{v}_1 = (1, 1, 1)$, $\vec{v}_2 = (1, 0, 0)$, $\vec{v}_3 = (0, 1, 0)$ and $\vec{v}_4 = (2, 1, 1)$ are linearly dependent.
11. Determine for which value of k the following vectors in \mathbb{R}^4 are linearly independent

$$\vec{v} = (2, -1, 2, -1); \quad \vec{w} = (1, 1, 1, 1); \quad \vec{z} = (k, 0, k, 0).$$

12. For any $k \in \mathbb{R}$ compute the rank of the following matrix:

$$\begin{pmatrix} k & 1 & 8 & k \\ k & 1 & 8 & 1 \\ 2k & 2 & 16 & 2 \end{pmatrix}$$