Mathematics II

Fourth Practice: Solutions

1. Find the eigenvalues and the eigenvectors of the following matrices:

$a)\ \begin{pmatrix} 1 & -5 \\ -5 & 1 \end{pmatrix};\qquad b)\ \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix};\qquad c)\ \begin{pmatrix} 2 & 4 \\ 5 & 3 \end{pmatrix};$

$d)\ \begin{pmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{pmatrix};\ e)\ \begin{pmatrix} 3 & -1 & 1 \\ 2 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix};\ f)\ \begin{pmatrix} 3 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & -1 & -2 \end{pmatrix};$

Solution:

	Eigenvalues	Corresponding eigenvectors
$a)$	$\lambda_1 = -4;$	$v_1 = (t, t)$
	$\lambda_2 = 6;$	$v_2 = (t, -t)$

	Eigenvalues	Corresponding eigenvectors
$b)$	$\lambda_1 = -3;$	$v_1 = (t, -2t)$
	$\lambda_2 = 2;$	$v_2 = (2t, t)$

	Eigenvalues	Corresponding eigenvectors
$c)$	$\lambda_1 = -2;$	$v_1 = (t, -t)$
	$\lambda_2 = 7;$	$v_2 = (\frac{4}{5}t, t)$

	Eigenvalues	Corresponding eigenvectors
$d)$	$\lambda_1 = -2;$	$v_1 = (\frac{1}{3}t, -t, -\frac{1}{3}t)$
	$\lambda_2 = 0;$	$v_2 = (t, s, t)$

	Eigenvalues	Corresponding eigenvectors
$e)$	$\lambda = 1;$	$v_1 = (t, 2t + s, s)$

	Eigenvalues	Corresponding eigenvectors
	$\lambda_1 = -1;$	$v_1 = (\frac{1}{4}t, -t, t)$
$f)$	$\lambda_2 = 1;$	$v_2 = (\frac{5}{2}t, -3t, t)$
	$\lambda_3 = 3;$	$v_3 = (t, 0, 0)$

2. Determine if the matrices in the previous exercise are diagonalizable. For each diagonalizable matrix find a basis in which the matrix is diagonal and write it in its diagonal form.

   Solution: a) The matrix is diagonalizable. A basis of eigenvectors is given by $v_1 = (1, 1)$ and $v_2 = (1, -1)$. With respect to this basis the matrix has diagonal form:

$$\begin{pmatrix} -4 & 0 \\ 0 & 6 \end{pmatrix}$$

   b) The matrix is diagonalizable. A basis of eigenvectors is given by $v_1 = (1, -2)$ and $v_2 = (2, 1)$. With respect to this basis the matrix has diagonal form:

$$\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$$

   c) The matrix is diagonalizable. A basis of eigenvectors is given by $v_1 = (1, -1)$ and $v_2 = (4/5, 1)$. With respect to this basis the matrix has diagonal form:

$$\begin{pmatrix} -2 & 0 \\ 0 & 7 \end{pmatrix}$$

   d) The matrix is diagonalizable. A basis of eigenvectors is given by $v_1 = (1, -3, -1)$, $v_2 = (1, 0, 1)$ and $v_3 = (0, 1, 0)$. With respect to this basis the matrix has diagonal form:

$$\begin{pmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

   e) Since $m_a(1) = 3 \neq 2 = m_g(1)$ the matrix is not diagonalizable.

f) The matrix is diagonalizable. A basis of eigenvectors is given by $v_1 = (1, -4, 4)$, $v_2 = (5, -6, 2)$ and $v_3 = (1, 0, 0)$. With respect to this basis the matrix has diagonal form:

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$