Mathematics II

Sixth Practice: Solutions

1. Find all the stationary points of the following functions and classify them:

$$a) \quad f(x,y) = ye^x - 3x - y + 5$$

$$Solution: \quad \begin{array}{c} (0,3) \\ saddle \\ point \end{array}$$

$$b) \quad f(x,y) = x^3 - y^2 - 3x + 2y$$

$$Solution: \quad \begin{array}{cc} (-1,1) & (1,1) \\ max & saddle \\ & point \end{array}$$

$$c) \quad f(x,y) = x^2 + 2y^2 - 3y^3$$

$$Solution: \quad \begin{array}{cc} (0,0) & (0,4/9) \\ min & saddle \\ & point \end{array}$$

$$d) \quad f(x,y) = x^4 + y^4 - 4xy$$

$$Solution: \quad \begin{array}{ccc} (0,0) & (-1,-1) & (1,1) \\ saddle & min & min \\ point & & \end{array}$$

$$e) \quad f(x,y) = x - x^3 - 4xy^2$$

$$Solution: \quad \begin{array}{cccc} (0,-\frac{1}{2}) & (0,\frac{1}{2}) & (-\frac{\sqrt{3}}{3},0) & (\frac{\sqrt{3}}{3},0) \\ saddle & saddle & min & max \\ point & point & & \end{array}$$

$f)\quad f(x,y,z) = z^2 e^{xy}$

Solution: $(x,y,0)$, $\forall\ x,y \in \mathbb{R}^2$ absolute minima

2. Find maxima and minima of the following functions f under the corresponding constraint V:

$a)\quad f(x,y) = x + y, \quad V = \{(x,y) \in \mathbb{R}^2 :\ x^2 + y^2 = 1\}$

Solution: $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ absolute maximum

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ absolute minimum

$b)\quad f(x,y) = \sqrt{x^2 + y^2} + y^2 - 1, \quad V = \{(x,y) \in \mathbb{R}^2 :\ x^2 + y^2 = 9\}$

Solution: $(0, \pm 3)$ absolute maxima

$(\pm 3, 0)$ absolute minima

$c)\quad f(x,y) = xy, \quad V = \{(x,y) \in \mathbb{R}^2 :\ x^2 + y^2 = 1\}$

Solution: $\left(\pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{2}}{2}\right)$ absolute maxima

$\left(\pm\frac{\sqrt{2}}{2}, \mp\frac{\sqrt{2}}{2}\right)$ absolute minima

$d)\quad f(x,y,z) = y\sqrt{1 + z^2}, \quad V = \{(x,y,z) \in \mathbb{R}^3 :\ (x-1)^2 + y^2 + z^2 = 4\}$

Solution: $\left(1, \sqrt{\frac{5}{2}}, \pm\sqrt{\frac{3}{2}}\right)$ absolute maxima

$\left(1, -\sqrt{\frac{5}{2}}, \pm\sqrt{\frac{3}{2}}\right)$ absolute minima