Mathematics I A

First Practice: Solutions

1. Determine the truth values of $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{P} \wedge \mathcal{Q}$ and $\mathcal{P} \vee \mathcal{Q}$ for each of the following pairs of logical propositions:

   a) $\mathcal{P}$: A whale is an animal.
   $\mathcal{Q}$: A whale is a plant.

   b) $\mathcal{P}$: Dublin is the capital of Ireland.
   $\mathcal{Q}$: $2 > 3$.

   c) $\mathcal{P}$: 3 is an even number.
   $\mathcal{Q}$: $4 \in (-\infty, 5) \cup [6, +\infty]$.

   Solution:

	a)	*b)*	*c)*
$\mathcal{P}$	$True$	$True$	$False$
$\mathcal{Q}$	$False$	$False$	$True$
$\mathcal{P} \wedge \mathcal{Q}$	$False$	$False$	$False$
$\mathcal{P} \vee \mathcal{Q}$	$True$	$True$	$True$

2. Determine the truth values of $\mathcal{P}$, $\neg\mathcal{Q}$, $\mathcal{P} \wedge (\neg\mathcal{Q})$ and $\mathcal{P} \vee (\neg\mathcal{Q})$ for each of the following pairs of propositions:

   a) $\mathcal{P}$: 3 is a rational number.
   $\mathcal{Q}$: $\sqrt{2}$ is a rational number.

   b) $\mathcal{P}$: $0 \notin (-3, 0) \cap [0, 3]$.
   $\mathcal{Q}$: $(-\infty, 2] \cap [1, +\infty] = [1, 2]$.

Solution:

	a)	*b)*
$\mathcal{P}$	$True$	$True$
$\neg\mathcal{Q}$	$True$	$False$
$\mathcal{P} \wedge (\neg\mathcal{Q})$	$True$	$False$
$\mathcal{P} \vee (\neg\mathcal{Q})$	$True$	$True$

3. Determine if the following subsets of real numbers are open, closed or neither; for each of them describe the set of interior, exterior, and boundary points:

$$A := [0,1]; \qquad B := [-1,1]; \qquad C := (-1,1);$$

$$D := [-1,2); \qquad E := (-\infty,1]; \qquad F := [-1,3] \cup [4,5];$$

Solution:

		Interior points	*Exterior points*	*Boundary points*
A	*closed*	$(0,1)$	$(-\infty,0) \cup (1,\infty)$	$\{0,1\}$
B	*closed*	$(-1,1)$	$(-\infty,-1) \cup (1,\infty)$	$\{-1,1\}$
C	*open*	$(-1,1)$	$(-\infty,-1) \cup (1,\infty)$	$\{-1,1\}$
D	–	$(-1,2)$	$(-\infty,-1) \cup (2,\infty)$	$\{-1,2\}$
E	*closed*	$(-\infty,1)$	$(1,\infty)$	$\{1\}$
F	*closed*	$(-1,3) \cup (4,5)$	$(-\infty,-1) \cup (3,4) \cup (5,\infty)$	$\{-1,3,4,5\}$

4. For each of the following subsets in $\mathbb{R}$ find the infimum (*inf*) and the supremum (*sup*); describe the minimum (*min*) and the maximum (*max*) too, if they exist:

$$A := [1,2]; \qquad B := [1,2); \qquad C := (0,+\infty);$$

$$D := \left\{ \frac{1}{n} \mid n \in \mathbb{N} \right\}; \quad E := \left\{ \frac{(-1)^n}{n} \mid n \in \mathbb{N} \right\}; \quad F := \left\{ x \in \mathbb{R} \mid x^2 < 9 \right\};$$

Solution:

	inf	sup	min	max
A	1	2	1	2
B	1	2	1	$\nexists$
C	0	$+\infty$	$\nexists$	$\nexists$
D	0	1	$\nexists$	1
E	-1	$1/2$	-1	$1/2$
$F = (-3, 3)$	-3	3	$\nexists$	$\nexists$

5. Given the following subsets of $\mathbb{R}$:

$$A := (-\infty, 1) \qquad B := (-\infty, 1]$$

$$C := (1, +\infty) \qquad D := [1, +\infty)$$

$$E := [0, 3] \qquad F := (-6, 6)$$

compute the set operations:

$a)$ $A \cup B$; $b)$ $A \cap B$; $c)$ $B \cup F$; $d)$ $B \cup C$;
$e)$ $A \cup C$; $f)$ $B \backslash A$; $g)$ $\mathbb{R} \backslash A$; $h)$ $D \cap E$;

Solution:

$a)$ $(-\infty, 1]$; $b)$ $(-\infty, 1)$; $c)$ $(-\infty, 6)$; $d)$ $\mathbb{R}$;
$e)$ $\mathbb{R} \backslash \{1\}$; $f)$ $\{1\}$; $g)$ $[1, +\infty)$; $h)$ $[1, 3]$;