# MATHEMATICS I-Part A

## October, 10, 2019



## Classwork #3

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1) *(1 point)* Consider the functions $f(x) = \sqrt{1+x}$ and $g(x) = \frac{1}{x}$. Find the domains and images of $f$ and $g$. Compute the equations of $f \circ g$ and of $g \circ f$. **Motivate your answers**.

The domain of $f$ is $D_f = [-1, +\infty)$ since the square root is defined only for positive values. The domain of $g$ is $D_g = \mathbb{R} \setminus \{0\}$ since division by zero is not defined. Since $\sqrt{1+x} \geq 0$ for all $x \in D_f$ the image of $f$ is $I_f = [0, +\infty)$. Similarly, since $1/x \neq 0$ for all $x \in D_g$ then the image of $g$ is $I_g = \mathbb{R} \setminus \{0\}$. To complete the exercise, note that

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \sqrt{1 + \frac{1}{x}} = \sqrt{\frac{x+1}{x}},$$

and

$$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{1+x}\right) = \frac{1}{\sqrt{1+x}}.$$

2) *(1 point)* As in the previous point consider the functions $f(x) = \sqrt{1+x}$ and $g(x) = \frac{1}{x}$. Compute the domains and the images of $f \circ g$ and of $g \circ f$. **Motivate your answers**.

From the previous exercise we see that

$$\begin{aligned} D_{(f \circ g)} &= \left\{ x \in \mathbb{R} \,\middle|\, x \neq 0 \wedge \frac{x+1}{x} \geq 0 \right\} = (-\infty, -1] \cup (0, +\infty), \\ I_{(f \circ g)} &= [0, 1) \cup (1, +\infty], \\ D_{(g \circ f)} &= (-1, +\infty], \\ I_{(g \circ f)} &= (0, +\infty). \end{aligned}$$

3) *(1 point)* Consider the set $A = \{1, 2\}$ and the set $B = \{1, 2, 3\}$. Is it possible to define a function

$$f : A \to B$$

which is both surjective and injective? **Motivate your answers**.

A one-to-one correspondence from $A$ to $B$ is not possible since $B$ has one element more than $A$. Whence is not possible to define a function which is surjective.