BAE Math 1 Exercises

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Exercise 1.
Write the equation of the line that passes through the point $C(-2;1)$ and that is parallel to the line that passes through the points $A(4;-3)$ and $B(-5;0)$

Exercise 2.
For which values of k the line of equation $(k+2)x+(k+3)y-1=0$ is

i) parallel to the x-axis;

ii) parallel to the y-axis;

iii) parallel to the line of equation $x-2y=0$

iv) perpendicular to the line of equation $4x-2y+1=0$.

Exercise 3.
Write the equation of a parabola with axis of symmetry parallel to the y axis, that passes through points $A(-3;4)$ and $B(0;1)$, knowing that in the point B it is tangent to a line with slope $m=2$.

Exercise 4.
Describe the domain of each of the following functions:

$$a)\ f(x)=\log(\log(x)) \qquad b)\ f(x)=\sqrt{x\log(x)}$$

$$c)\ f(x)=\log(1-3^x) \qquad d)\ f(x)=\sqrt{\left(\frac{1}{2}\right)^x-1}$$

$$e)\ f(x)=\frac{1}{3^{x^2}-4} \qquad f)\ f(x)=\frac{\sqrt{x}e^{-x}}{2^{x^2+1}-3}$$

Exercise 5.
Establish if the following functions are even, odd or neither

$$\text{(i)}\quad y=3x^2+2x-1; \quad \text{(ii)}\quad y=x^2-|5x|;$$

$$\text{(iii)}\quad y=\frac{\sqrt{7-x^2}}{x}; \quad \text{(ii)}\quad y=\frac{x^3-1}{1-x^2};$$

$$\text{(iv)}\quad y=\frac{e^x+e^{-x}}{x^2}; \quad \text{(v)}\quad y=\frac{|x|+x^2}{2x};$$

Exercise 6.
Compute the following limits:

$a)$ $\lim_{n\to+\infty} \dfrac{n^3-6n+11}{9-7n^4}$,

$b)$ $\lim_{n\to+\infty} \dfrac{n^2+9n+3}{4-n^2}$,

$c)$ $\lim_{n\to+\infty} \dfrac{3n^5+6n+1}{1-3n^4}$,

$d)$ $\lim_{n\to+\infty} (-1)^n \cdot \dfrac{n}{n^2+1}$,

$e)$ $\lim_{n\to+\infty} (\sqrt{n^2+n}-n)$,

$f)$ $\lim_{n\to+\infty} \dfrac{1+\cos(n)-n\sqrt{n}}{2+\sin(n)-3n}$,

$g)$ $\lim_{n\to+\infty} (\log(n)-n^3+2^n-e^n)$,

$h)$ $\lim_{n\to+\infty} \dfrac{3^{2n+1}+1}{8^n+1}$,

$i)$ $\lim_{n\to+\infty} \dfrac{7^n n^9}{8^n}$,

$j)$ $\lim_{n\to+\infty} \dfrac{2^n+\log(n^2)+e^n}{n^8+8^n+\cos(n)+1}$,

$k)$ $\lim_{n\to+\infty} \left(\dfrac{n^2+n}{n^2-n+2}\right)^n$,

$l)$ $\lim_{n\to+\infty} \left(1+\dfrac{1}{5n}\right)^{2n}$,

$m)$ $\lim_{n\to+\infty} n \cdot \sin\left(\dfrac{3}{n}\right)$,

$n)$ $\lim_{n\to+\infty} n \cdot \tan\left(\dfrac{1}{n}\right)$,

$o)$ $\lim_{n\to+\infty} \dfrac{\sin\left(\frac{1}{n}\right)}{\sin\left(\frac{3}{n}\right)}$,

$p)$ $\lim_{n\to+\infty} \dfrac{\tan^2\left(\dfrac{1}{n}\right)}{1-\cos\left(\dfrac{1}{n}\right)}$,

$q)$ $\lim_{n\to+\infty} \dfrac{1-\cos\left(\dfrac{3}{n}\right)}{\sin\left(\dfrac{3}{n^2}\right)}$,

$r)$ $\lim_{n\to+\infty} (n-\sin(n))$,

Exercise 7.
Prove the following limits do not exist:

$\lim_{n\to+\infty} \cos(\pi n)$,

$\lim_{n\to+\infty} \dfrac{(-1)^n n}{n+3}$