



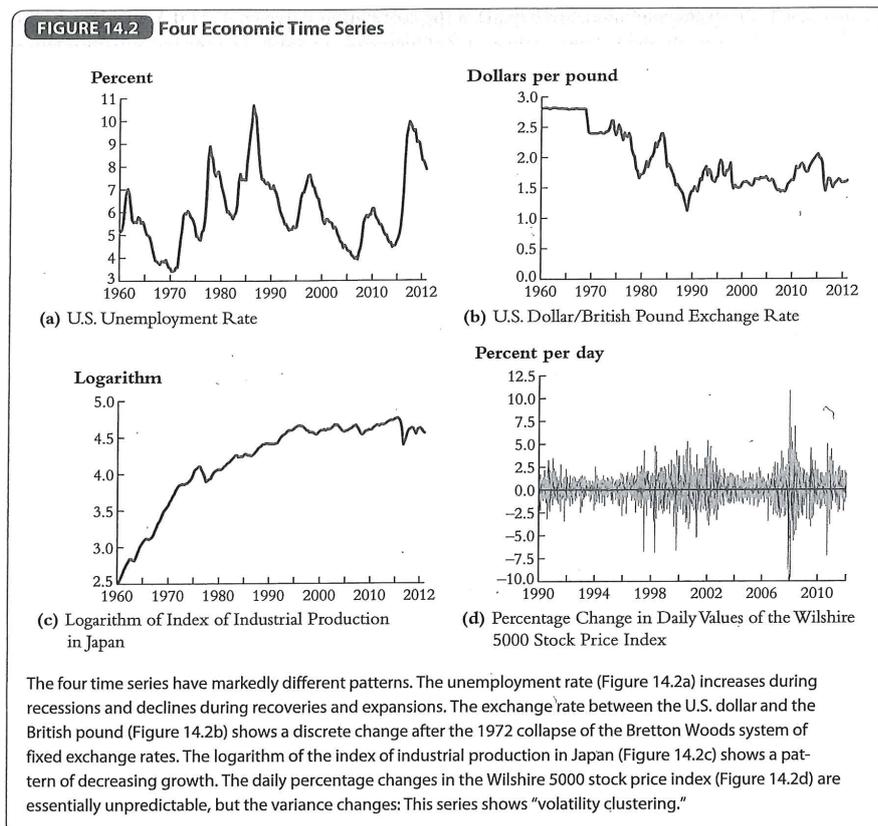
## Time Series

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### 1 Time Series Regressions

**Exercise 1.1** (Stock and Watson, Review the Concepts 14.1) Look at the four plots in Figure 14.2—the US unemployment rate, the dollar-pound exchange rate, the logarithm of the index of industrial production, and the percentage change in stock prices.

- Which of these series appears to be non-stationary?
- Which of these series appears to resemble a random walk?



**Exercise 1.2** In this exercise, we will use the ADF test to examine whether the time series for log GDP has a stochastic trend.

- Explain in words (i.e., not equations) what the null and alternative hypotheses are in an ADF test.

(b) Suppose that the alternative hypothesis we would like to use is that log GDP has a stationary trend around a deterministic linear time trend. Further, suppose that after using AIC, we have chosen a lag length of 2 for the ADF regression.

- (i) Write down the regression model for the ADF test.
- (ii) What are the mathematical expressions for  $H_0$  and  $H_1$  in the ADF test? Is this a one-sided or two-sided test?

(c) Now, we estimate the regression in part (b) using OLS and quarterly data from 1962:Q1-2012:Q4. The OLS estimates are:

$$\begin{aligned} \Delta \log(GDP_t) = & \frac{0.244}{(0.09)} + \frac{0.0002}{(0.0001)} t - \frac{0.030}{(0.011)} \log(GDP_{t-1}) \\ & + \frac{0.269}{(0.078)} \Delta \log(GDP_{t-1}) + \frac{0.178}{(0.083)} \Delta \log(GDP_{t-2}) + u_t. \end{aligned}$$

What is the ADF statistic? Is this a one-sided or two-sided test?

(d) Suppose that we are carrying out the ADF test at the 10% level. What is the conclusion of the hypothesis test? Do you find evidence that log GDP has a stochastic trend?

**Exercise 1.3** Consider an ADL(2,2) model to forecast the GDP growth rate, where we include two lags of the GDP growth rates ( $GDPGR$ ) and two lags of the term spread ( $TS_{spread}$ ). Specifically, the ADL(2,2) regression model is given by

$$GDPGR_t = \beta_0 + \beta_1 GDPGR_{t-1} + \beta_2 GDPGR_{t-2} + \delta_1 TS_{spread}_{t-1} + \delta_2 TS_{spread}_{t-2} + u_t.$$

Suppose that we have estimated the above regression using the period 1962:Q1 to 2012:Q4.

(a) Explain how we can use the QLR test to check whether the predictive power of the term spread over the GDP growth rate has been stable over time.

(b) Suppose that having implemented the QLR test, you find that the largest  $F$ -statistic is 6.39 which occurs in 1980:Q4. Is there evidence of a break in the relationship between GDP growth and lagged term spread? Use the 1% significance level.

**Exercise 1.4** (Stock and Watson, Review the Concepts 14.2) Many financial economists believe that the random walk model is a good description of the logarithm of stock prices. It implies that the percentage changes in stock prices are unforecastable. A financial analyst claims to have a new model that makes better predictions than the random walk model. Explain how you would examine the analyst's claim that his model is superior.

**Exercise 1.5** (Stock and Watson, Review the Concepts 14.3) A researcher estimates an AR(1) with an intercept and finds that the OLS estimate of  $\beta_1$  is 0.88, with a standard error of 0.03. Does a 95% confidence interval include  $\beta_1 = 1$ ? Explain.

**Exercise 1.6** (Stock and Watson, Review the Concepts 14.4) Suppose that you suspected that the intercept in Equation 14.16 changed in 1992:Q1. How would you modify the equation to incorporate this change? How would you test for a change in the intercept? How would you test for a change in the intercept if you did not know the date of the change?

**Exercise 1.7** (Stock and Watson, Exercise 14.2e, 14.3) The index of industrial production ( $IP_t$ ) is a monthly time series that measures the quantity of industrial commodities produced in a given month. This problem uses data on this index for the United States. All regressions are estimated over the sample period 1986:M1 (January 1986) to 2013:M12 (December 2013). Let  $Y_t = 1200 * \ln(IP_t/IP_{t-1})$ .

(a) The forecaster then estimates the following AR(4) model for  $Y_t$ :

$$\hat{Y}_t = \underset{(0.539)}{0.787} + \underset{(0.093)}{0.052} Y_{t-1} + \underset{(0.053)}{0.185} Y_{t-2} + \underset{(0.078)}{0.234} Y_{t-3} + \underset{(0.066)}{0.164} Y_{t-4}$$

Worried about a potential break, she computes a QLR test (with 15% trimming) on the constant and AR coefficients in the above AR(4) model. The resulting QLR statistic was 3.94. Is there evidence of a break? Explain. If you are conducting any hypothesis test, use the 10% level.

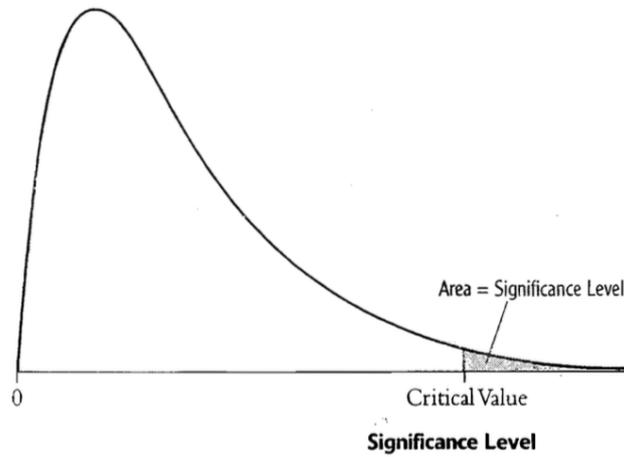
(b) The researcher tests for a stochastic trend in  $\ln(IP_t)$  using the following regression:

$$\begin{aligned} \widehat{\Delta \ln(IP_t)} = & \underset{(0.015)}{0.030} + \underset{(0.000009)}{0.000014t} - \underset{(0.0044)}{0.0085 \ln(IP_{t-1})} \\ & + \underset{(0.054)}{0.050} \Delta \ln(IP_{t-1}) + \underset{(0.053)}{0.186} \Delta \ln(IP_{t-2}) + \underset{(0.053)}{0.240} \Delta \ln(IP_{t-3}) + \underset{(0.054)}{0.173} \Delta \ln(IP_{t-4}) \end{aligned}$$

where the standard errors are shown in parentheses are computed using the homoskedasticity-only formula and the regressor  $t$  is a linear time trend. Use the ADF statistic to test for a stochastic trend (unit root) in  $\ln(IP)$ .

(c) Do the results in part (b) support the AR(4) specification used in part (a)? Explain.

**TABLE 4** Critical Values for the  $F_{m,\infty}$  Distribution



Degrees of Freedom	10%	5%	1%
1	2.71	3.84	6.63
2	2.30	3.00	4.61
3	2.08	2.60	3.78
4	1.94	2.37	3.32
5	1.85	2.21	3.02
6	1.77	2.10	2.80
7	1.72	2.01	2.64
8	1.67	1.94	2.51
9	1.63	1.88	2.41
10	1.60	1.83	2.32
11	1.57	1.79	2.25
12	1.55	1.75	2.18
13	1.52	1.72	2.13
14	1.50	1.69	2.08
15	1.49	1.67	2.04
16	1.47	1.64	2.00
17	1.46	1.62	1.97
18	1.44	1.60	1.93
19	1.43	1.59	1.90
20	1.42	1.57	1.88
21	1.41	1.56	1.85
22	1.40	1.54	1.83
23	1.39	1.53	1.81
24	1.38	1.52	1.79
25	1.38	1.51	1.77
26	1.37	1.50	1.76
27	1.36	1.49	1.74
28	1.35	1.48	1.72
29	1.35	1.47	1.71
30	1.34	1.46	1.70

This table contains the 90<sup>th</sup>, 95<sup>th</sup>, and 99<sup>th</sup> percentiles of the  $F_{m,\infty}$  distribution. These serve as critical values for tests with significance levels of 10%, 5%, and 1%.

**TABLE 14.4** Large-Sample Critical Values of the Augmented Dickey–Fuller Statistic

Deterministic Regressors	10%	5%	1%
Intercept only	-2.57	-2.86	-3.43
Intercept and time trend	-3.12	-3.41	-3.96

**TABLE 14.5** Critical Values of the QLR Statistic with 15% Trimming

Number of Restrictions ( $q$ )	10%	5%	1%
1	7.12	8.68	12.16
2	5.00	5.86	7.78
3	4.09	4.71	6.02
4	3.59	4.09	5.12
5	3.26	3.66	4.53
6	3.02	3.37	4.12
7	2.84	3.15	3.82
8	2.69	2.98	3.57
9	2.58	2.84	3.38
10	2.48	2.71	3.23
11	2.40	2.62	3.09
12	2.33	2.54	2.97
13	2.27	2.46	2.87
14	2.21	2.40	2.78
15	2.16	2.34	2.71
16	2.12	2.29	2.64
17	2.08	2.25	2.58
18	2.05	2.20	2.53
19	2.01	2.17	2.48
20	1.99	2.13	2.43

*Note:* These critical values apply when  $\tau_0 = 0.15T$  and  $\tau_1 = 0.85T$  (rounded to the nearest integer), so the  $F$ -statistic is computed for all potential break dates in the central 70% of the sample. The number of restrictions  $q$  is the number of restrictions tested by each individual  $F$ -statistic. Critical values for other trimming percentages are given in Andrews (2003).