Mathematics II

Practice 2
03/04/2022

1. Given the matrices

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 1 \\ 2 & -1 & -1 \\ 1 & 2 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 3 & 1 \\ -4 & 2 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

   (a) calculate $A + B$
   (b) calculate A^T
   (c) verify that $(A + B)^T = A^T + B^T$
   (d) calculate $C = 4A$

2. Recognize if it is possible to realize the product between the indicated matrices, and in these cases calculate it:

   (a) $A = \begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & -1 & 3 & 5 \\ 1 & -3 & 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 0 \\ 4 & -5 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 0 & 1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$

   (c) $A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \\ 1 & 1 \end{pmatrix}$

   (d) $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

   (e) $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$

3. Given the matrices $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$, calculate AB and BA. Is it $AB = BA$?

4. Given the matrices $A = \begin{pmatrix} 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} \beta & 2 \\ 1 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$, calculate β such that $A \cdot B \cdot C = 0$
5. Given the matrices $A = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, find a matrix $X = \begin{pmatrix} x \\ y \end{pmatrix}$ such that $A \cdot X = B$
6. Evaluate k such that the matrix $A = \begin{pmatrix} 1 & 0 \\ 2 & k \end{pmatrix}$ satisfies $A^2 = A$ (a matrix that satisfies such property is called IDEMPOTENT)
7. For which value $t \in \mathbb{R}$ the following matrix is symmetric?

$$A = \begin{pmatrix} 0 & 3t-2 & -1 \\ t^2 & 3 & t^2+4 \\ -1 & 4t & 1 \end{pmatrix}$$

8. Given the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ -1 & 5 & -1 & 3 \\ 4 & 3 & 2 & -2 \\ 2 & 0 & 1 & 8 \end{pmatrix}$$

write the minors M_{11}, M_{23}, M_{33}, M_{42}.

9. Calculate the determinant of the following matrices
   - $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$
   - $B = \begin{pmatrix} -11 & 3 \\ 2 & 0 \end{pmatrix}$
   - $C = \begin{pmatrix} 2 & 3 & -2 \\ 1 & -2 & 0 \\ 0 & -1 & 2 \end{pmatrix}$
   - $D = \begin{pmatrix} 2 & -2 & -2 \\ 1 & 1 & 0 \\ -3 & 4 & 0 \end{pmatrix}$
   - $E = \begin{pmatrix} 7 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 4 & -3 \end{pmatrix}$

- $F = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 5 \end{pmatrix}$
- $G = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$
- $H = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 4 & 1 & -1 & 2 \\ 3 & 6 & 6 & 8 \\ 3 & 2 & 1 & 3 \end{pmatrix}$

10. Given the matrix

$$D = \begin{pmatrix} t & 0 & 4 & -1 \\ 0 & 2 & 1 & 3 \\ -2 & 1 & 0 & t-1 \\ 4 & 0 & 0 & 1 \end{pmatrix}$$

for which value of $t \in \mathbb{R}$ the determinant of D is zero?

11. Calculate the inverse matrix of the following matrices

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -1 & 3 \\ 1 & 1 & 2 \\ 2 & 0 & 7 \end{pmatrix}$$

12. For which values of $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{R}$ the matrices

$$A = \begin{pmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} \beta & 1 & 0 \\ 1 & \beta & 1 \\ 0 & 1 & \beta \end{pmatrix}$$

are invertible?

13. Calculate the rank of the given matrices using the minors' method

(a) $A = \begin{pmatrix} 1 & 0 & 1 & 5 \\ 1 & 2 & 0 & 1 \\ 2 & 2 & 1 & 6 \end{pmatrix}$

(b) $A = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$