Mathematics II

Practice 1
03/11/2022

1. Determine if these matrices are invertible and, if so, compute the inverse:

$$\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 4 \\ -3 & 3 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ -1 & 3 & 0 \end{pmatrix}$$

2. Compute the rank of the following matrices:

$$\begin{pmatrix} 0 & -1 & 3 \\ 1 & 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 4 & 2 \\ 5 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & -1 \\ 0 & 0 & -3 & 0 \end{pmatrix}$$

3. Compute the rank of the following matrix for any value $k \in \mathbb{R}$:

$$A = \begin{pmatrix} 1 & 2 & k-1 & -1 \\ 3 & -2 & k & 1 \\ 1 & -3k & 0 & 3 \end{pmatrix}$$

4. Determine if the following vectors

$$v_1 = (1, 1, -1) \qquad v_2 = (0, -1, 2) \qquad v_3 = (1, 2, -3)$$

are linearly independent and say if they are a basis of the linear space $\mathbb{R}^3$.

5. Determine for what values of $k \in \mathbb{R}$ the following vectors

$$v_1 = (0, k, -k, 0) \qquad v_2 = (1, 0, 1, k) \qquad v_3 = (k+1, 2, 0, 2k)$$

are linearly independent. For what values of k are they a basis of $\mathbb{R}^4$?

6. For the following vectors of the Euclidean space $\mathbb{R}^3$:

$$\vec{u} = (1, 0, -1) \qquad \vec{v} = (3, 3, 2) \qquad \vec{w} = (1, -1, -2)$$

compute:

$a)$ the vectors $\vec{v} + 3\vec{w} - 2\vec{u}$ and $\sqrt{2}\vec{u} - 2\vec{v}$;

$b)$ the inner products $\vec{u} \cdot \vec{v}$, $\vec{u} \cdot \vec{w}$ and $\vec{v} \cdot \vec{w}$;

$c)$ the lengths $\| \vec{u} \|$, $\| \vec{v} \|$ and $\| \vec{w} \|$;

$d)$ the corresponding versor for each vector;

$e)$ the vector $\| \vec{w} \| \vec{u} - 2(\vec{u} \cdot \vec{w})\vec{w} + \| \vec{u} \| \vec{v}$

7. Find the Cartesian equation and the parametric equation of the line in $\mathbb{R}^2$ that passes through the point $P = (2, 1)$ and is parallel to the vector $\vec{v} = (1, -1)$.

8. Find the Cartesian equations and the parametric equations of the line in $\mathbb{R}^3$ that passes through the point $P = (1, 0, 1)$ and is parallel to the vector $\vec{v} = (2, -1, -1)$.

9. Determine the relative position of the lines r and s in $\mathbb{R}^3$ of equations:

$$r : \begin{cases} x = 3 \\ z = 2 \end{cases} \qquad\qquad s : \begin{cases} y = 1 \\ x + z = 2 \end{cases}$$

10. For the following lines in $\mathbb{R}^3$:

$$r : \begin{cases} x - y + z = 0 \\ x + 2y + 2z = 1 \end{cases} \qquad\qquad s : \begin{cases} x - 4y - 4 = 0 \\ 3y + z + 3 = 0 \end{cases}$$

$a)$ Determine if the line r passes through the point $P = (2, 1, -1)$;

$b)$ Determine if the line s passes through the point $Q = (-4, -2, 3)$;

$c)$ Prove that the two lines r and s are parallel;

$d)$ Find the Cartesian equation of the plane containing both r and s;

$e)$ Find the Cartesian equations of all the planes orthogonal to r and s;