

Mathematics II

Practice 4 03/18/2022

1. Given these systems

$$(a) \begin{cases} x_1 - x_2 - x_3 = 0 \\ 3x_1 + x_2 + 2x_3 = 0 \\ 4x_1 + x_3 = 0 \end{cases}$$

$$(b) \begin{cases} -x_1 + x_2 + x_3 = 2 \\ x_1 - x_2 = -1 \\ x_1 - 2x_2 - 2x_3 = 0 \end{cases}$$

$$(c) \begin{cases} x_1 + 2x_2 + 3x_3 + x_4 = 0 \\ -x_1 + x_2 + 2x_3 + x_4 = 0 \\ 3x_2 + 5x_3 + 2x_4 = 0 \end{cases}$$

- Verify if the system admits solutions and how many solutions it has
- Write down the solutions

2. Solve these parametric systems in dependence from $h \in \mathbb{R}$

$$(a) \begin{cases} x - y - z = 0 \\ 3x + y + 2z = 0 \\ 4x + hz = 0 \end{cases}$$

$$(b) \begin{cases} -x + y + z = 2 \\ x - y = 1 \\ x - 2y - 2z = h \end{cases}$$

3. Through the characteristic polynomial find the eigenvalues of these matrices

$$(a) \quad A = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$$

$$(b) \quad A = \begin{pmatrix} 3 & 0 & 0 \\ -4 & -1 & -8 \\ 0 & 0 & -3 \end{pmatrix}$$

$$(c) \ A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$(d) \ A = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 3 & 4 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$(e) \ A = \begin{pmatrix} 1 & 2 & 2 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

4. Determine the eigenvectors for each eigenvalues of the matrices in the previous exercise and if the matrices are diagonalizable.
5. Determine if this parametric matrix is diagonalizable in dependence of the parameter $h \in \mathbb{R}$:

$$A = \begin{pmatrix} 1 & 0 & h^2 \\ 0 & h & 0 \\ 1 & 0 & 1 \end{pmatrix}$$