**1** IN ORDER TO DETERMINE IF A MATRIX IS INVERTIBLE, WE HAVE TO VERIFY IF THE DETERMINANT IS NOT EQUAL TO ZERO.

$$A_1 = \begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} \Rightarrow \det A_1 = -1$$

$$A_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 4 \\ -3 & 3 & 0 \end{pmatrix} \Rightarrow \det A_2 = (-1)^{3+1}(-3)(8-6) + (-1)^{3+2} \cdot 3 \cdot (4-6) =$$

$$= -6 + 6 = 0$$

$$A_3 = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ -1 & 3 & 0 \end{pmatrix} \Rightarrow \det A_3 = (-1)^{3+1}(-1)(2-2) + (-1)^{3+2} \cdot 3 (1-0) =$$

$$= 0 - 3 = -3$$

SO WE HAVE TO INVERT $A_1$ AND $A_3$.

STEP 0: COMPUTE THE DETERMINANT OF THE MINORS

STEP 1: ADJOINT OF A

- $A_1 \Rightarrow$ $M_{11} = 0$ $\quad M_{12} = -1$ $\quad \Rightarrow adj(A_1) = \begin{pmatrix} 0 & +1 \\ +1 & 1 \end{pmatrix}$

  $M_{21} = -1 \quad M_{22} = 1$

STEP 2: TRANSPOSED

$$\Rightarrow adj(A_1)^T = \begin{pmatrix} 0 & +1 \\ +1 & 1 \end{pmatrix} \Rightarrow A_1^{-1} = \frac{adj(A_1)^T}{\det A_1} = \begin{pmatrix} 0 & -1 \\ -1 & -1 \end{pmatrix}$$

STEP 3: DIVISION BY THE DETERMINANT

REMEMBER: $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \Rightarrow adj(A) = \begin{pmatrix} (-1)^{1+1} M_{11} & (-1)^{1+2} M_{12} \\ (-1)^{2+1} M_{21} & (-1)^{2+2} M_{22} \end{pmatrix}$

STEP 0

- $A_3 \Rightarrow \det M_{11} = \det \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$ $\quad \det M_{12} = \det \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\quad \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ -1 & 3 & 0 \end{pmatrix}$

  $\det M_{13} = \det \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix}$ $\quad \det M_{21} = \det \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$

$\det M_{22} = \det \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$ $\det M_{23} = \det \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

$\det M_{31} = \det \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}$ $\det M_{32} = \det \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$\det M_{33} = \det \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

STEP 1

$$\mathrm{adj}(A_3) = \begin{pmatrix} -3 & -1 & 2 \\ 3 & 1 & -5 \\ 0 & -1 & 2 \end{pmatrix}$$

STEP 2

$$\mathrm{adj}(A_3)^T = \begin{pmatrix} -3 & 3 & 0 \\ -1 & 1 & -1 \\ 2 & -5 & 2 \end{pmatrix}$$

STEP 3

$$A_3^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -2/3 & 5/3 & -2/3 \end{pmatrix}$$

$\boxed{2}$

$$A_1 = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 2 & 1 \end{pmatrix} \qquad rk(A_1) \leq 2$$

$$M = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \Rightarrow \det(M) \neq 0$$

WE CAN'T HAVE A 3x3 MATRIX CONTAINING M

$$\Rightarrow \boxed{rk(A_1) = 2}$$

$$A_2 = \begin{pmatrix} 1 & 0 & 4 & 2 \\ 5 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 \end{pmatrix} \qquad rk(A_2) \leq 3$$

$$M = \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{pmatrix} \Rightarrow \det M = (-1)^{2+1} \cdot 1 \cdot \det \begin{pmatrix} 4 & 2 \\ 1 & -1 \end{pmatrix} =$$

$$= -(-4-2) = 6 \neq 0$$

$$\Rightarrow \boxed{rk(A_2) = 3}.$$

$$A_3 = \begin{pmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & -1 \\ 0 & 0 & -3 & 0 \end{pmatrix} \qquad r_k(A_3) \leq 4$$

WE HAVE 5 DIFFERENT MINORS $4 \times 4$ :

$$M_1 = \begin{pmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & -1 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 0 \end{pmatrix} \qquad M_3 = \begin{pmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & -2 & 0 \\ 1 & 2 & 1 & -1 \\ 0 & 0 & -3 & 0 \end{pmatrix}$$

$\Rightarrow$ FIRST AND SECOND ROWS ARE PROPORTIONAL $\rightarrow \det M_1 = 0$

$\Rightarrow \det M_2 = 0$ LIKE BEFORE

$\det M_3 = 0$ LIKE BEFORE

$$M_4 = \begin{pmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & -1 \\ 0 & 0 & -3 & 0 \end{pmatrix} \qquad M_5 = \begin{pmatrix} -1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & -1 \\ 0 & 0 & -3 & 0 \end{pmatrix}$$

$\Rightarrow$ SECOND AND LAST ROWS ARE PROPORTIONAL $\rightarrow \det M_4 = 0$

$\Rightarrow \det M_5 = 0$ LIKE BEFORE

$$M^* = \begin{pmatrix} -1 & 1 & -2 \\ 0 & 0 & 1 \\ 1 & 2 & 1 \end{pmatrix} \Rightarrow \det M^* = (-1)^{2+3} \cdot 1 \det \begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix} =$$

$$= 3 \neq 0$$

$$\Rightarrow r_k(A_3) = 3$$

$\boxed{3}$

$$A = \begin{pmatrix} 1 & 2 & k-1 & -1 \\ 3 & -2 & k & 1 \\ 1 & -3k & 0 & 3 \end{pmatrix} \qquad r_k(A) \leq 3$$

WE HAVE 4 $3 \times 3$ MINORS:

$$M_1 = \begin{pmatrix} 1 & 2 & k-1 \\ 3 & -2 & k \\ 1 & -3k & 0 \end{pmatrix} \Rightarrow \det M_1 = (-1)^{3+1} \cdot 1 \det \begin{pmatrix} 2 & k-1 \\ -2 & k \end{pmatrix} + (-1)^{3+2} (-3k) \det \begin{pmatrix} 1 & k-1 \\ 3 & k \end{pmatrix} =$$

$$= 2k + 2k - 2 + 3k(k - 3k + 3) =$$

$$= 4k - 2 - 6k^2 + 9k =$$

$$= -6k^2 + 13k - 2$$

(•) IF $-6k^2 + 13k - 2 \neq 0 \Rightarrow r_k A = 3$

(•) IF $-6k^2 + 13k - 2 = 0 \Rightarrow$ WE HAVE TO VERIFY

WE HAVE TO COMPUTE $6k^2 - 13k + 2 = 0$ $\to k = \frac{1}{6}$, $\to k = 2$

$\boxed{k=2} \Rightarrow A = \begin{pmatrix} 1 & 2 & 1 & -1 \\ 3 & -2 & 2 & 1 \\ 1 & -6 & 0 & 3 \end{pmatrix}$

$M = \begin{pmatrix} 1 & 2 \\ 3 & -2 \end{pmatrix} \Rightarrow \det M = -2 - 6 = -8 \neq 0$

MOREOVER $\det \begin{pmatrix} 1 & 2 & 1 \\ 3 & -2 & 2 \\ 1 & -6 & 0 \end{pmatrix} = 0$ AND $\det \begin{pmatrix} 1 & 2 & -1 \\ 3 & -2 & 1 \\ 1 & -6 & 3 \end{pmatrix} = 0 \Rightarrow r_k A = 2$

$\boxed{k = \frac{1}{6}} \Rightarrow A = \begin{pmatrix} 1 & 2 & -5/6 & -1 \\ 3 & -2 & 1/6 & 1 \\ 1 & -\frac{1}{2} & 0 & 3 \end{pmatrix}$ $\det \begin{pmatrix} 1 & 2 & -5/6 \\ 3 & -2 & 1/6 \\ 1 & -\frac{1}{2} & 0 \end{pmatrix} = 0$

BUT $\det \begin{pmatrix} 1 & 2 & -1 \\ 3 & -2 & 1 \\ 1 & -\frac{1}{2} & 3 \end{pmatrix} \neq 0$

SUMMING UP: $\Rightarrow \boxed{r_k A = 3}$

- $k \neq 2 \Rightarrow r_k A = 3$
- $k = 2 \Rightarrow r_k A = 2$

$\boxed{4}$ WE HAVE TO BUILD THE MATRIX WITH THE VECTORS LIKE COLUMNS:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 2 \\ -1 & 2 & -3 \end{pmatrix}$$

$$\det A = (-1)^{1+1} \cdot 1 \cdot \det \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix} + (-1)^{3+1} \cdot 1 \det \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} =$$

$$= -1 + 1 = 0$$

SO THE VECTORS ARE NOT INDEPENDENT.

$\boxed{5}$ WITH THE SAME METHOD:

$$A = \begin{pmatrix} 0 & 1 & k+1 \\ -k & 0 & 2 \\ -k & 1 & 0 \\ 0 & k & 2k \end{pmatrix}$$

$$M = \begin{pmatrix} -k & 0 & 2 \\ -k & 1 & 0 \\ 0 & k & 2k \end{pmatrix} \Rightarrow$$

$$\det M = -k \det \begin{pmatrix} 1 & 0 \\ k & 2k \end{pmatrix} + 2 \det \begin{pmatrix} -k & 1 \\ 0 & k \end{pmatrix} =$$

$$= -k(2k - 0) + 2(-k^2) = -4k^2$$

(•) So if $k \neq 0 \Rightarrow \det M \neq 0 \Rightarrow r_k A = 3 \Rightarrow v_1, v_2, v_3$ ARE INDEPENDENT

(•) IF $k = 0 \Rightarrow A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

BECAUSE A ROW IS COMPOSED ONLY BY ZEROS, THE ONLY MINOR TO STUDY IS $M = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} \Rightarrow \det M = (-1)^{3+2} \cdot 2 \cdot \det \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = 0$

IN THIS CASE $R_k A \leq 2$, SO THE VECTORS ARE NOT INDEPENDENT

$\boxed{6}$ $\vec{u} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$ $\vec{w} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$

(a) $\vec{v} + 3\vec{w} - 2\vec{u} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} =$

$$= \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$$

$$\sqrt{2}\,\vec{u} - 2\vec{v} = \sqrt{2}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - 2\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \sqrt{2}-6 \\ -6 \\ -\sqrt{2}-4 \end{pmatrix}$$

(b) $\vec{u} \cdot \vec{v} = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} = 3 - 2 = 1$

$$\vec{u} \cdot \vec{w} = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = 1 + 2 = 3$$

$$\vec{v} \cdot \vec{w} = \begin{pmatrix} 3 & 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = 3 - 3 - 4 = -4$$

(c) $\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}} = \sqrt{\begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}} = \sqrt{1+1} = \sqrt{2}$

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{\begin{pmatrix} 3 & 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}} = \sqrt{9+9+4} = \sqrt{22}$$

$$\|\vec{w}\| = \sqrt{\vec{w} \cdot \vec{w}} = \sqrt{\begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}} = \sqrt{1+1+4} = \sqrt{6}$$

(d) $\hat{u} = \dfrac{\vec{u}}{\|\vec{u}\|} = \left(\dfrac{1}{\sqrt{2}}, 0, -\dfrac{1}{\sqrt{2}}\right) = \left(\dfrac{\sqrt{2}}{2}, 0, -\dfrac{\sqrt{2}}{2}\right)$

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|} = \left(\frac{3}{\sqrt{22}}, \frac{3}{\sqrt{22}}, \frac{2}{\sqrt{22}}\right) = \left(\frac{3\sqrt{22}}{22}, \frac{3\sqrt{22}}{22}, \frac{2\sqrt{22}}{22}\right)$$

$$\hat{w} = \frac{\vec{w}}{\|\vec{w}\|} = \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right) = \left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, \frac{2\sqrt{6}}{6}\right)$$

$\boxed{7}$ $P = (2, 1)$ $\vec{v} = (1, -1)$

PARAMETRIC EQUATION: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad t \in \mathbb{R}$

$$\boxed{\begin{aligned} x &= 2 + t \\ y &= 1 - t \end{aligned} \quad t \in \mathbb{R}}$$

CARTESIAN EQUATION

WE HAVE TO ELIMINATE THE PARAMETER

$$\begin{cases} x = 2 + t \rightarrow t = x - 2 \\ y = 1 - t \rightarrow y = 1 - (x - 2) \end{cases}$$

$\Rightarrow y = 1 - x + 2$

$$\boxed{y + x - 3 = 0}$$

$\boxed{8}$ $P = (1, 0, 1)$ $\vec{v} = (2, -1, -1)$

PARAMETRIC EQUATION

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \quad t \in \mathbb{R}$$

$$\boxed{\begin{aligned} x &= 1 + 2t \\ y &= -t \\ z &= 1 - t \end{aligned} \quad t \in \mathbb{R}}$$

$$\begin{cases} x = 1+2t \\ y = -t \\ z = 1-t \end{cases} \longrightarrow \begin{cases} x = 1-2y \\ t = -y \\ z = 1+y \end{cases}$$

CARTESIAN EQUATION

$$\longrightarrow \boxed{\begin{cases} x+2y-1=0 \\ z-y-1=0 \end{cases}}$$

REMEMBER THIS RULE:

LINE IN $\mathbb{R}^2$ $\longrightarrow$ DIMENSION $= 1$ $\longrightarrow$ 1 PARAMETER $\longrightarrow$ 1 CARTESIAN EQUATION

LINE IN $\mathbb{R}^3$ $\longrightarrow$ DIMENSION $= 1$ $\longrightarrow$ 1 PARAMETER $\longrightarrow$ 2 CARTESIAN EQUATION

IN GENERAL:

SUBSPACE IN $\mathbb{R}^n$ $\longrightarrow$ DIMENSION OF THE SUBSPACE $= m$ $\longrightarrow$ $m$ PARAMETER $\longrightarrow$ $n-m$ CARTESIAN EQUATION

FOR EXAMPLE A PLANE HAS DIMENSION = 2.

**9**

$$r: \begin{cases} x = 3 \\ z = 2 \end{cases} \xrightarrow{\text{PARAMETRIC EQUATION}} \begin{cases} x = 3 \\ y = t \\ z = 2 \end{cases} \longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$s: \begin{cases} y = 1 \\ x+z = 2 \end{cases} \xrightarrow{\text{PARAMETRIC EQUATION}} \begin{cases} x = t \\ y = 1 \\ z = 2-t \end{cases} \longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

THE TWO DIRECTOR VECTORS ARE NOT PROPORTIONAL.

FOR THIS REASON $r$ AND $s$ ARE NOT PARALLEL

IF $r$ AND $s$ ARE NOT PARALLEL, THEY CAN BE INCIDENT OR SKEW.

BUT
$$r \cap s: \begin{cases} x=3 \\ z=2 \\ y=1 \\ x+z=2 \end{cases} \to \begin{cases} x=3 \\ z=2 \\ y=1 \\ 3+1=2 \to \text{IMPOSSIBLE} \end{cases}$$

SO $r \cap s = \emptyset$

THEN $r$ AND $s$ ARE SKEW.

$\boxed{10}$
$$r: \begin{cases} x-y+z=0 \\ x+2y+2z=1 \end{cases} \qquad s: \begin{cases} x-4y-4=0 \\ 3y+z+3=0 \end{cases}$$

(a) $P=(2,1,-1) \to r: \begin{cases} x-y+z=0 \\ x+2y+2z=1 \end{cases}$

$$\Rightarrow \begin{cases} 2-1-1=0 \\ 2+2-2=1 \end{cases} \to \begin{cases} 0=0 \\ 2=1 \end{cases} \text{IMPOSSIBLE}$$

(b) $Q=(-4,-2,3) \to s: \begin{cases} x-4y-4=0 \\ 3y+z+3=0 \end{cases}$

$$\to \begin{cases} -4+8-4=0 \\ -6+3+3=0 \end{cases} \to \begin{cases} 0=0 \\ 0=0 \end{cases}$$

$\to \boxed{Q \in s}$

(c) $r: \begin{cases} x-y+z=0 \\ x+2y+2z=1 \end{cases} \qquad s: \begin{cases} x-4y-4=0 \\ 3y+z+3=0 \end{cases}$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & 2 & 2 & 1 \end{array}\right)$$

MINOR WITH DETERMINANT DIFFERENT FROM 0

$$\begin{cases} x - y = -z \\ x + 2y = 1 - 2z \end{cases} \longrightarrow \quad \Delta_x = \begin{vmatrix} -z & -1 \\ 1-2z & 2 \end{vmatrix} \quad \Delta_y = \begin{vmatrix} 1 & -z \\ 1 & 1-2z \end{vmatrix} \quad \Delta = \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$$

$$\Rightarrow \quad x = \frac{-2z + 1 - 2z}{3} = -\frac{4}{3} z + \frac{1}{3}$$

$$y = \frac{1 - 2z + z}{3} = -\frac{z}{3} + \frac{1}{3}$$

GETTING $z = t$, WE HAVE

$$x = -\frac{4}{3} t + \frac{1}{3}$$
$$y = -\frac{1}{3} t + \frac{1}{3}$$
$$z = t$$

$$\Rightarrow \boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 0 \end{pmatrix} + t \begin{pmatrix} -4/3 \\ -\frac{1}{3} \\ 1 \end{pmatrix} \quad r}$$

$$s: \begin{cases} x - 4y - 4 = 0 \\ 3y + z + 3 = 0 \end{cases} \longrightarrow \begin{cases} x - 4y = 4 \\ 0 \cdot x + 3y = -z - 3 \end{cases}$$

$$x = \frac{\Delta x}{\Delta} = \frac{\begin{vmatrix} 4 & -4 \\ -z-3 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & -4 \\ 0 & 3 \end{vmatrix}} \qquad y = \frac{\Delta y}{\Delta} = \frac{\begin{vmatrix} 1 & 4 \\ 0 & -z-3 \end{vmatrix}}{\begin{vmatrix} 1 & -4 \\ 0 & 3 \end{vmatrix}}$$

$$x = \frac{12 + 4(-z-3)}{3} \qquad y = \frac{-z-3}{3}$$

$$x = \cancel{4} - 4/3\, z - \cancel{4} \qquad y = -z/3 - 1$$

GETTING $z = t$

$$x = -\frac{4}{3} t$$

$$y = -\frac{1}{3} t - 1$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4/3 \\ -\frac{1}{3} \\ 1 \end{pmatrix}$$

r AND s HAVE PROPORTIONAL DIRECTOR'S VECTOR, SO THEY ARE PARALLEL.

(d) THE GENERAL EQUATION OF A PLANE IS

$$ax + by + cz + 1 = 0$$

SO WE NEED THREE POINTS (1 FROM THE FIRST LINE, 2 FROM THE SECOND LINE).

$$\begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 0 \end{pmatrix} ; \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} ; \begin{pmatrix} -4/3 \\ -4/3 \\ 1 \end{pmatrix}$$

$t=0$ FROM r; $t=0$ FROM s; $t=1$ FROM s

$$a/3 + b/3 + 1 = 0 \qquad -b + 1 = 0 \qquad -4/3\, a - 4/3\, b + c + 1 = 0$$

$$\Rightarrow b = 1; \quad a/3 + \frac{1}{3} + 1 = 0 \quad 16/3 - 4/3 + c + 1 = 0$$

$$a = -4 \qquad \Rightarrow c = -5$$

$\Rightarrow \quad a = -4 \quad b = 1 \quad c = -5$

$$\Rightarrow \boxed{-4x + y - 5z + 1 = 0}$$

(e) THE DIRECTOR VECTOR OF $r$ AND $s$ IS $\begin{pmatrix} -4/3 \\ -1/3 \\ 1 \end{pmatrix}$

$\Rightarrow$ THE EQUATION OF THE ORTHOGONAL PLANE TO $r$ IS

$$-\frac{4}{3}x - \frac{1}{3}y + z + k = 0$$

IN GENERAL:

GIVEN A LINE $r$ OF DIRECTOR VECTOR $\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$

THEN THE ORTHOGONAL PLANE TO $r$ IS

$$\boxed{v_x \cdot x + v_y \cdot y + v_z \cdot z + k = 0}$$

THERE ARE INFINITE PLANES (TRANSLATIONS OF THE SAME)