MEETING 11/03/2022

EX. 5 of 1^{st} PRACTICAL CLASS

$$F(x) = \int_0^{x^2-x} \frac{e^{t^2+2t}}{\sqrt{t^2+3}}\, dt.$$

the Fundamental Thm of Calculus says that if f is continuous in $[a,b]$ and if we define the INTEGRAL FUNCTION as follows

$$F(x) = \int_a^x f(t)\, dt$$

$\Longrightarrow$ $F(x)$ is differentiable and

$$F'(x) = f(x)$$

Who is $F(x)$ in ex 5 ?

$$G(x) = \int_0^x \frac{e^{t^2+2t}}{\sqrt{t^2+3}}\, dt \qquad g(x) = x^2 - x$$

$$g \in C^\infty$$

$$F(x) = G(g(x))$$

Let us analyze the differentiability of $G(x)$

$$f(t)=\frac{e^{t^2+2t}}{\sqrt{t^2+3}}$$ is continuous $\forall t\in\mathbb{R}$

$f(t)$ is continuous in $\mathbb{R}$

$D_f=\mathbb{R} \Longrightarrow$ by FTC surely

$G(x)$ is differentable and

$$G'(x)=\frac{e^{x^2+2x}}{\sqrt{x^2+3}}$$

$$g(x)=x^2-x\in C^\infty$$

hence

$F(x)=G(g(x))$ as a

composition of 2 differentable functions, is differentable itself. And its derivative is

$$F'(x)=G'(g(x))\cdot g'(x)$$

$$=\frac{e^{(x^2-x)^2+2(x^2-x)}}{\sqrt{(x^2-x)^2+3}}\cdot(2x-1)$$

FIND CRITICAL POINTS:

$$F'(x) = 0 \iff$$

$$\frac{\underbrace{e^{(x^2-x)^2+2(x^2-x)}}_{>0}}{\underbrace{\sqrt{(x^2-x)^2+3}}_{>0}} \cdot (2x-1) = 0$$

$$2x-1=0 \iff x=\frac{1}{2}$$ ONLY CRITICAL POINT

$D_F = \mathbb{R}$, in D_F the only critical point is $x=\frac{1}{2}$, to evaluate its nature I study the sign of the derivative

$$F'(x) = \underbrace{\frac{e^{(x^2-x)^2+2(x^2-x)}}{\sqrt{(x^2-x)^2+3}}}_{>0 \text{ always}} \cdot (2x-1)$$

THIS FACTOR DETERMINES THE SIGN OF $F'(x)$

if and only if

$F'(x)>0$ iff $2x-1>0 \iff x>\frac{1}{2}$

$1/2$

F' − − − − − − − 0 + + + + + + + +

Local Minimum

in $\left(-\infty, \frac{1}{2}\right)$ F is DECREASING

in $\left(\frac{1}{2}, +\infty\right)$ F is INCREASING

in $x=\frac{1}{2}$ F has a local Minimum.

EXERCISE:

Determine if the following pairs of planes intersect

i) $x+2y-3z=6$ and

$x+3y-2z=6$

$$\begin{cases} x+2y-3z=6 \\ x+3y-2z=6 \end{cases} \Longrightarrow \cancel{x}+2y-3z=\cancel{x}+3y-2z$$

$$-3y+2y=3z-2z$$

$$-y=z \Longrightarrow y=-z$$

THE 2 PLANES INTERSECT AND THEIR INTERSECTION IS THE LINE OF EQUATION $y=-z$

ii) $x+2y-3z=6$ and

$-2x-4y+6z=10$

$$n \cdot p = n \cdot p_0$$

$$n = (1, 2, -3)$$

$$\begin{cases} x + 2y - 3z = 6 \\ -2x - 4y + 6z = 10 \end{cases}$$

$-2 \times$ I member gives I member of 1^{st} equation

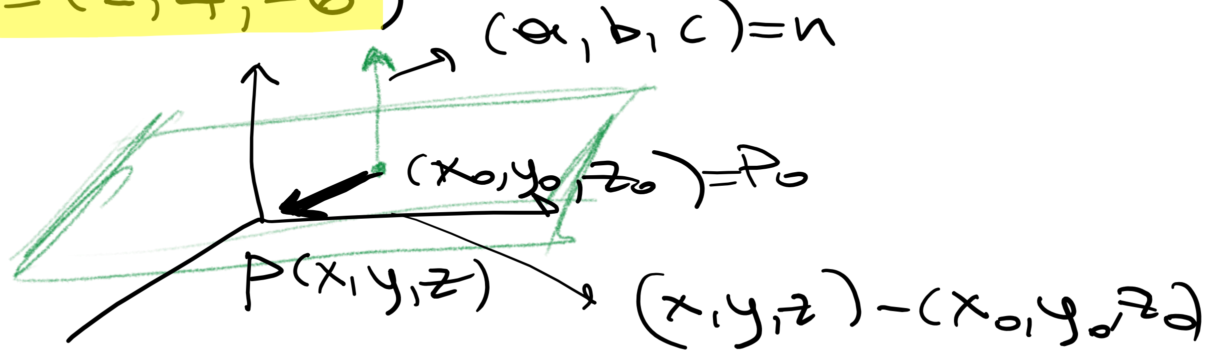
$$\begin{cases} 2x + 4y - 6z = 12 \\ -2x - 4y + 6z = 10 \end{cases}$$

$$0 = 22$$

IMPOSSIBLE

$n = (-2, -4, 6)$

$n = (2, 4, -6)$



$$(x - x_0, y - y_0, z - z_0) \cdot (a, b, c) = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz = ax_0 + by_0 + cz_0$$

$$n \cdot p = n \cdot P_0$$

$V = \mathbb{R}^n$ m vectors

$v_1, v_2, \dots, v_m$ are

LINEARLY INDEPENDENT if the only LINEAR COMBINATION of $v_1, v_2, \dots, v_m$ that generates the null vector is the trivial combination where all coefficients are null

$v_1, v_2, \dots, v_m$ are Linearly Ind

iff $\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n = \mathbf{0}$

if $\alpha_1 = \alpha_2 = \dots = \alpha_n = 0$

$v_1, v_2, \dots, v_m$ are LINEARLY DEPENDENT if in the linear combination

$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_m = \mathbf{0}$$

at least one $\alpha_j \neq 0$

say for example $\alpha_1 \neq 0$

$$\frac{\cancel{\alpha_1} v_1}{\cancel{\alpha_1}} = -\frac{\alpha_2}{\alpha_1} v_2 - \dots - \frac{\alpha_n}{\alpha_1} v_n$$

$$v_1 = -\frac{\alpha_2}{\alpha_1} v_2 - \dots - \frac{\alpha_n}{\alpha_1} v_n$$

EXERCISE:

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad v_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \quad v_3 = \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}$$

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0$$

$$\alpha_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} \alpha_1 + 2\alpha_2 = 0 & \rightarrow \alpha_1 = -2\alpha_2 \\ -\alpha_1 - 2\alpha_3 = 0 & \rightarrow \alpha_1 = -2\alpha_3 \\ \alpha_1 - \alpha_2 + 3\alpha_3 = 0 \end{cases}$$

$$\Longrightarrow -2\alpha_2 = -2\alpha_3 \Longrightarrow \alpha_2 = \alpha_3$$

$$\cancel{-2\alpha_3 - \alpha_3 + 3\alpha_3 = 0}$$

$$0 = 0$$

$$\begin{cases} \alpha_1 = -2\alpha_2 \\ \alpha_2 = \alpha_3 \\ \end{cases} \Rightarrow \begin{cases} \alpha_1 = -2t \\ \alpha_2 = t \\ \alpha_3 = t \end{cases} \quad \forall t \in \mathbb{R}$$

if $t = 1$ you get as a solution

$$\begin{cases} \alpha_1 = -2 \\ \alpha_2 = 1 \\ \alpha_3 = 1 \end{cases}$$

ALTERNATIVE WAY OF CHECKING IF A SET OF VECTORS IS DEP OR INDEP IS TO CALCULATE THE RANK OF THE MATRIX THAT HAS AS ROWS OR COLUMNS THE COMPONENTS OF THE VECTORS

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad v_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \quad v_3 = \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}$$

$$
M = \begin{array}{c} \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \end{pmatrix} \end{array} \quad 3\times 3
$$

$= 2$

$\det M = -4 - 2 + 6 = 0$

hence the $\mathrm{rk} M \neq 3$ and the vectors are dependent

$\mathrm{rk} M = 2$ this means that among v_1, v_2, v_3 only two of them are independent. That choice of minor tells me that v_1 and v_2 are independent

whereas v_3 can be written as a linear combination of v_1 and v_2

EX. For the following lines in $\mathbb{R}^3$

$$r: \begin{cases} x - y + z = 0 \\ x + 2y + 2z = 1 \end{cases} \qquad s: \begin{cases} x - 4y - 4 = 0 \\ 3y + z + 3 = 0 \end{cases}$$

a) Determine if the line r passes through the point $P(2; 1, -1)$

$$r \begin{cases} x - y + z = 0 & 2 - 1 - 1 = 0 \text{ TRUE} \\ x + 2y + 2z = 1 & 2 + \cancel{2} - \cancel{2} = 1 \text{ FALSE} \end{cases}$$

P doesn't belong to r

b) Determine if the line s passes through the point $Q(-4, -2, 3)$

$$s: \begin{cases} x - 4y - 4 = 0 & -4 + 8 - 4 = 0 \text{ TRUE} \\ 3y + z + 3 = 0 & -6 + 3 + 3 = 0 \text{ TRUE} \end{cases}$$

$\Rightarrow$ Q belongs to s

c) Prove that the lines r and s are parallel

$$r\begin{cases} x-y+z=0 \\ x+2y+2z=1 \end{cases} \qquad s\begin{cases} x-4y-4=0 \\ 3y+z+3=0 \end{cases}$$

$$r\begin{cases} x-y=-t \\ x+2y=1-2t \\ z=t \end{cases} \qquad \begin{cases} x=y-t \\ y-t+2y=1-2t \\ z=t \end{cases}$$

$$\begin{cases} x=y-t \\ 3y=1-t \\ z=t \end{cases} \qquad \begin{cases} x=\frac{1}{3}-\frac{1}{3}t-t \\ y=\frac{1}{3}-\frac{1}{3}t \\ z=t \end{cases}$$

$$\begin{cases} x=\frac{1}{3}-\frac{4}{3}t \\ y=\frac{1}{3}-\frac{1}{3}t \\ z=t \end{cases} \qquad P = P_0 + t\,u$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 0 \end{pmatrix} + t\begin{pmatrix} -\frac{4}{3} \\ -\frac{1}{3} \\ 1 \end{pmatrix}$$

$$s\begin{cases} x-4y-4=0 \\ 3y+z+3=0 \end{cases}$$

$$\begin{cases} x= 4+4\left(-1-\frac{1}{3}t\right) \\ y=-1 \quad -\frac{1}{3}t \\ z=t \end{cases}$$

$$\begin{cases} x= \quad -\frac{4}{3}t \\ y=-1-\frac{1}{3}t \\ z=t \end{cases}$$

$$\overset{P}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \overset{P_0}{\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}} + t \overset{u}{\begin{pmatrix} -\frac{4}{3} \\ -\frac{1}{3} \\ 1 \end{pmatrix}}$$