

Mathematics II

Practice 1 03/25/2022

1. For any of these matrices compute all the eigenvalues and the corresponding eigenvectors:

$$\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}, \quad \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 3 & 0 & 0 \\ -4 & -1 & -8 \\ 0 & 0 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 3 & 4 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 2 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

2. Determine if the following matrices

$$A = \begin{pmatrix} 4 & -1 & 1 \\ 2 & 0 & 1 \\ -2 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \end{pmatrix}$$

are diagonalizable and, if so, find diagonal matrices D_A and D_B and invertible matrices P_A and P_B such that respectively $A = P_A D_A P_A^{-1}$ and $B = P_B D_B P_B^{-1}$.

3. Determine for what values of $h \in \mathbb{R}$ the following matrix is diagonalizable:

$$\begin{pmatrix} 1 & 0 & h^2 \\ 0 & h & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

4. Describe the domain of each of the following functions of two variables:

$$\begin{array}{ll}
 a) f(x, y) = \frac{x^3 y^2 + 1}{\sqrt{x^2 + y^2}} & b) f(x, y) = \frac{x + 1}{y + 3} \\
 c) f(x, y) = x^2 + xy + \sqrt{y} & d) f(x, y) = \log(2x - y) \\
 e) f(x, y) = \sqrt{1 - x^2 - y^2} & f) f(x, y) = x e^{\frac{y}{\sqrt{x^2 - 5x + 6}}} \\
 g) f(x, y) = \frac{\log(x^2 + y^4) + \sin(x^2 y)}{\sqrt{x^2 + y^2 + 3}} & h) f(x, y) = \frac{\log(x^2 + y^2 - 1)}{\sqrt{9 - x^2 - y^2}}
 \end{array}$$

5. Describe the level curves of the following functions:

$$\begin{array}{ll}
 a) f(x, y) = 3x - y & b) f(x, y) = 2x^2 + y \\
 c) f(x, y) = \log(x + y - 3) & d) f(x, y) = \frac{1}{2 - x^2 - y^2}
 \end{array}$$

6. Compute the partial derivatives of the following functions:

$$\begin{array}{ll}
 a) f(x, y) = x^3 y + 5xy^2 - 3x\sqrt{y} + 7y & b) f(x, y) = \frac{y + 1}{x^2 + 3} \\
 c) f(x, y) = \frac{xy - 1}{y + 2} & d) f(x, y) = x \sin^2(xy^3) \\
 e) f(x, y) = x e^y + \log\left(\frac{y}{x}\right) & f) f(x, y) = x^2 e^{y/x} \\
 g) f(x, y) = x \log(y^2) \cos(3x - 1) & h) f(x, y) = \frac{x \sin(y)}{x^2 + y^4 + 2}
 \end{array}$$

7. Find all the stationary points of the following functions:

$$\begin{array}{ll}
 a) f(x, y) = 3x^2 + 3x^2 y + xy - 1 & b) f(x, y) = x^2 y + 3xy + y^3 \\
 c) f(x, y) = x^3 + 3x^2 y + y^2 & d) f(x, y) = x^2 y^3 + xy^2 - 3y \\
 e) f(x, y) = x y e^{x-y} & f) f(x, y) = \sqrt{x^3 + 2xy^2} \\
 g) f(x, y) = \sqrt{x^2 + 3x^2 y^2 + 2y^4 + 1} & h) f(x, y) = y \sin(x)
 \end{array}$$