MATHEMATICS

2nd Midterm Simulation

April 1, 2022, A.Y. 2021/2022

Any kind of electronic device (calculators, smartphones, smartwatches etc..) is FORBIDDEN. Put all your devices in the location indicated by the examiners. If an attendee violates this rule (even if the device is switched off or in offline mode), she/he will be immediately expelled from the exam session and her/his exam will be invalidated.

Examiners will mark only THIS A3 paper SHOWING THE UNIVERSITY LOGO on the upper left corner. Any other additional papers (drafts, notes, scribbles or anything else) will not be taken into consideration. Use a clear and clean handwriting. Unclear or ambiguous sentences may result in a negative impact on the final grade of the exam.

Remember to always double check the consistency of your results. EVEN IF CORRECT, inconsistent statements will result in a negative impact on the final grade of the exam.

MARKS: you get 3 points for each correct answer, 0 points for unanswered questions, -1 points for each incorrect answers. Open questions are not penalized.

Time for the test: 1 hour and 15 minutes

MATRICOLA .... *Lastname* .... *Name* ....

1. *Compute the following integral*

$$\int \frac{3x^2+2-4x}{3x} dx$$

☐ *A.* $\frac{x^2}{2}+\frac{2}{3}\ln|x|-\frac{4}{3}x+c$

☐ *B.* $\frac{x^2}{2}+2\ln|x|-\frac{4}{3}x+c$

☐ *C.* $x+\frac{1}{x}-\frac{4}{3}+c$

☐ *D.* $x^2+\ln|x|-\frac{4}{3}x+c$

2. *Compute the following definite integral:*

$$\int_0^{\sqrt{\ln 3}} xe^{x^2} dx$$

☐ *A.* 1

☐ *B.* 2

☐ *C.* 0

☐ *D.* e

3. *Compute the domain and the range of the following function of two variables*

$$z = f(x, y) = \frac{\sqrt{x+y+1}}{x-1}$$

☐ A. $D_f = \{(x, y) \in \mathbb{R}^2 \ ; \ y < -x-1\} \quad R_f = (0, +\infty)$

☐ B. $D_f = \{(x, y) \in \mathbb{R}^2 \ ; \ y \geq -x-1 \text{ and } x \neq 1\} \quad R_f = (-\infty, +\infty)$

☐ C. $D_f = \{(x, y) \in \mathbb{R}^2 \ ; \ x \geq y-1 \ ; \ x \neq 1\} \quad R_f = (-\infty, +\infty)$

☐ D. $D_f = \{(x, y) \in \mathbb{R}^2 \ ; \ y \geq -x-1 \text{ and } x \neq 1\} \quad R_f = [0, +\infty)$

4. *Discuss if the following system is consistent as k changes and specify the number of solutions.*

$$\begin{cases} x+y+kz & = 2 \\ x+y+3z & = 2 \\ 2x+ky-z & = 1 \end{cases}$$

☐ A. *The system is consistent with a unique solution if $k \neq 2$ and $k \neq 3$, if $k = 3$ the system is consistent with ∞^1 solutions, if $k = 2$ the system is inconsistent*

☐ B. *The system is consistent with a unique solution if $k \neq 2$ and $k \neq 3$, if $k = 2$ the system is consistent with ∞^1 solutions, if $k = 3$ the system is inconsistent*

☐ C. *The system is consistent with a unique solution if $k \neq -2$ and $k \neq -3$, if $k = -3$ and $k = -2$ the system is consistent*

☐ D. *The system is consistent with a unique solution if $k \neq 2$ and $k \neq 3$, if $k = 3$ and $k = 2$ the system is consistent with ∞^1 solutions.*

5. *Evaluate the following limit*

$$\lim_{x \to 0} \frac{1}{x^3} \int_0^x (1 - \cos^2 t) dt$$

Then,

☐ A. 1 ☐ B. 3

☐ C. 0 ☐ D. $\frac{1}{3}$

6. *Identify the cartesian equation of the plane passing through the point $A(1, 2, 0)$ and orthogonal to the vector $\mathbf{u} = (1, 1, 1)$*

☐ A. $x+y+z=3$

☐ B. $x-y-z+3=0$

☐ C. $2x+2y-z=3$

☐ D. $x-y+z=2$

7. *For which values of k are the following three vectors dependent?*

$$\mathbf{v} = (1,1,1) \quad \mathbf{u} = (3,2,k) \quad \mathbf{w} = (0,k,k)$$

- ☐ A. $k = 1, \;\; k = 0$
- ☐ B. $k = -1, \;\; k = 2$
- ☐ C. $k = 0, \;\; k = 2$
- ☐ D. $k = 1, \;\; k = 2$

8. *Compute all eigenvalues of the matrix*

$$A = \begin{pmatrix} -1 & 0 & 0 \\ -2 & -1 & -2 \\ 2 & 0 & 1 \end{pmatrix}$$

and indicate its algebraic multiplicity (AM).

- ☐ A. $\lambda_1 = 1$ *with* $AM(\lambda_1) = 1$, $\lambda_2 = 2$ *with* $AM(\lambda_2) = 2$
- ☐ B. $\lambda_1 = -1$ *with* $AM(\lambda_1) = 1$, $\lambda_2 = 1$ *with* $AM(\lambda_2) = 1$, $\lambda_3 = 2$ *with* $AM(\lambda_3) = 1$,
- ☐ C. $\lambda_1 = 1$ *with* $AM(\lambda_1) = 1$, $\lambda_2 = -1$ *with* $AM(\lambda_2) = 2$
- ☐ D. $\lambda_1 = -2$ *with* $AM(\lambda_1) = 1$, $\lambda_2 = -1$ *with* $AM(\lambda_2) = 1$, $\lambda_3 = 2$ *with* $AM(\lambda_3) = 1$,

9. *(Open question: please answer in the space below. Indicate all necessary steps to solve the exercise)*
Describe the integration by parts method

10. *(Open question: please answer in the space below) Give definition of similar matrices and prove that similar matrices have same eigenvalues*

11. *(Open question: please answer in the space below. Indicate all necessary steps to solve the exercise) Compute all stationary points of the function below and determine their nature:*

$$f(x, y) = y^2[x^2 + y^2 - 2(x + y) + 2]$$