# Quantitative Methods – I (Statistics)

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# Chapter 2

## Organizing and Graphing Data

# Organizing and Graphing data: Road Map

1. Distributions
2. Frequency Distributions: Absolute, Relative, Percentage
3. Cumulative Distributions: Absolute, Relative, Percentage
4. Appropriate Graphs: Pie charts, Bar charts, Pareto charts, Histogram
5. Shapes of the distributions

# Univariate and multivariate distributions

Raw statistical information takes the form of a **unit distribution** **simple** (univariate) o **multiple** (multivariate), depending on the number of variables that are considered.

Univariate distribution: a single variable is considered.

| Units | Values of $X$ |
|---|---|
| $u_1$ | $x_1$ |
| $u_2$ | $x_2$ |
| $\vdots$ | $\vdots$ |
| $u_i$ | $x_i$ |
| $\vdots$ | $\vdots$ |
| $u_n$ | $x_n$ |

We wish to describe and summarize the main facts about $X$.

# Univariate and multivariate distributions

## Multivariate distribution:

| Units | Values of $X$ | Values of $Y$ | ... | Values of $Z$ |
|---|---|---|---|---|
| $u_1$ | $x_1$ | $y_1$ | ... | $z_1$ |
| $u_2$ | $x_2$ | $y_2$ | ... | $z_1$ |
| $\vdots$ | $\vdots$ | $\vdots$ | ... | $\vdots$ |
| $u_i$ | $x_i$ | $y_i$ | ... | $z_i$ |
| $\vdots$ | $\vdots$ | $\vdots$ | ... | $\vdots$ |
| $u_n$ | $x_n$ | $y_n$ | ... | $z_n$ |

We can also discuss dependence and association among the variables.

# Frequency Distributions

**Raw data** (i.e. in their original form): typically so large to look meaningless ➔ immense importance of Descriptive Statistics.

**Ex:** age of 50 students…

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 21 | 19 | 24 | 25 | 29 | 34 | 26 | 27 | 37 | 33 |
| 18 | 20 | 19 | 22 | 19 | 19 | 25 | 22 | 25 | 23 |
| 25 | 19 | 31 | 19 | 23 | 18 | 23 | 19 | 23 | 26 |
| 22 | 28 | 21 | 20 | 22 | 22 | 21 | 20 | 19 | 21 |
| 25 | 23 | 18 | 37 | 27 | 23 | 21 | 25 | 21 | 24 |

# Frequency Distributions

A frequency distribution is a tabular way of summarizing the distribution of a character.

Collection of Raw Data: ex. Age of 50 students

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 21 | 19 | 24 | 25 | 29 | 34 | 26 | 27 | 37 | 33 |
| 18 | 20 | 19 | 22 | 19 | 19 | 25 | 22 | 25 | 23 |
| 25 | 19 | 31 | 19 | 23 | 18 | 23 | 19 | 23 | 26 |
| 22 | 28 | 21 | 20 | 22 | 22 | 21 | 20 | 19 | 21 |
| 25 | 23 | 18 | 37 | 27 | 23 | 21 | 25 | 21 | 24 |

*From raw data to frequency distribution*

Frequency Distribution

| Age | Nr. Of Students |
|---|---|
| 18 | 3 |
| 19 | 8 |
| 20 | 3 |
| 21 | 6 |
| 22 | 5 |
| 23 | 6 |
| 24 | 2 |
| 25 | 6 |
| 26 | 2 |
| 27 | 2 |
| 28 | 1 |
| 29 | 1 |
| 31 | 1 |
| 33 | 1 |
| 34 | 1 |
| 37 | 2 |
| Total | 50 |

# Frequency Distribution – Absolute ($f_i$)

Lists all values/categories ($x_i$) and associated number of elements ($f_i$)

**Ex:** age of 50 students…

**Variable**

**Values ($x_i$) (Categories/ Classes)**

| Age | Nr. Of Students |
|---|---|
| 18 | 3 |
| 19 | 8 |
| 20 | 3 |
| 21 | 6 |
| 22 | 5 |
| 23 | 6 |
| 24 | 2 |
| 25 | 6 |
| 26 | 2 |
| 27 | 2 |
| 28 | 1 |
| 29 | 1 |
| 31 | 1 |
| 33 | 1 |
| 34 | 1 |
| 37 | 2 |
| Total | 50 |

**Absolute Frequencies of $i$-thValue (or $i$–th Category / Class) ➔ ($f_i$)**

**$N=\sum f_i$ = total nr of elements**

# Frequency Distribution – Absolute ($f_i$)

The frequency distribution is used also for qualitative and quantitative variables.

Lists all values/categories ($x_i$) and associated number of elements ($f_i$)

**Ex:** worries about reaching the end of the month…

Variable → | **Response** | **Number of Adults** ← Frequency column

| Response | Number of Adults |
|---|---|
| Very worried | 162 |
| Moderately worried | 203 |
| Category → Not too worried | 305 ← Frequency |
| Not worried at all | 325 |
| Others | 20 |
| | Sum = 1015 |

# Organizing data

**1.6** The following table lists the number of billionaires in eight countries as of February 2011, as reported in The New York Times of July 27, 2011.

| Country | Number of Billionaires |
|---|---|
| United States | 413 |
| China | 115 |
| Russia | 101 |
| India | 55 |
| Germany | 52 |
| Britain | 32 |
| Brazil | 30 |
| Japan | 26 |

Variable

Observation

Character (or category)

Data set

*Source:* Forbes, International Monetary Fund.

Briefly explain the meaning of a member, a variable, a measurement, and a data set with reference to this table.

a. What is the variable for this data set?
b. How many observations are in this data set?
c. How many characters does this data set contain?

a. Number of Billionaires by Country
b. n=413+115+101+55+52+32+30+26=824
c. 8 *(United States, China, etc.)*

# Frequency distributions for quantitative variables

In general, a frequency table provides a useful summary if the number of values $x_j$ is small.
For discrete variable with a large number of values and for continuous variables we have to group the values into classes to achieve an adequate level of synthesis. However, the grouping is by and large arbitrary and the mapping of the unit distribution $(u_i, x_i)$ into a frequency distribution carries with it an information loss.
The construction of the frequency distribution entails:

i subdividing the range of values that $X$ can take into nonoverlapping intervals (aka classes)

ii counting the number of observations falling within each class

# Frequency distributions for quantitative variables

For continuous variables there are several alternative ways of defining the classes:

Left-open, right-closed classes:

$x_j \dashv x_{j+1}, \quad (x_j, x_{j+1}], \quad x_j < x \leq x_{j+1}$

Left-closed, right-open classes:

$x_j \vdash x_{j+1}, \quad [x_j, x_{j+1}), \quad x_j \leq x < x_{j+1}$

The *size* of the class is defined by the difference between the upper and the lower bounds $(x_{j+1} - x_j)$.

# Distribution in classes

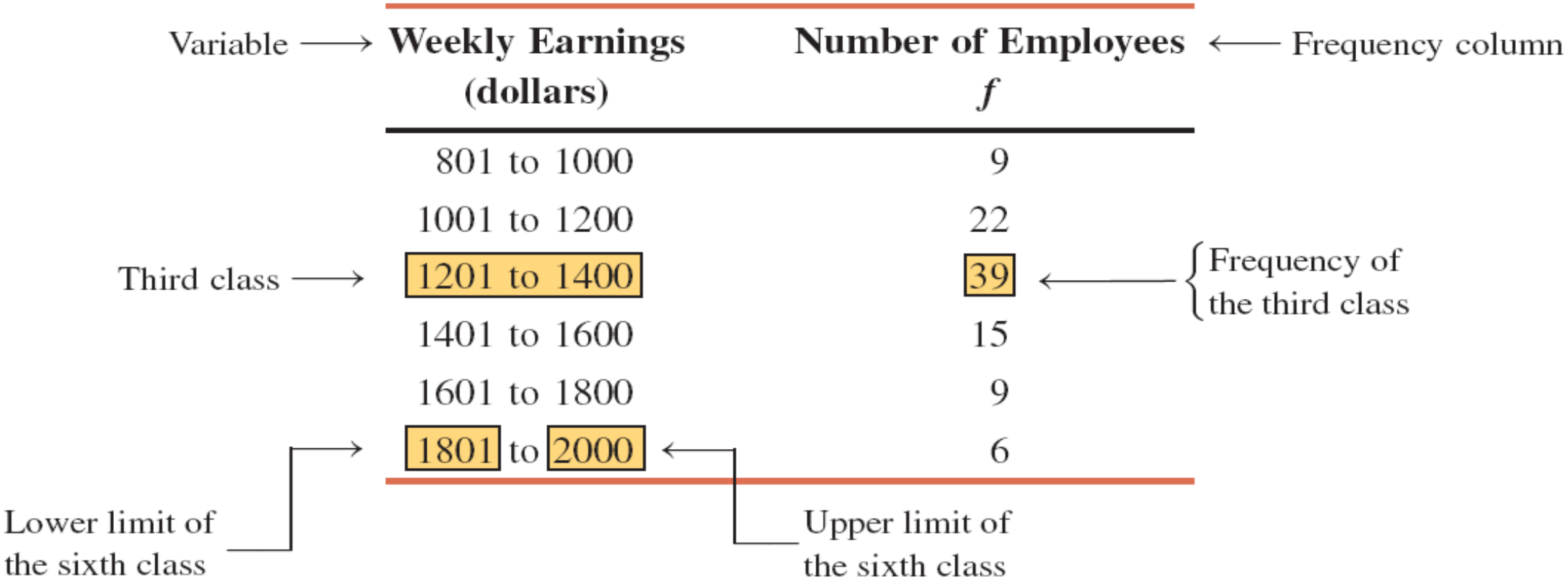
More generally, for quantitative variables is useful to subdivide the range of values that X can take into mutually exclusive and exhaustive intervals or classes

Classes ($x_i$) and associated number of elements ($f_i$ or $m_i$)

# Distribution in classes

Classes ($x_i$) and associated number of elements ($f_i$ or $m_i$)

**Ex:** weekly earnings (grouped into classes)



# Frequency Distribution – Excercise

The following data give the total number of iPads® sold by an internet store in 30 days.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 25 | 11 | 15 | 29 | 22 | 10 | 5 | 17 | 21 | 22 | 13 | 26 |
| 16 | 18 | 12 | 9 | 26 | 20 | 16 | 23 | 14 | 19 | 23 | 20 | 16 |
| 27 | 16 | 21 | 14 | | | | | | | | | |

Construct a frequency distribution table using the following classes:

5-9, 10-14, 15-19, 20-24, 25-29

# Frequency Distribution – Excercise

The following data give the total number of iPads® sold by an internet store in 30 days.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 25 | 11 | 15 | 29 | 22 | 10 | 5 | 17 | 21 | 22 | 13 | 26 |
| 16 | 18 | 12 | 9 | 26 | 20 | 16 | 23 | 14 | 19 | 23 | 20 | 16 |
| 27 | 16 | 21 | 14 | | | | | | | | | |

| Classes | $m_i$ |
|---|---|
| 5-9 | |
| 10-14 | |
| 15-19 | |
| 20-24 | |
| 25-29 | |

# Frequency Distribution – Excercise

The following data give the total number of iPads® sold by an internet store in 30 days.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 25 | 11 | 15 | 29 | 22 | 10 | 5 | 17 | 21 | 22 | 13 | 26 |
| 16 | 18 | 12 | 9 | 26 | 20 | 16 | 23 | 14 | 19 | 23 | 20 | 16 |
| 27 | 16 | 21 | 14 | | | | | | | | | |

| Classes | $m_i$ |
|---|---|
| 5-9 | 3 |
| 10-14 | |
| 15-19 | |
| 20-24 | |
| 25-29 | |

# Frequency Distribution – Excercise

The following data give the total number of iPads® sold by an internet store in 30 days.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 25 | 11 | 15 | 29 | 22 | 10 | 5 | 17 | 21 | 22 | 13 | 26 |
| 16 | 18 | 12 | 9 | 26 | 20 | 16 | 23 | 14 | 19 | 23 | 20 | 16 |
| 27 | 16 | 21 | 14 | | | | | | | | | |

| Classes | $m_i$ |
|---|---|
| 5-9 | 3 |
| 10-14 | 6 |
| 15-19 | 8 |
| 20-24 | 8 |
| 25-29 | 5 |

# Frequency Distributions – Relative and Percentage

## Relative Frequencies

$$rf_i = \frac{f_i}{n} \qquad \text{with} \sum rf_i = 1$$

## Percentage Distribution

$$p_i = 100 \times rf_i \quad \text{with} \sum p_i = 100$$

# Frequency Distributions – Relative and Percentage

The relative frequency $f_j$ by which $x_j$ occurs si the ratio of $n_j$ to the total frequency, $n$:

$$f_j = \frac{n_j}{n}, \quad j = 1, \ldots, K.$$

The percentage frequency is obtained by multiplying $f_j$ by 100:

$$p_j = 100 \cdot \frac{n_j}{n} = 100 \cdot f_j, \quad j = 1, \ldots, K.$$

The following obviously hold:

$$0 \leq f_j \leq 1, \quad \sum_{j=1}^{K} f_j = 1; \quad 0 \leq p_j \leq 100, \quad \sum_{j=1}^{K} p_j = 100.$$

# Relative frequency

Relative frequency distribution:

| Values of $X$ | Frequency |
|---|---|
| $x_1$ | $f_1$ |
| $x_2$ | $f_2$ |
| $\vdots$ | $\vdots$ |
| $x_j$ | $f_j$ |
| $\vdots$ | $\vdots$ |
| $x_K$ | $f_K$ |
| Total | 1 |

# Percentage frequency

Percentage frequency distribution:

| Values of $X$ | Relative frequency | | Percentage freq. |
|---|---|---|---|
| $x_1$ | $f_1$ | x 100 | $p_1$ |
| $x_2$ | $f_2$ | x 100 | $p_2$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $x_j$ | $f_j$ | x 100 | $p_j$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $x_K$ | $f_K$ | x 100 | $p_K$ |
| Total | 1 | | 100 |

# Cumulative frequency

For variables that are measured on an ordinal or a quantitative scale we can count the number of cases which have a value not greater than $x_j$.
This is known as a cumulative frequency.
More formally, the absolute cumulative frequency is defined as

$$N_j = \sum_{k=1}^{j} n_k, \quad j = 1, \ldots, K.$$

# Cumulative frequency

| Values of $X$ | Frequency | Cumulative freq. |
|---|---|---|
| $x_1$ | $n_1$ | $N_1 = n_1$ |
| $x_2$ | $n_2$ | $N_2 = n_1 + n_2$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $x_j$ | $n_j$ | $N_j = n_1 + n_2 + \cdots + n_j$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $x_K$ | $n_K$ | $n$ |
| Totale | $n$ | |

# Cumulative relative and percentage frequency

Simularly, we define

- Cumulative relative frequency:

$$F_j = \sum_{k=1}^{j} f_k, j = 1, \ldots, K$$

(note that $F_K = 1$)

- Cumulative percentage frequency:

$$P_j = \sum_{k=1}^{j} p_k, j = 1, \ldots, K$$

(note that $P_K = 100$)

# Cumulative relative frequency

| Values of $X$ | Rel. freq. | Cumulative rel. freq. |
|---|---|---|
| $x_1$ | $f_1$ | $F_1 = f_1$ |
| $x_2$ | $f_2$ | $F_2 = f_1 + f_2$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $x_j$ | $f_j$ | $F_j = f_1 + f_2 + \cdots + f_j$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $x_K$ | $f_K$ | $1$ |
| Total | $1$ | |

# Frequency Distributions – Relative and Percentage

**Ex:** Federal Tax (in classes)

| Federal and State Tax (in cents) | Frequency | Relative Frequency | Percentage |
|---|---|---|---|
| 27 to less than 36 | 5 | .10 | 10 |
| 36 to less than 45 | 21 | .42 | 42 |
| 45 to less than 54 | 16 | .32 | 32 |
| 54 to less than 63 | 6 | .12 | 12 |
| 63 to less than 72 | 2 | .04 | 4 |
| | Sum = 50 | Sum = 1.00 | Sum = 100 |

# Cumulative Distribution – Absolute

For each value (category/class) gives the total number of observations taking that value or lower (or falling below the upper boundary of each class)

| Federal and State Tax (in cents) | Frequency $f_i$ | Cumulative Frequency |
|---|---|---|
| **27\|-36** | 5 | 5 |
| **36\|-45** | 21 | 26 |
| **45\|-54** | 16 | 42 |
| **54\|-63** | 6 | 48 |
| **63\|-72** | 2 | 50 |
| Total | 50 | |

# Cumulative Distribution: Relative and Percentage

| Federal and State Tax (in cents) | Frequency | Cumulative Frequency | Relative Frequency | Cumulative Relative Frequency | Percentage Distribution | Cumulative Percentage Distribution |
|---|---|---|---|---|---|---|
| **27\|-36** | 5 | 5 | 0.1 | 0.1 | 10.0 | 10.0 |
| **36\|-45** | 21 | 26 | 0.42 | 0.52 | 42.0 | 52.0 |
| **45\|-54** | 16 | 42 | 0.32 | 0.84 | 32.0 | 84.0 |
| **54\|-63** | 6 | 48 | 0.12 | 0.96 | 12.0 | 96.0 |
| **63\|-72** | 2 | 50 | 0.04 | 1 | 4.0 | 100.0 |
| Total | 50 | | 1.00 | | 100.0 | |

# Graphic Presentation

*It is important to choose the appropriate graphs to make statistical information coherent.*

- **The Pie Chart**
- **The Bar Graph**
- **The Statistical Map**
- **The Histogram**
- **Times Series Charts**
- **Distortions in Graphs**

# Graphing data

Appropriate Graphs:

- Pie charts, Bar charts, Pareto charts,

  ➔ for Qualitative and Quantitative Discrete

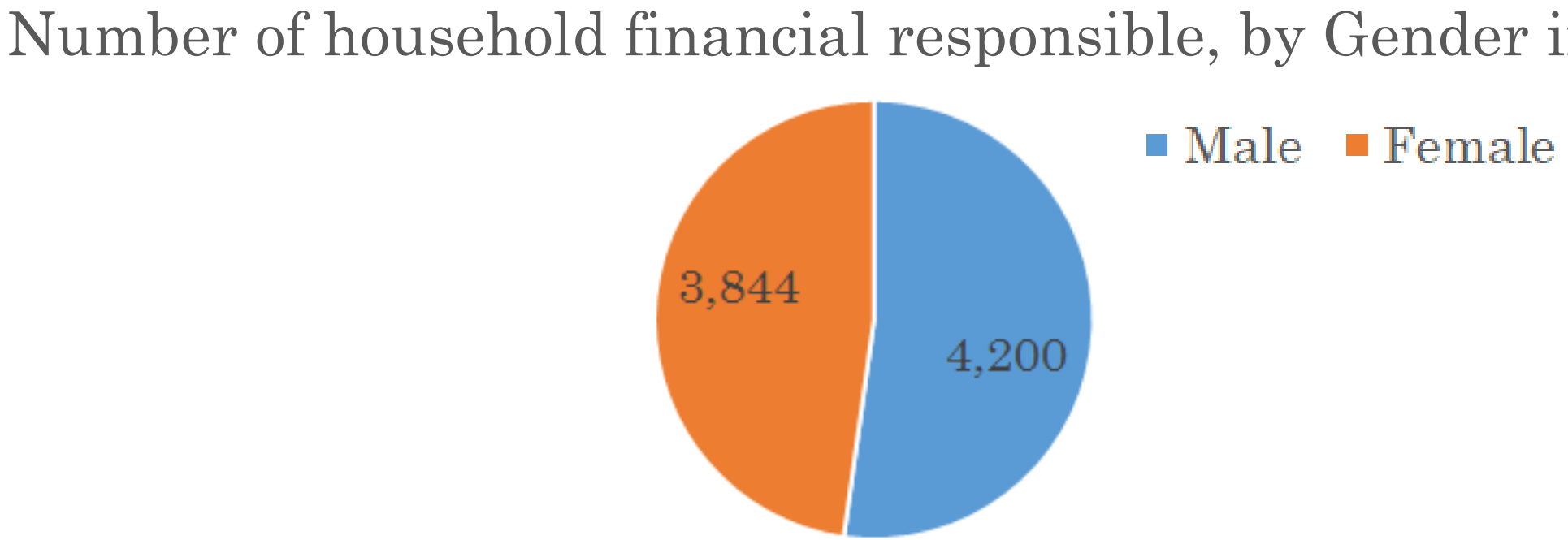
- Histogram

  ➔ for Quantitative Continuous (in classes)

# Graphs – Pie Chart

A circle divided into portions that represent the relative frequencies or percentages of each category/value.

Appropriate for: Qualitative, Quantitative Discrete (few values)



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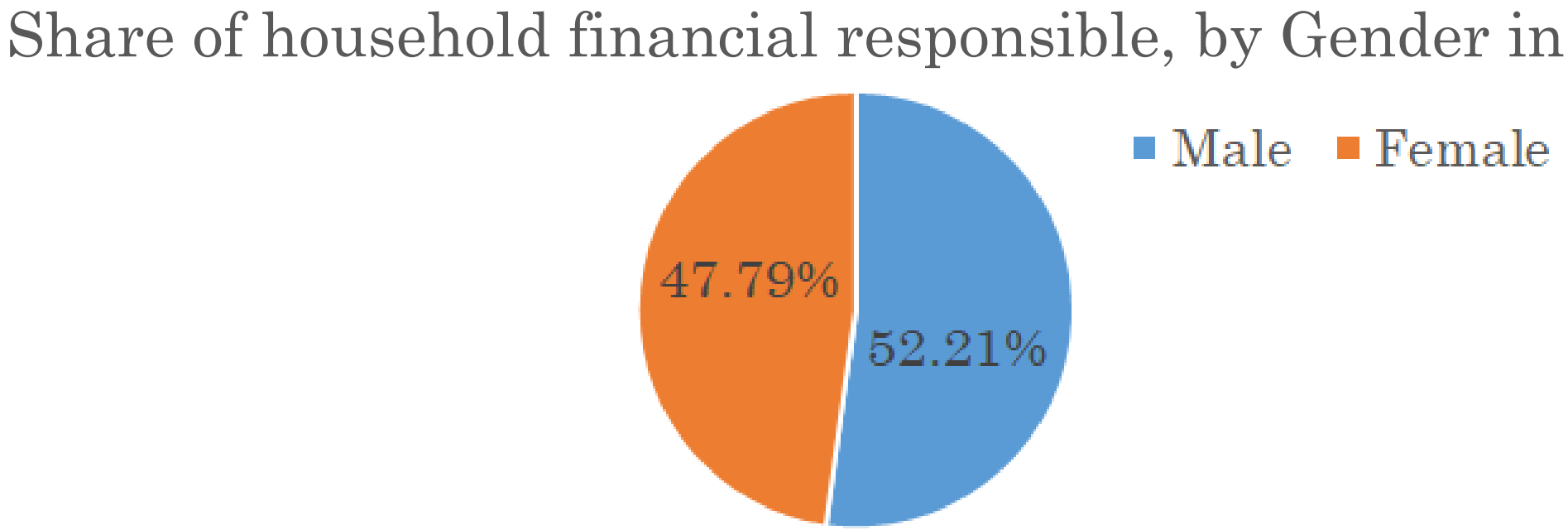
A circle divided into portions that represent the relative frequencies or percentages of each category/value.

Appropriate for: Qualitative, Quantitative Discrete (few values)



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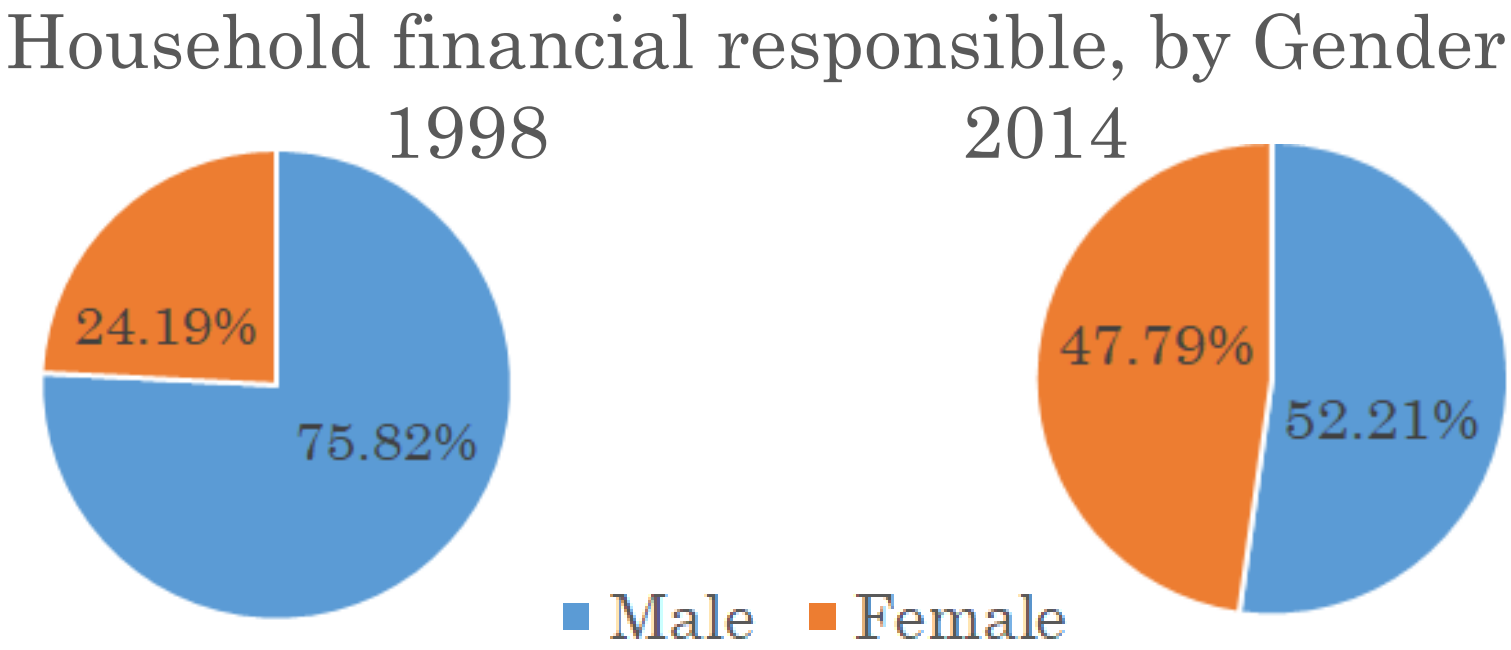
A circle divided into portions that represent the relative frequencies or percentages of each category/value.

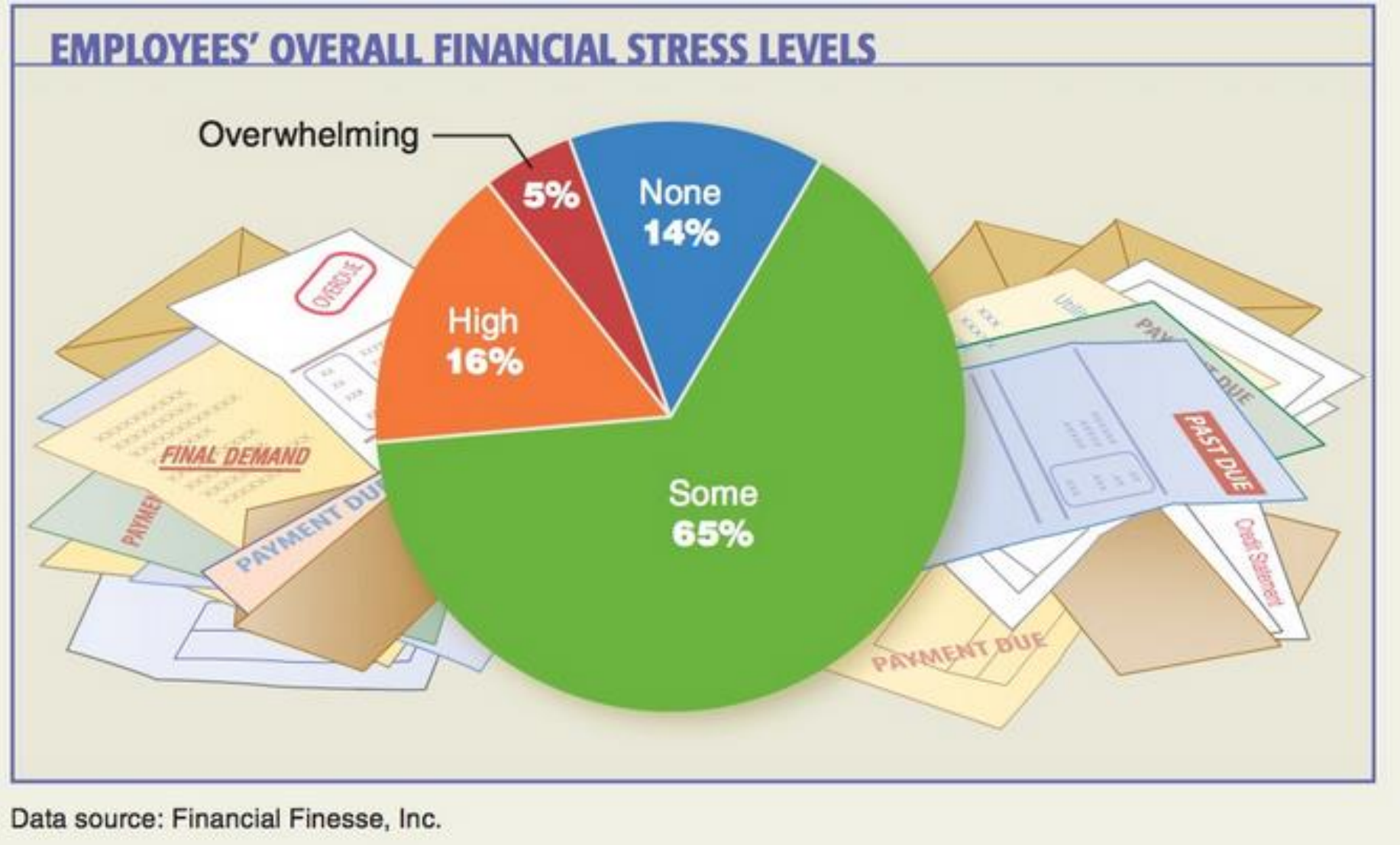
Appropriate for: Qualitative, Quantitative Discrete (few values)



# Graphs – Pie Chart


Data source: Gallup poll of U.S. adults aged 18 and older conducted July 9–12, 2012

# Graphs – Pie Chart



# Graphs – Pie Chart

- ***Pie chart:*** a graph showing the differences in frequencies or percentages among categories of a **nominal** or an **ordinal** variable. The categories are displayed as segments of a circle whose pieces add up to 100 percent of the total frequencies.

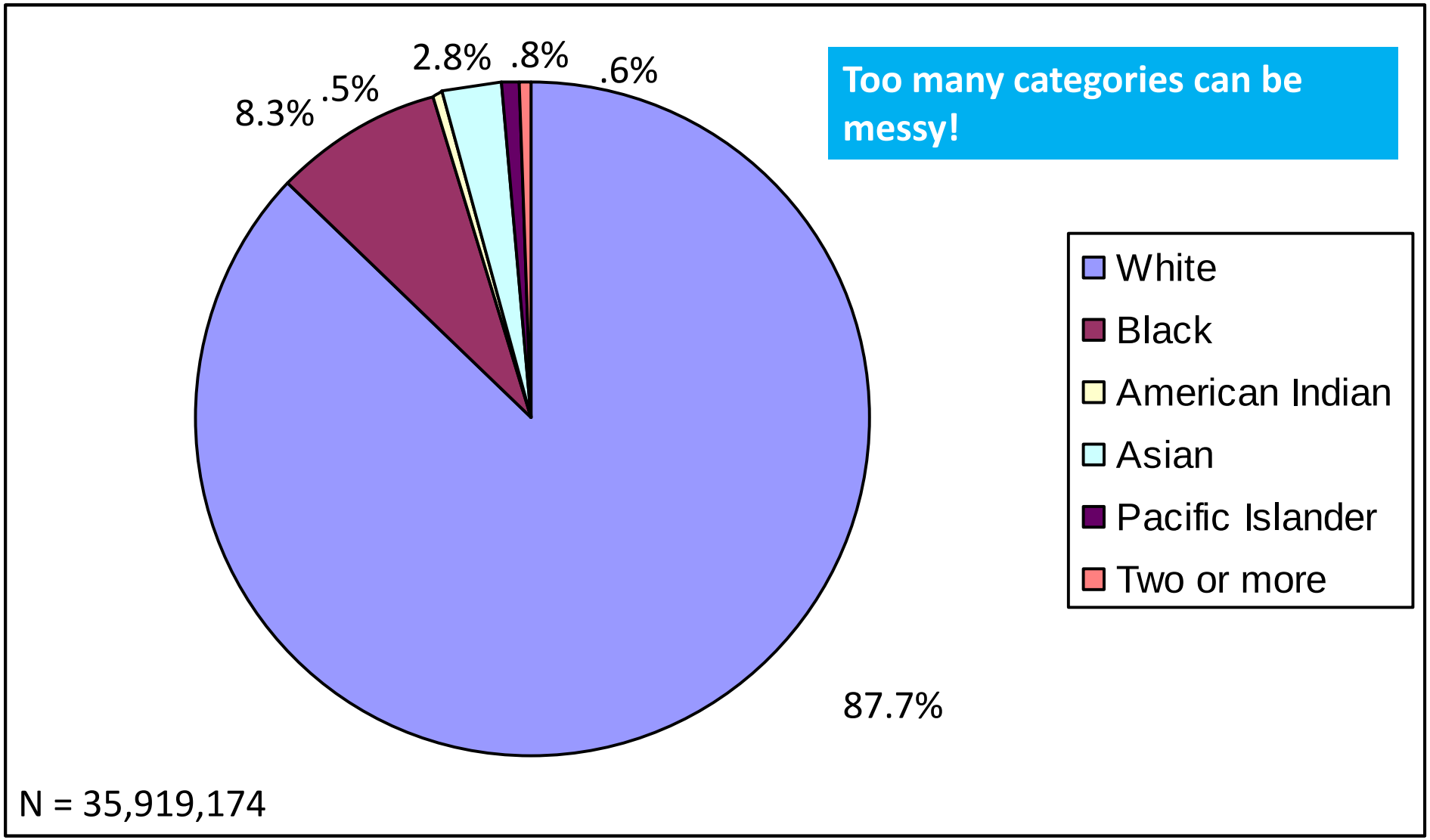



**Figure 3.1 Annual Estimates of U.S. Population 65 Years and Over by Race, 2003**

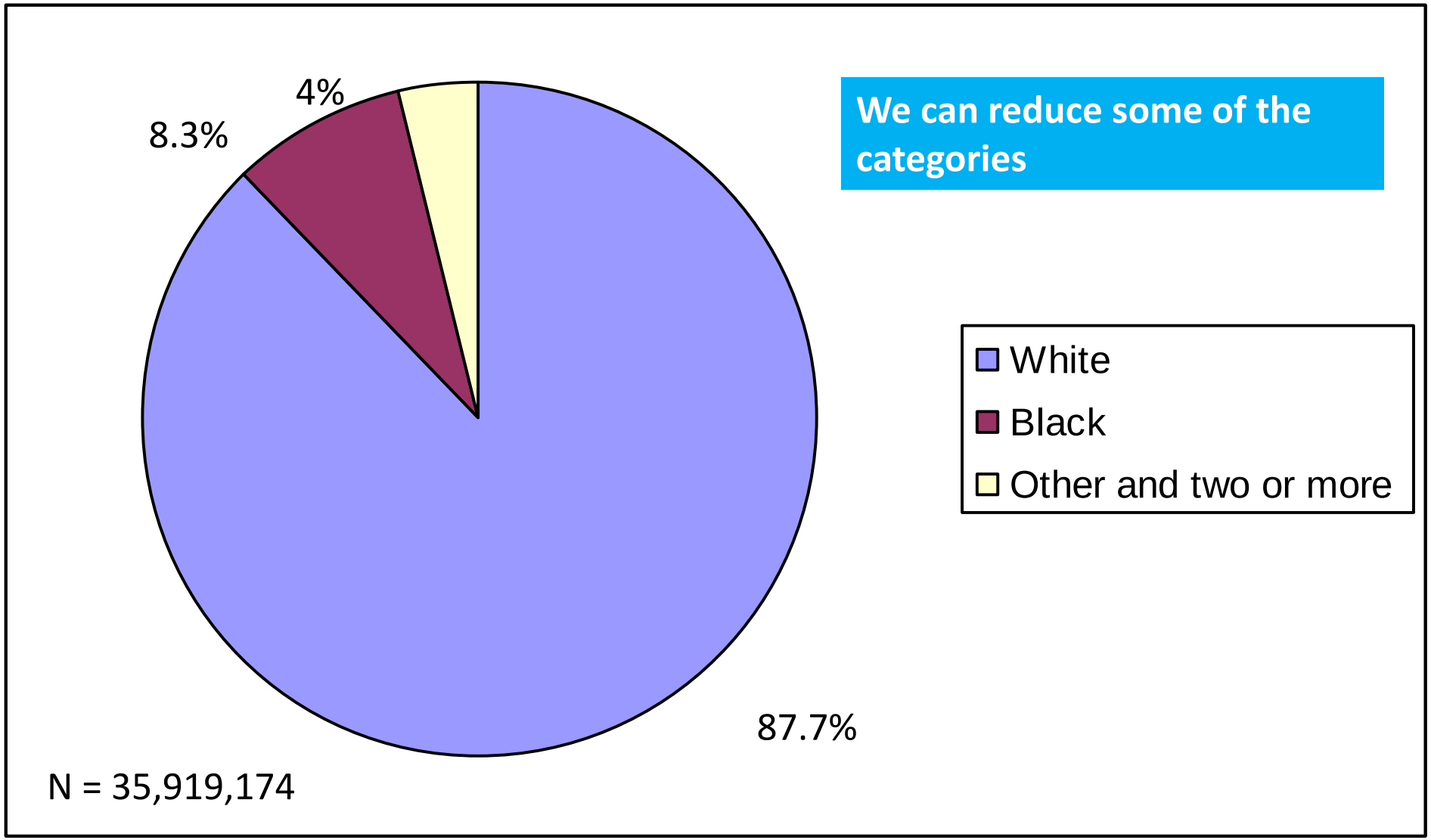
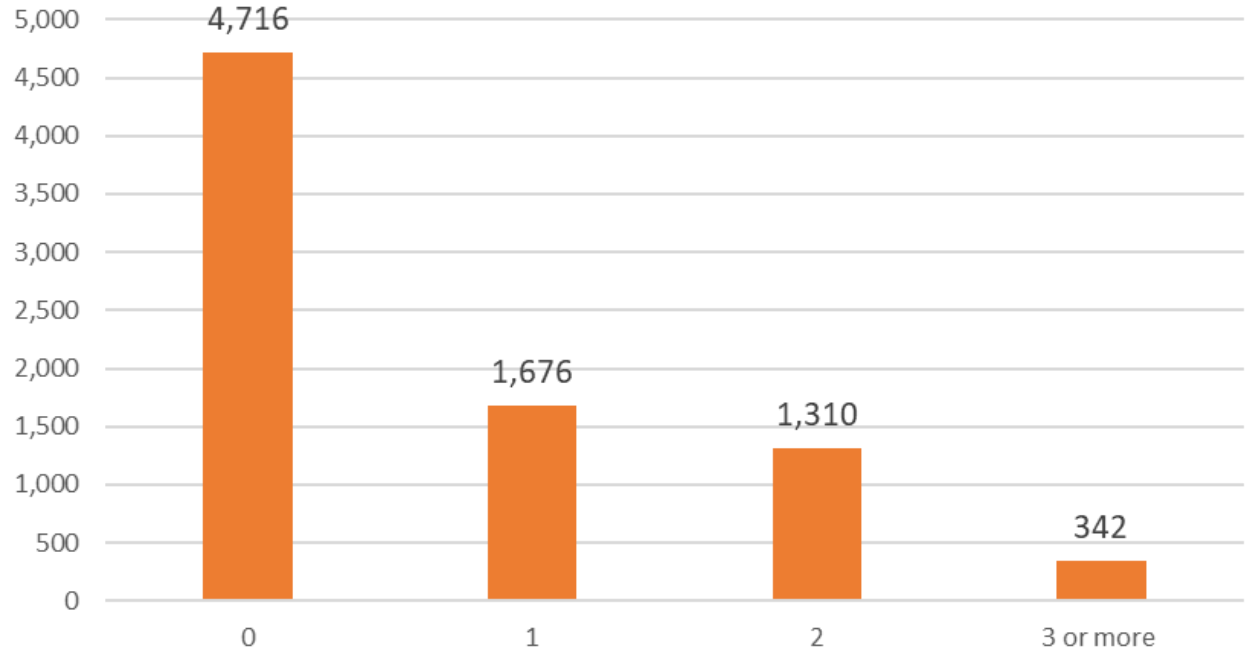



**Figure 3.2 Annual Estimates of U.S. Population 65 Years and Over, 2003**

# Graphs – Bar Chart

Each bar's height represents the frequencies (absolute, relative, percentage) of each category/value.

Appropriate for: Qualitative, Quantitative Discrete

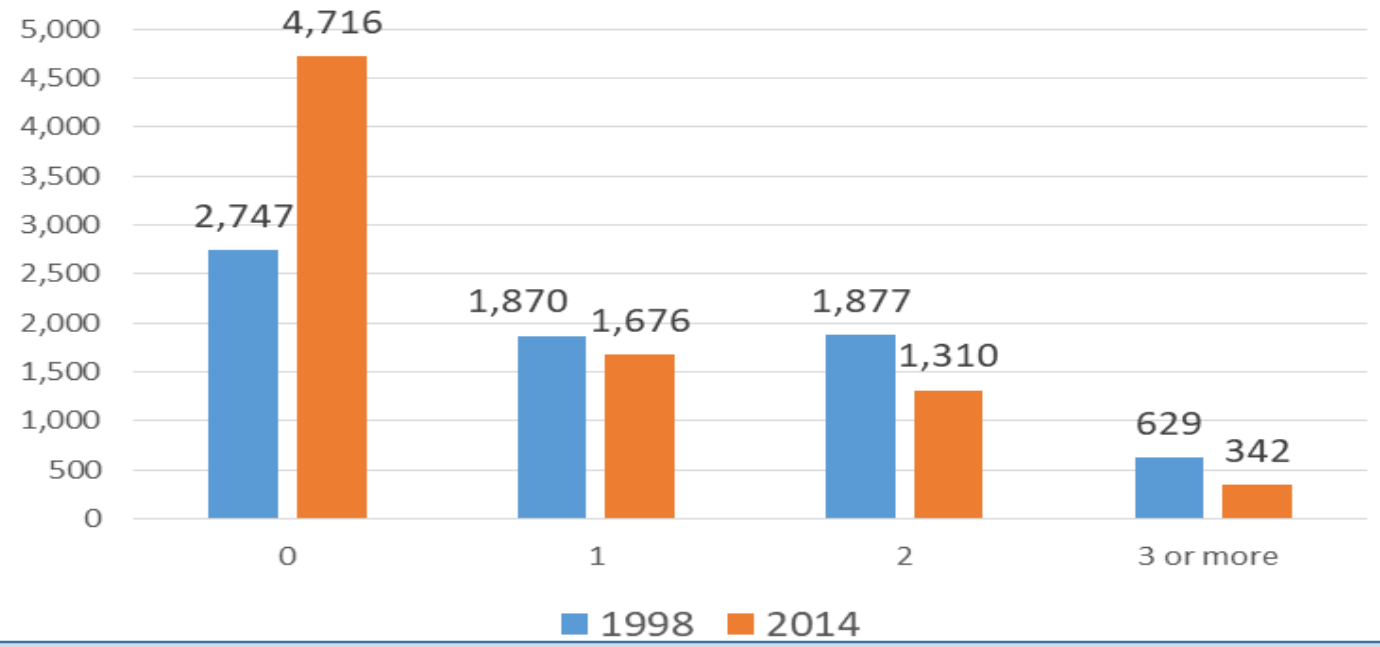


# Graphs – Bar Chart

Each bar's height represents the frequencies (absolute, relative, percentage) of each category/value.

Appropriate for: Qualitative, Quantitative Discrete

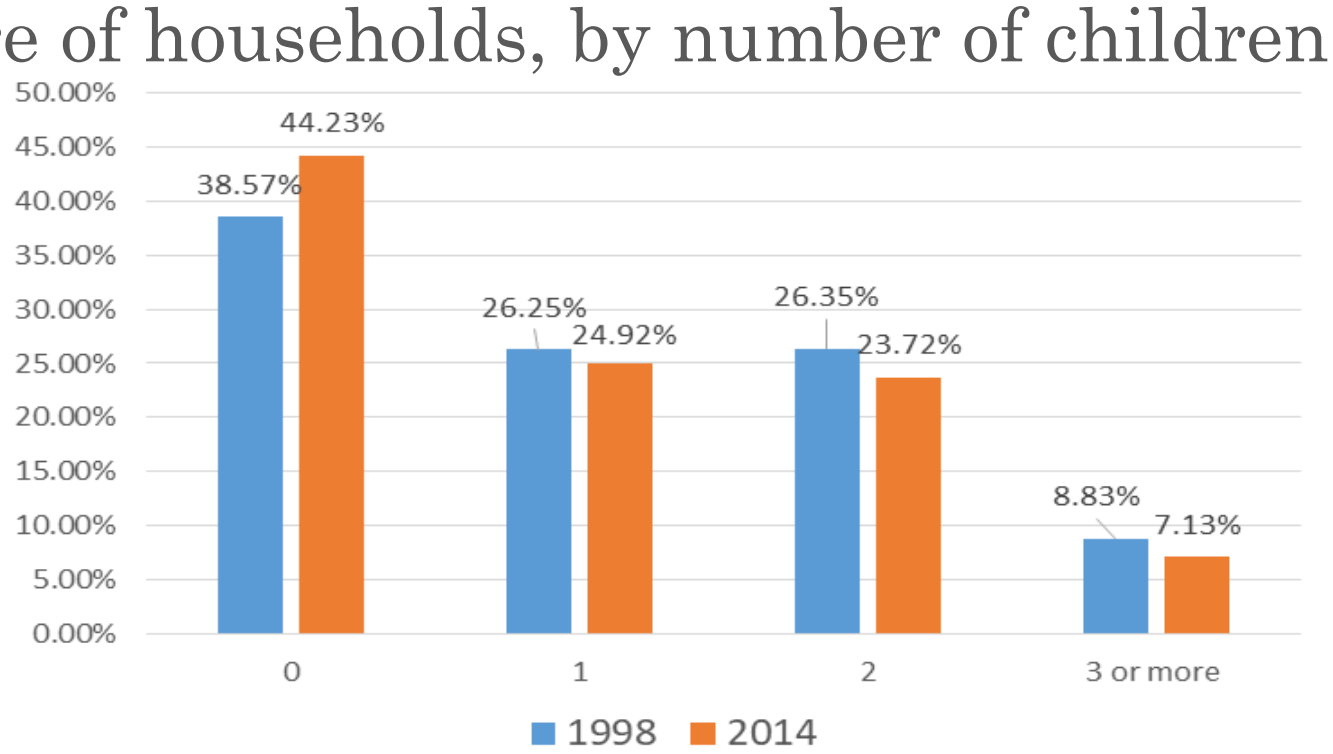
Number of households, by number of children: 1998 and 2014



# Graphs – Bar Chart

Each bar's height represents the frequencies (absolute, relative, percentage) of each category/value.

Appropriate for: Qualitative, Quantitative Discrete



# Graphs – Bar Chart

- ***Bar graph:*** a graph showing the differences in frequencies or percentages among categories of a **nominal** or an **ordinal** variable. The categories are displayed as rectangles of equal width with their height proportional to the frequency or percentage of the category.

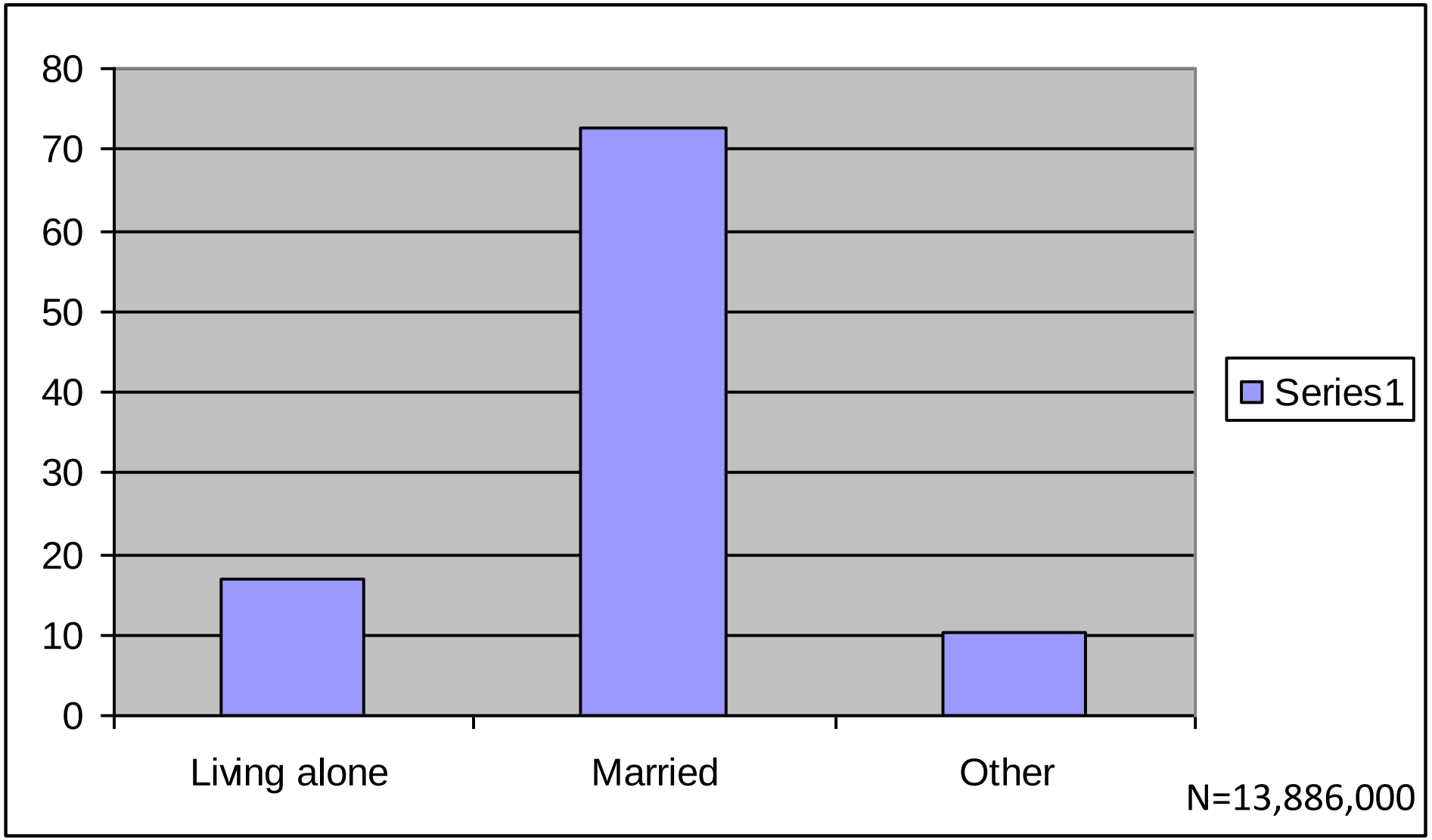



**Figure 3.3 Living Arrangements of Males (65 and Older) in the United States, 2000**

Can display more info by splitting sex

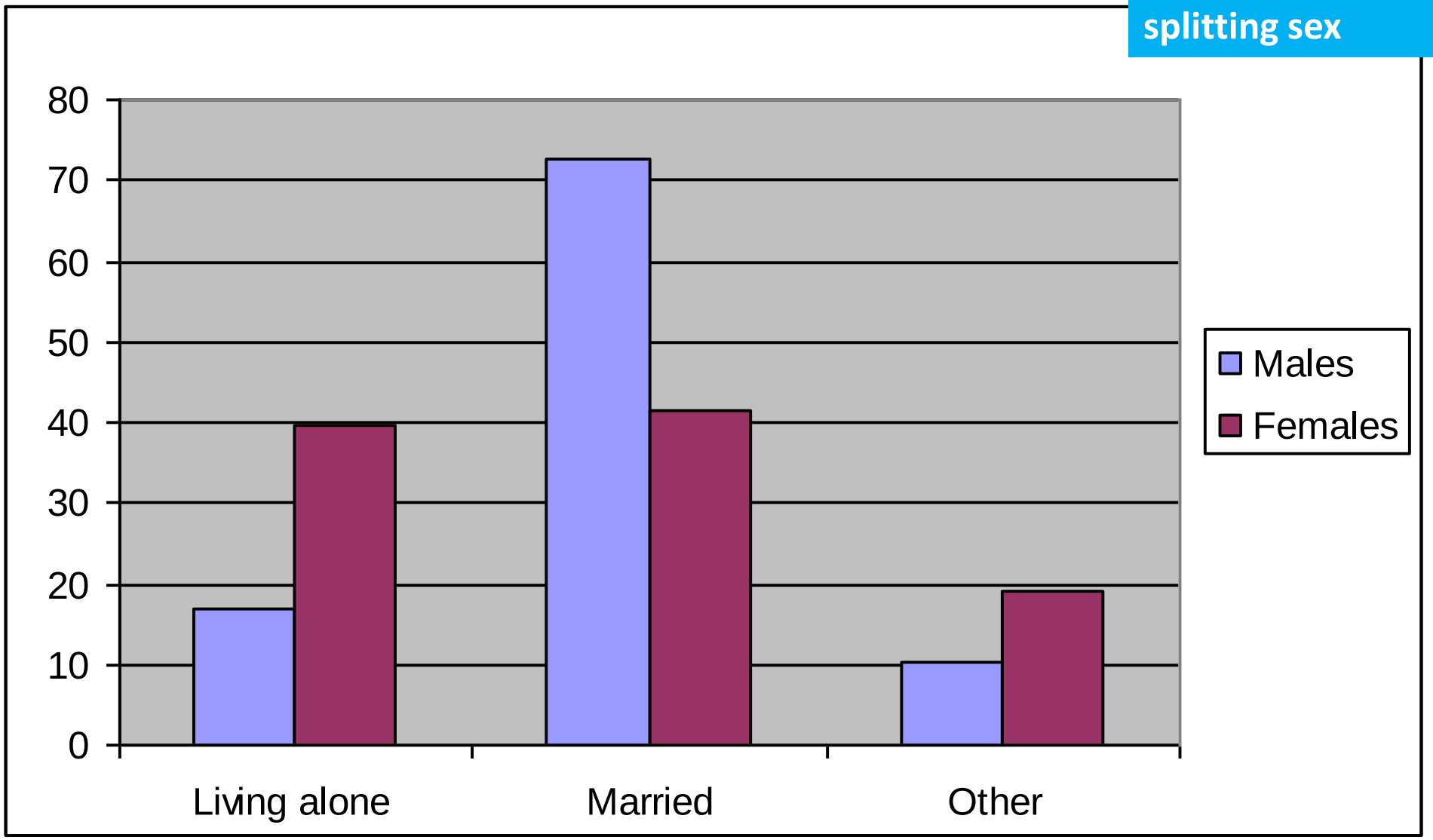
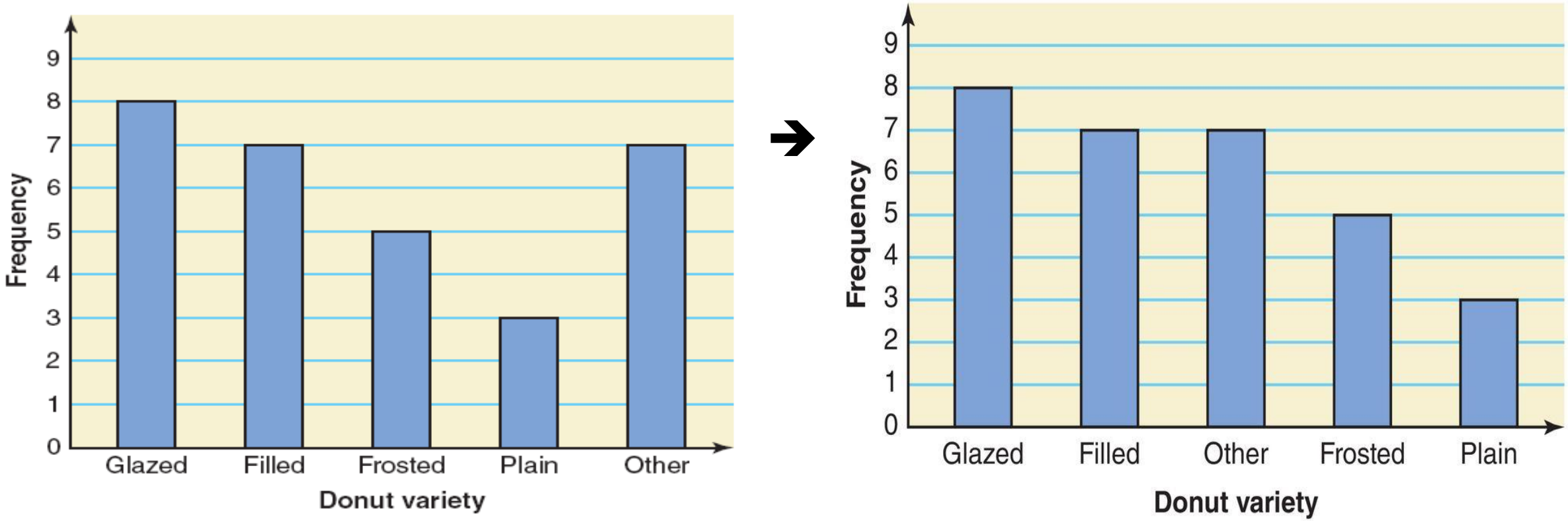



**Figure 3.4 Living Arrangement of U.S. Elderly (65 and Older) by Gender, 2003**

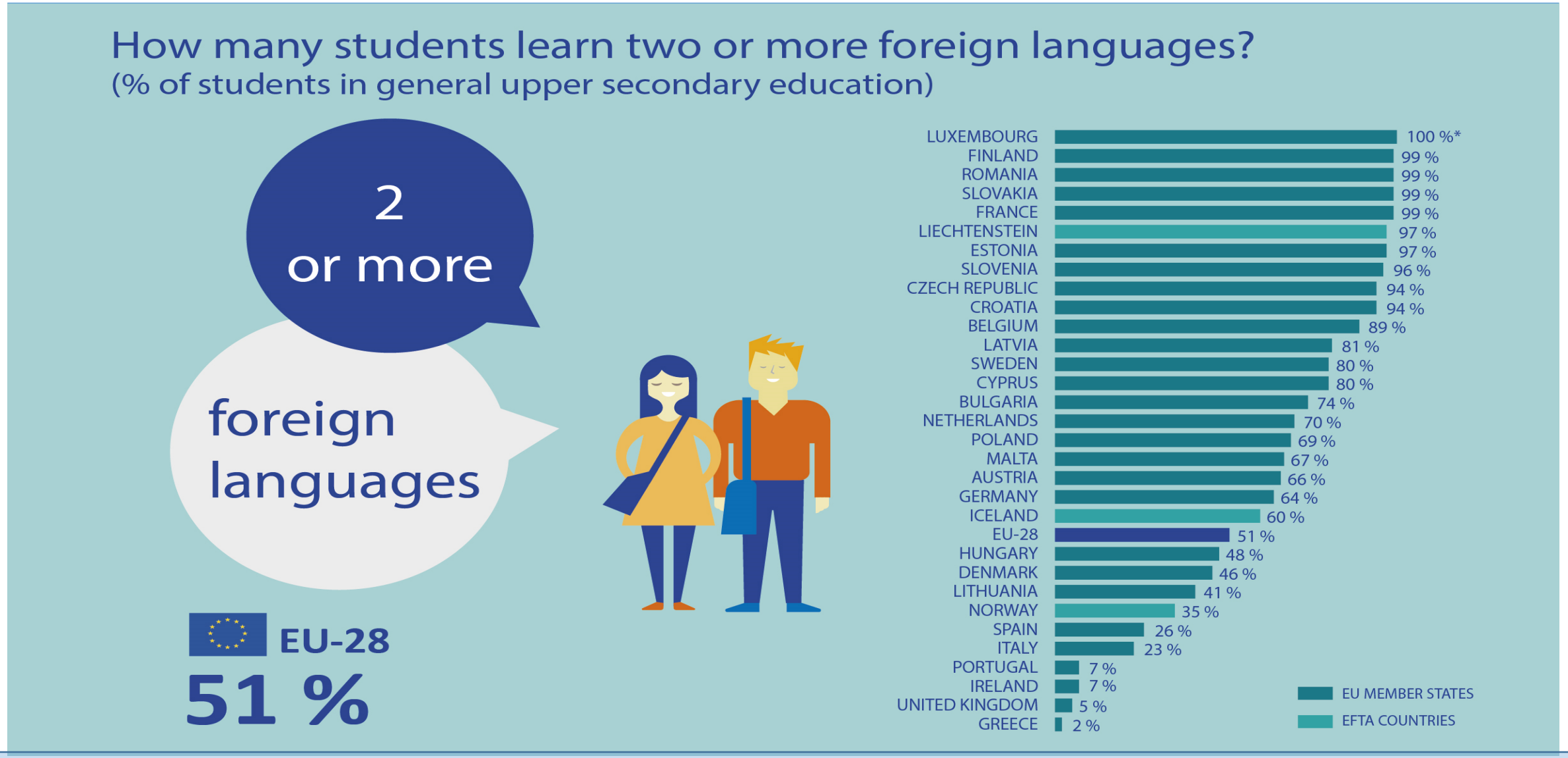
# Graphs – Pareto Chart

A Bar chart where the bars are in descending order

Ex: Bar Chart ➔ Pareto Chart



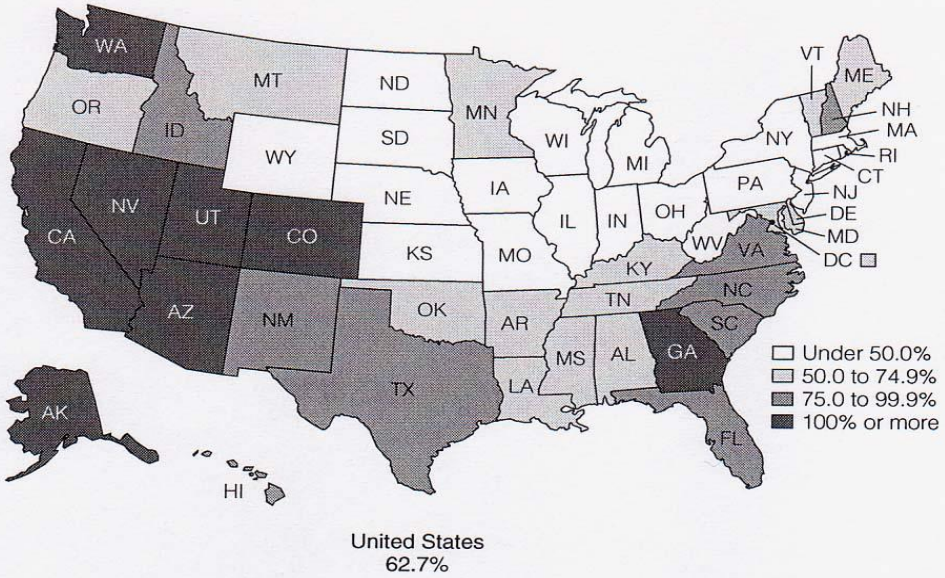
# Graphs – Pareto Chart



# The Statistical Map

We can display dramatic geographical changes in American society by using a statistical map. Maps are especially useful for describing geographical variations in variables, such as population distribution, voting patterns, crimes rates, or labor force participation.

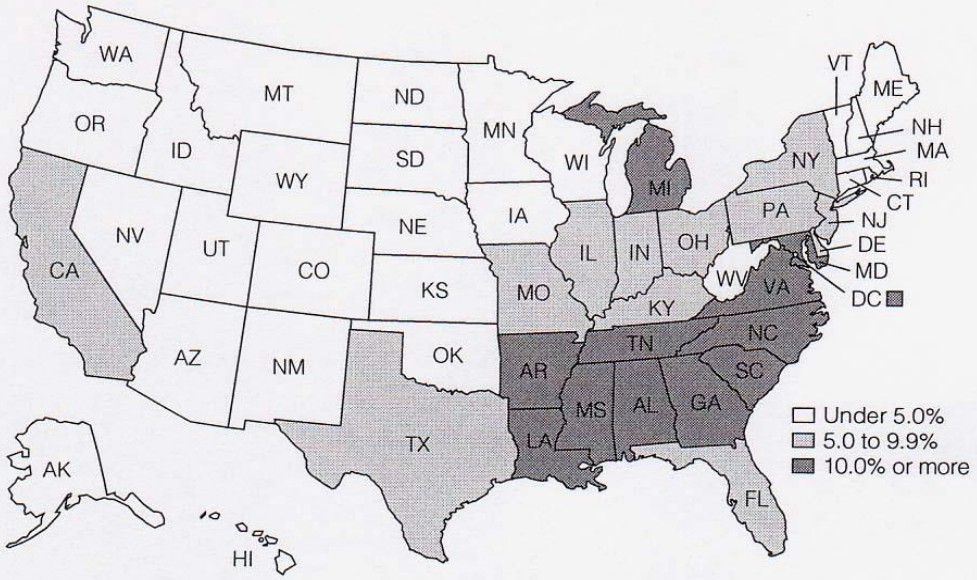
Figure 3.6 **Percentage Increase in Population 65 Years and Over, 1993 to 2020**




United States
62.7%

*Source:* U.S. Bureau of the Census, 1993 from 1994 Press Release, *Updated National/State Population Estimates,* CB94-43; 2020 from "Population Projections for States, by Age, Sex, Race, and Hispanic Origin: 1993 to 2020," *Current Population Reports,* P25-111, U.S. Government Printing Office, Washington, DC, 1994.

Figure 3.7 **Percentage Black of Total State Population 65 Years and Over, 1991**



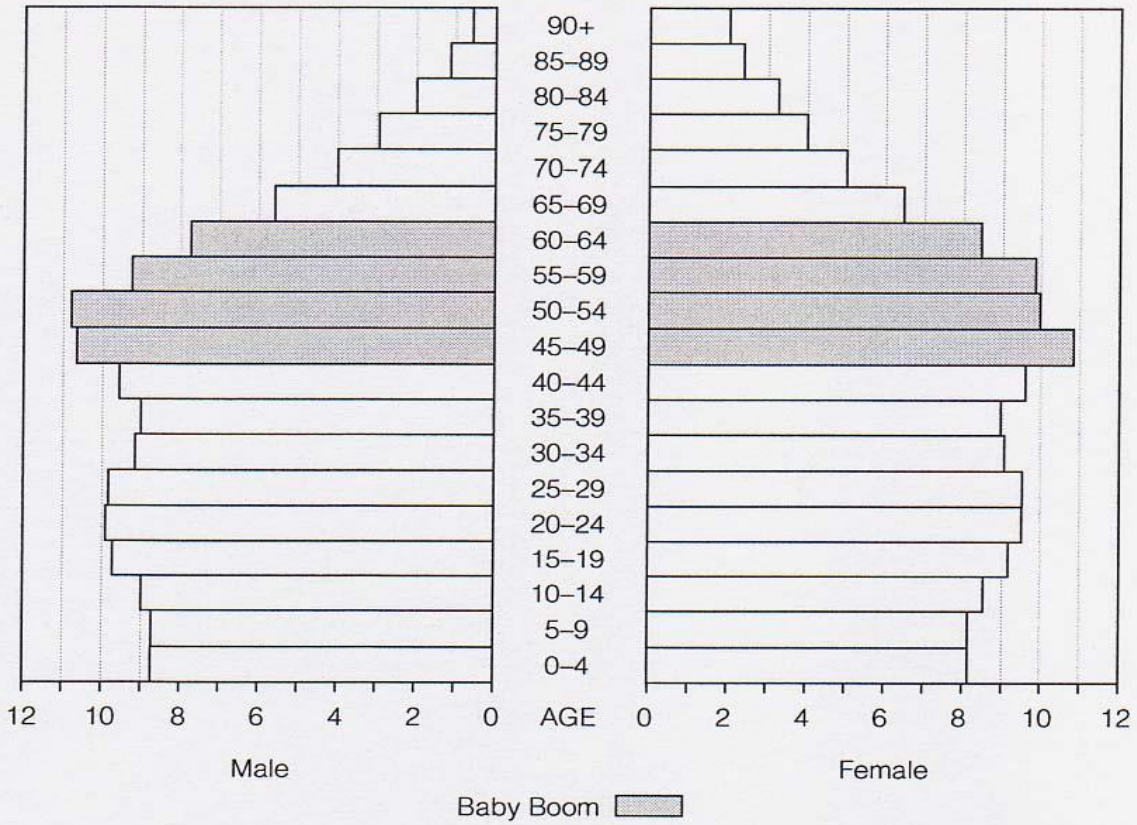

United States
8.0%

*Source:* U.S. Bureau of the Census, "1991 Estimates of the Population of States by Age, Sex, Race, and Hispanic Origin," PE-16.

# Graphs – Histogram

**The Histogram** is a graph showing the differences in frequencies or percentages among categories of an **interval-ratio** variable. The categories are displayed as contiguous bars, with width proportional to the width of the category and height proportional to the frequency or percentage of that category.

Figure 3.11 **U.S. Population by Gender and Age, 2010 (in millions)**




*Source:* U.S. Bureau of the Census, *Current Population Reports*, 1992, P23-178.

# Graphs – Histogram

A bar chart in which each contiguous bar represents a class:

1. the base is proportional to the class width
2. the area is proportional to the relative frequency, $rf_i$
3. ➔ the height is given by the *density*, $h_i$

Appropriate for: Quantitative Continuous (in class)

Steps:

1) Compute the relative frequency of each class, $rf_i = \frac{f_i}{n}$
2) Compute the width of each class, $W_i = upper\ limit\ - lower\ limit$
3) Derive the density, as $h_i = \frac{rf_i}{W_i}$

# Graphs – Histogram: Example

Sample of 400 households, by weekly fuel expenses.

| Weekly fuel expenses (in €) | Absolute Frequency fi |
|---|---|
| **0\|-20** | 20 |
| **20\|-50** | 80 |
| **50\|-100** | 210 |
| **100\|-200** | 50 |
| **200\|-300** | 25 |
| **300\|-350** | 15 |
| Total | 400 |

# Graphs – Histogram: Example

Sample of 400 households, by weekly fuel expenses.

| Weekly fuel expenses (in €) | Absolute Frequency fi | Relative Frequency rfi | Width Wi | Density hi= rfi/Wi |
|---|---|---|---|---|
| **0\|-20** | 20 | 20/400= | 20-0= | 0.05/20 = |
| **20\|-50** | 80 | | | |
| **50\|-100** | 210 | | | |
| **100\|-200** | 50 | | | |
| **200\|-300** | 25 | | | |
| **300\|-350** | 15 | | | |
| Total | 400 | | | |

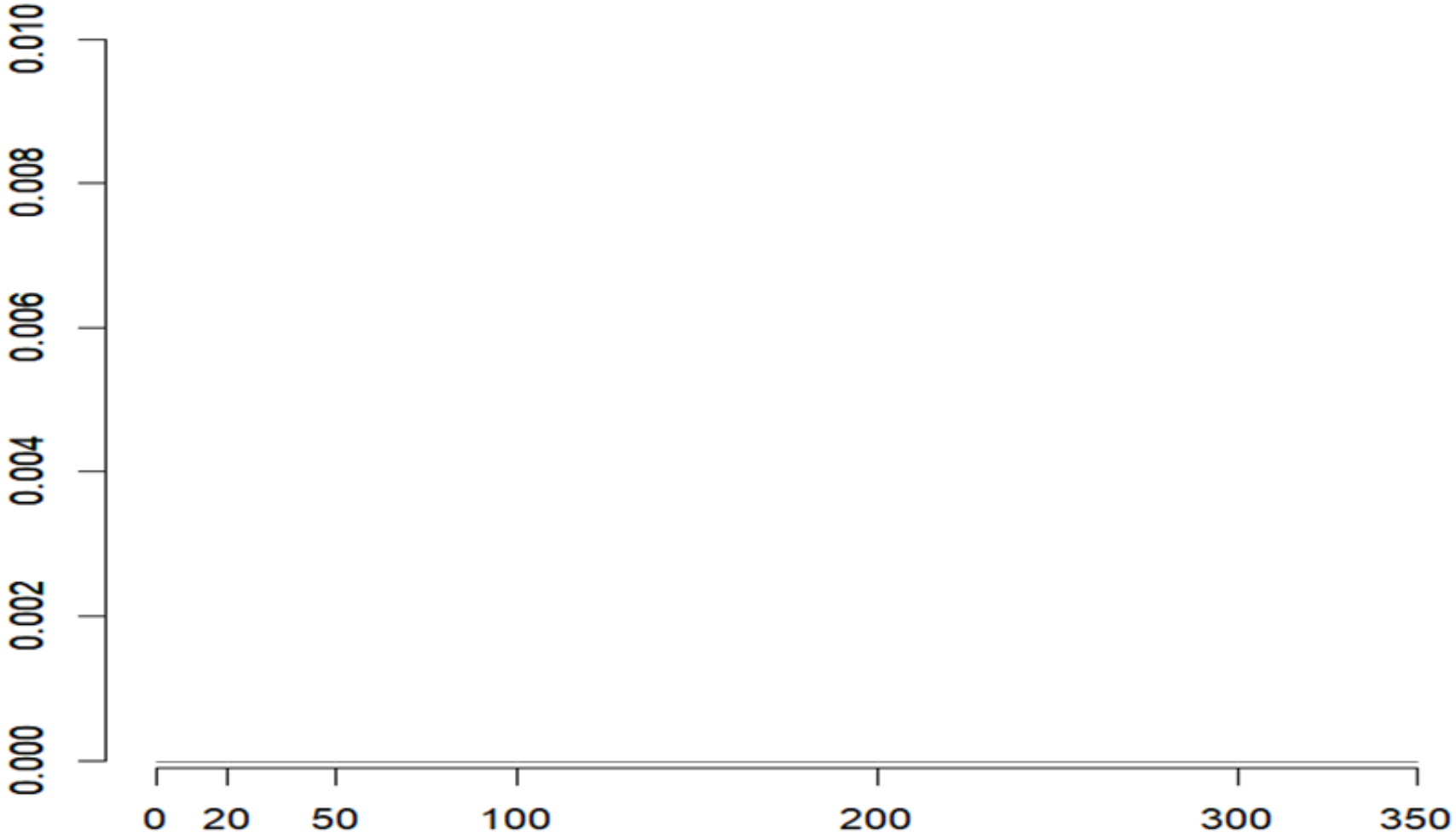
# Graphs – Histogram: Example

Sample of 400 households, by weekly fuel expenses.

| Weekly fuel expenses (in €) | Absolute Frequency fi | Relative Frequency rfi | Width Wi | Density hi |
|---|---|---|---|---|
| **0\|-20** | 20 | 20/400= 0.0500 | 20-0= 20 | 0.0025 |
| **20\|-50** | 80 | 80/400= 0.2000 | 50-20= 30 | 0.0067 |
| **50\|-100** | 210 | 210/400= 0.5250 | 100-50 50 | 0.0105 |
| **100\|-200** | 50 | 50/400= 0.1250 | 200-100 100 | 0.0013 |
| **200\|-300** | 25 | 25/400= 0.0625 | 300-200= 100 | 0.0006 |
| **300\|-350** | 15 | 15/400= 0.0375 | 350-300= 50 | 0.0008 |
| Total | 400 | 1.00 | | |

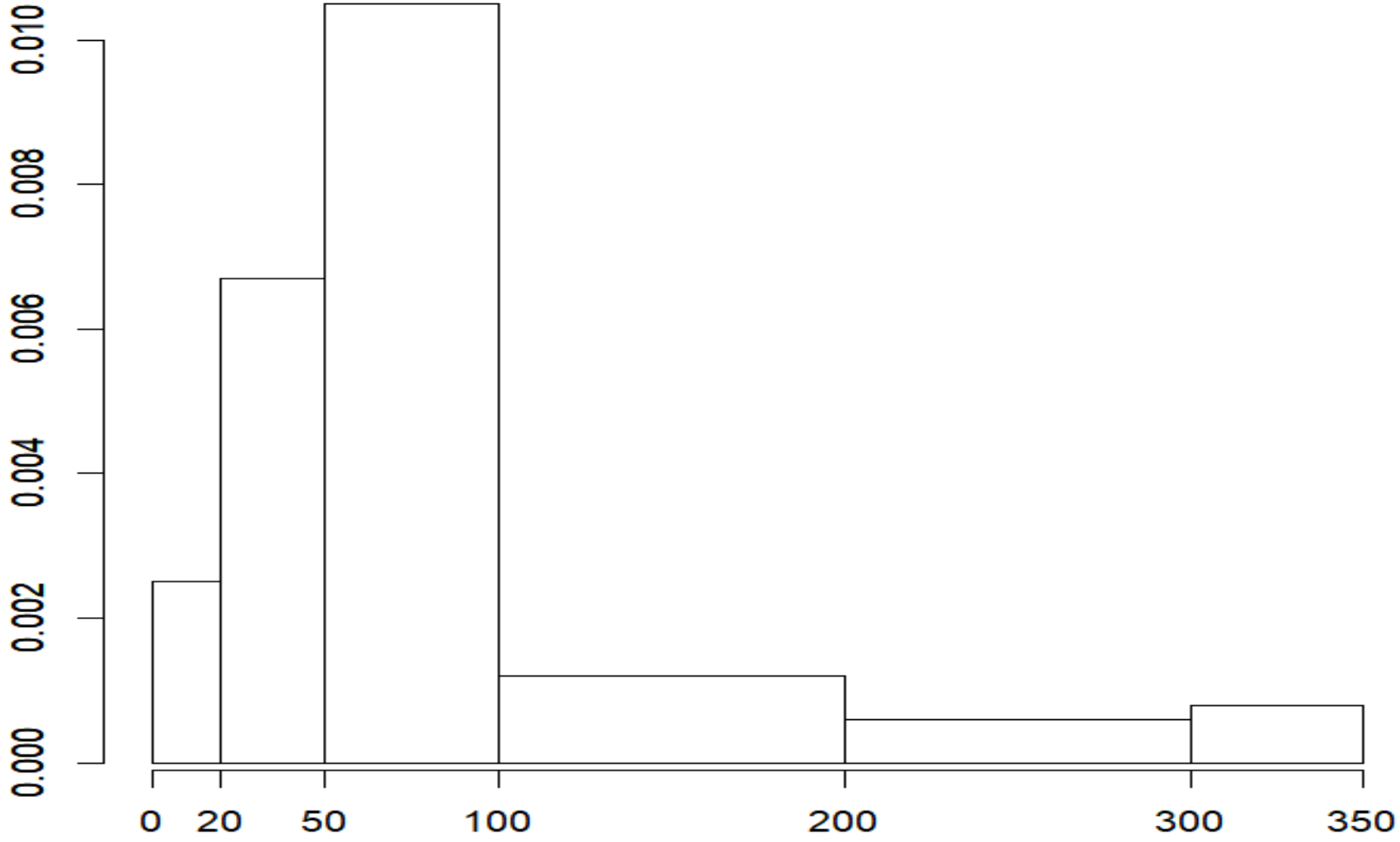
# Graphs – Histogram: Example

Sample of 400 households, by weekly fuel expenses.



# Graphs – Histogram: Example

Sample of 400 households, by weekly fuel expenses.



# Graphs – Histogram: another example

Draw the histogram for the following distribution:

| Weekly food expenses (in €) | Absolute Frequency fi |
|---|---|
| **0\|-10** | 20 |
| **10\|-50** | 120 |
| **50\|-80** | 90 |
| **80\|-100** | 20 |
| Total | 250 |

# Graphs – Histogram: another example

Steps: derive the relative frequency, the width and the density

| Weekly food expenses (in €) | Absolute Frequency fi | Relative Frequency rfi | Width Wi | Density hi |
|---|---|---|---|---|
| **0\|-10** | 20 | | | |
| **10\|-50** | 120 | | | |
| **50\|-80** | 90 | | | |
| **80\|-100** | 20 | | | |
| Total | 250 | | | |

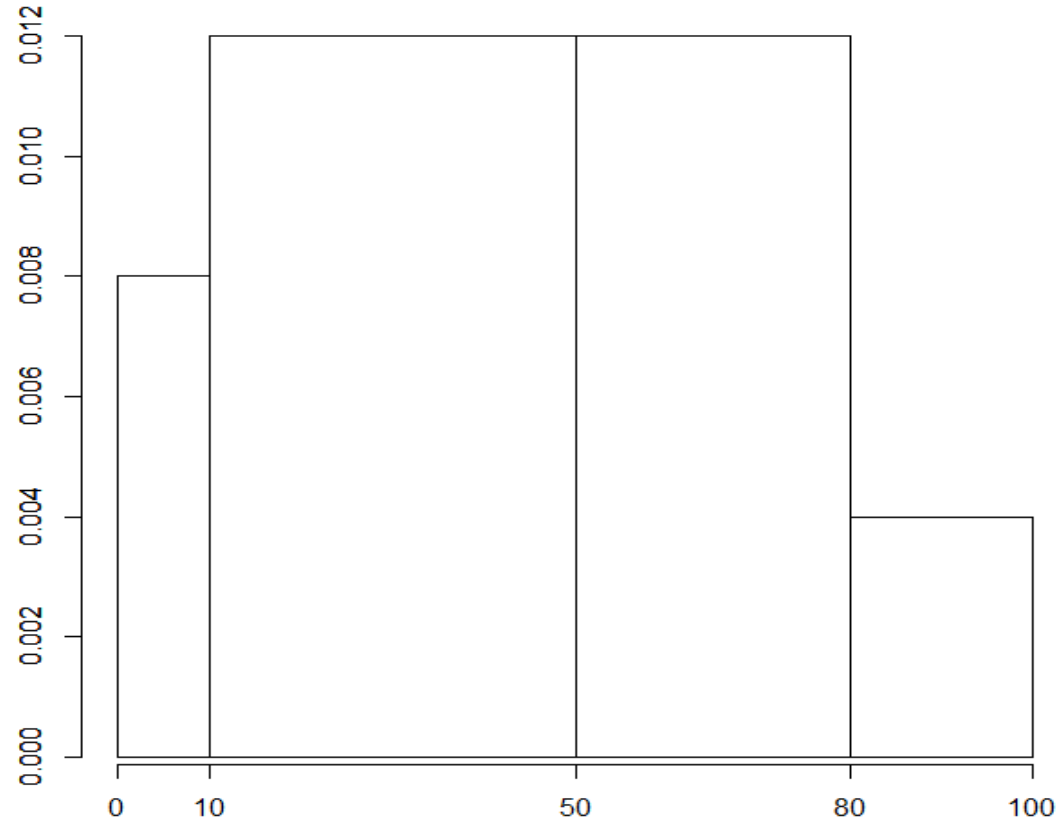
# Graphs – Histogram: another example

Steps: derive the relative frequency, the width and the density

| Weekly food expenses (in €) | Absolute Frequency fi | Relative Frequency rfi | Width Wi | Density hi |
|---|---|---|---|---|
| **0\|-10** | 20 | 0.0800 | 10 | 0.0080 |
| **10\|-50** | 120 | 0.4800 | 40 | 0.0120 |
| **50\|-80** | 90 | 0.3600 | 30 | 0.0120 |
| **80\|-100** | 20 | 0.0800 | 20 | 0.0040 |
| Total | 250 | 1.00 | | |

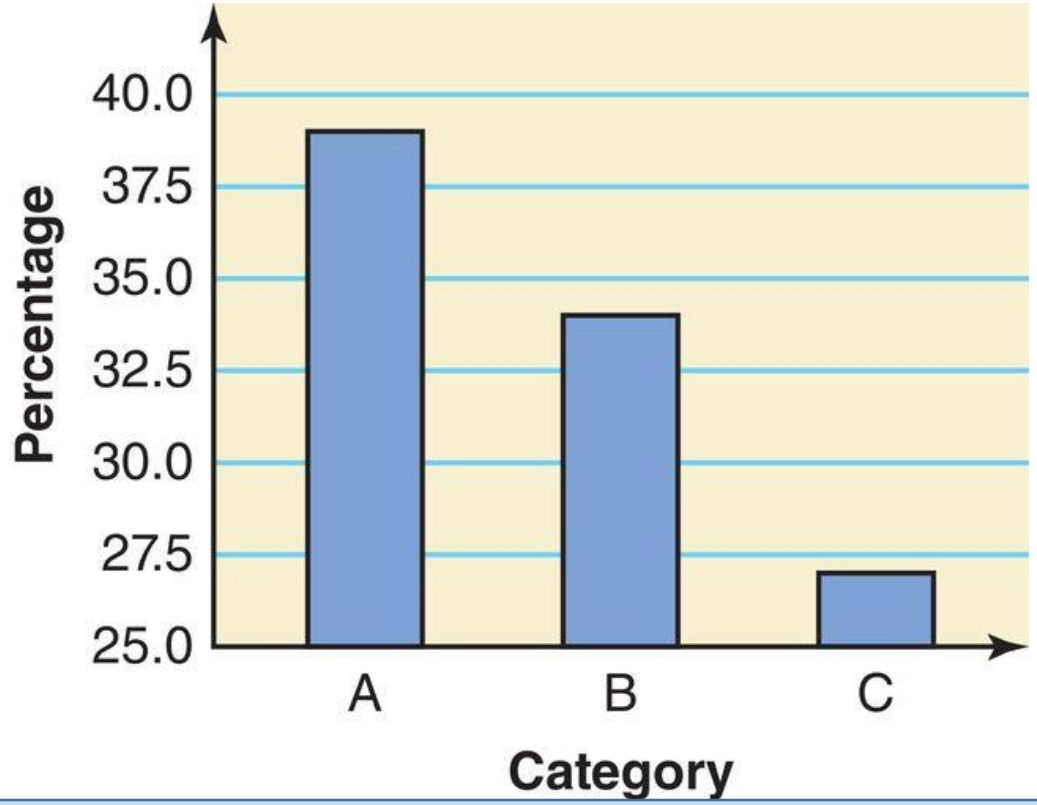
# Graphs – Histogram: Example

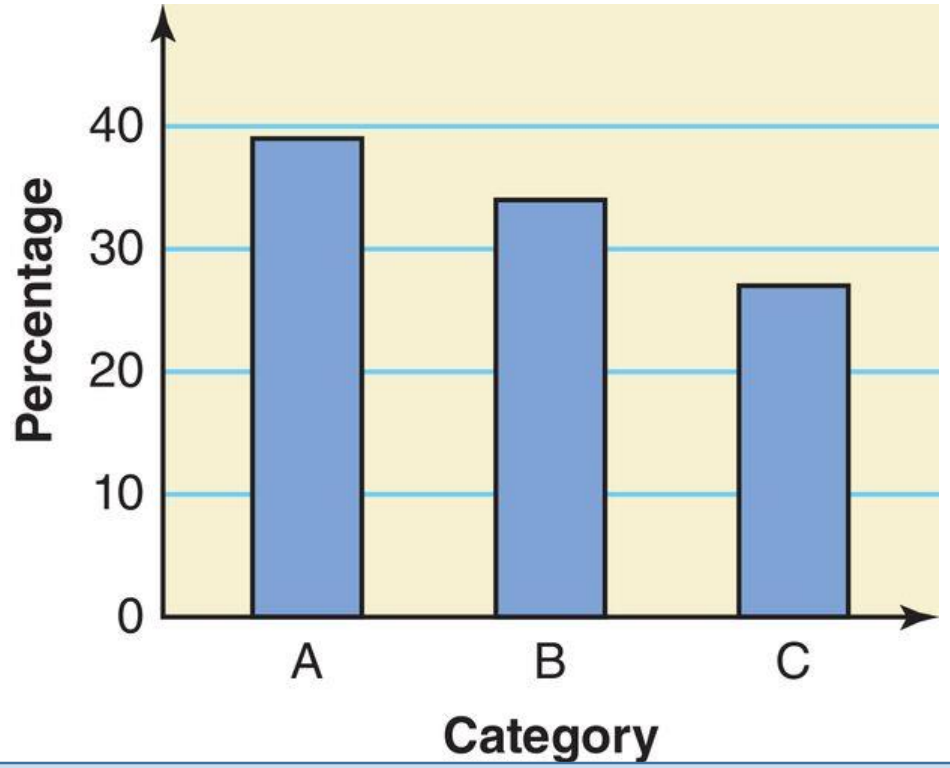
Sample of 250 individuals, by weekly food expenses.



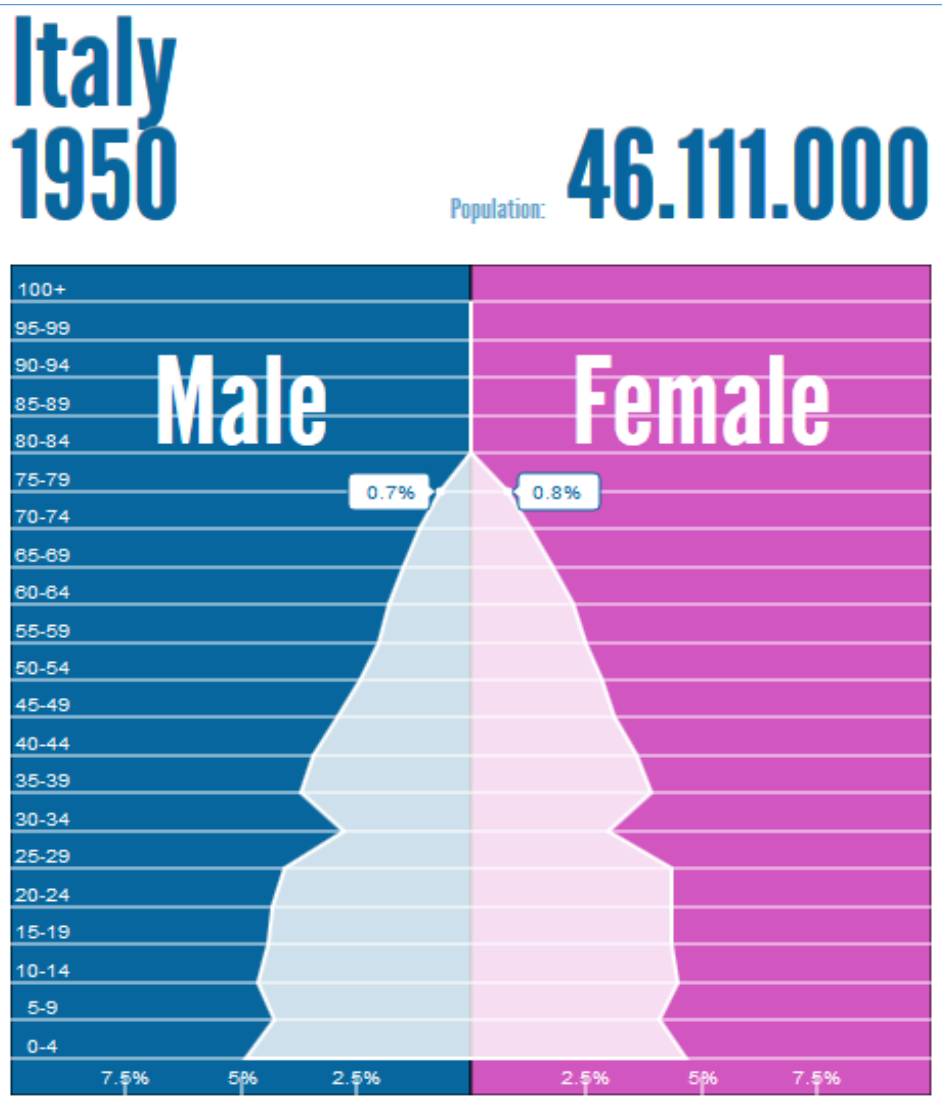
# Graphs – Warnings

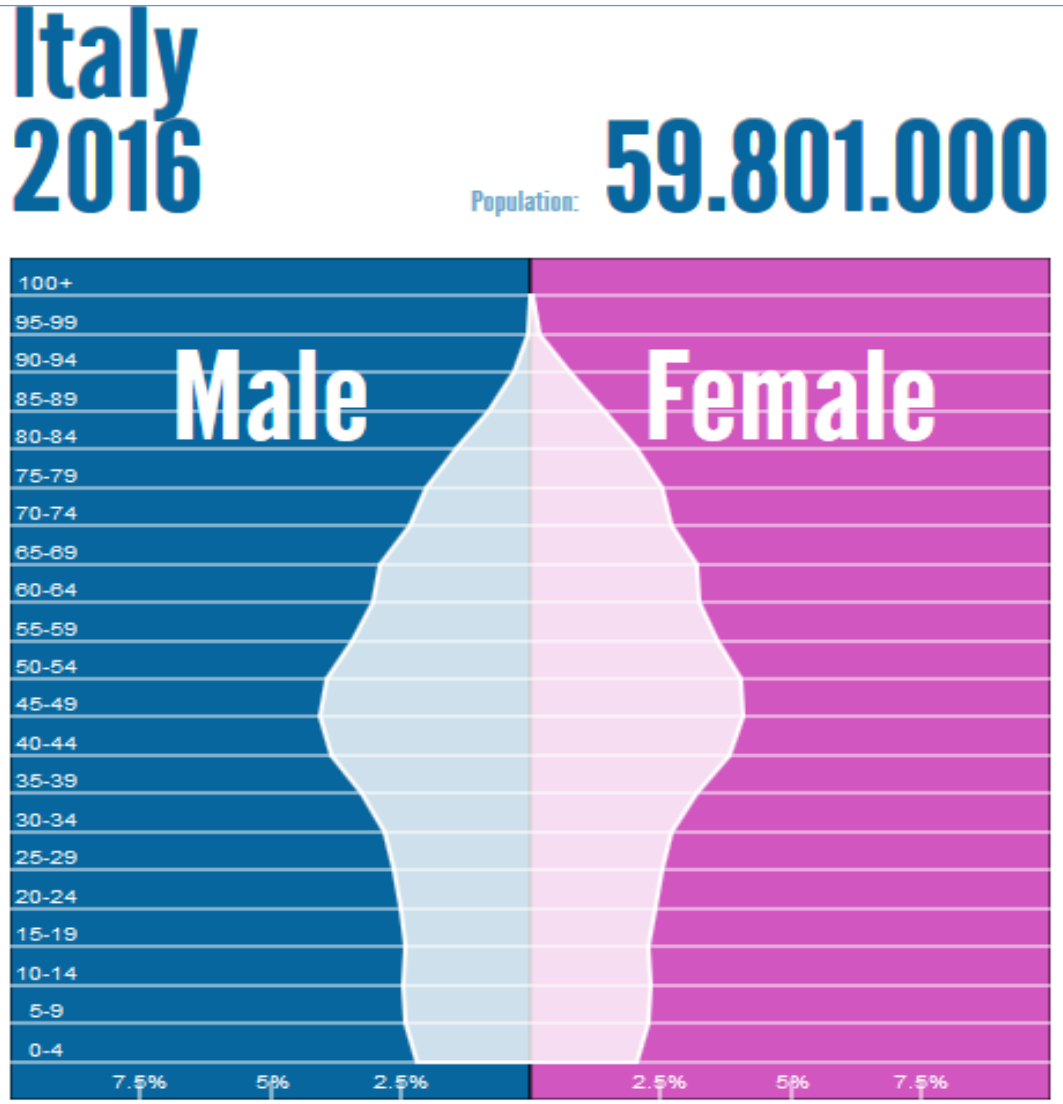
1. Use the most appropriate type of graph
2. Always apply labels to the axes
3. Pay attention to the scales!!!



# Exploring data: bar graph for age-classes



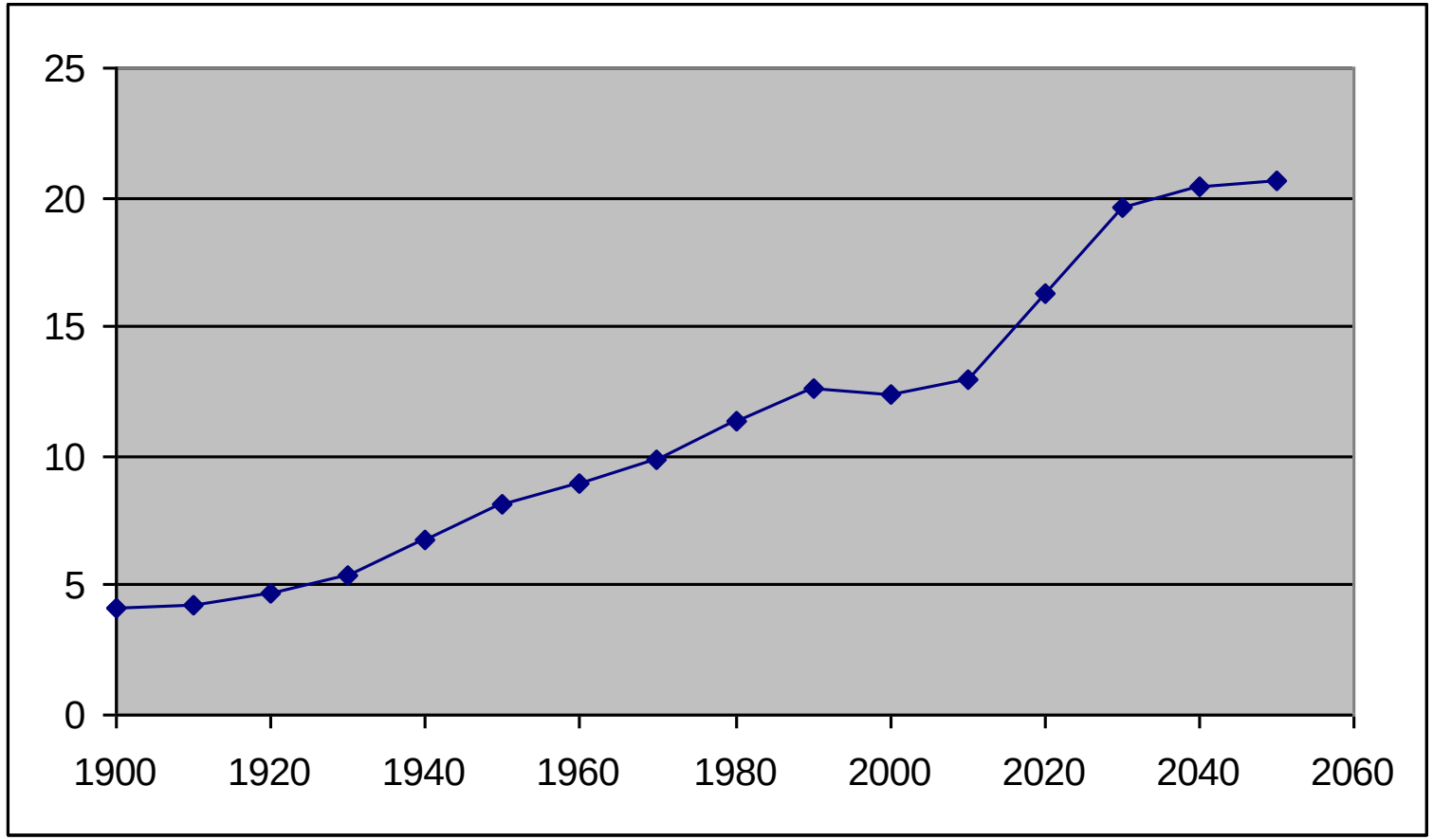



# Time Series Charts

**Time series chart:** a graph displaying **changes** in a variables at **different points in time**. It shows time (measured in units such as years or months) on the horizontal axis and the frequencies (percentages or rates) of another variable on the vertical axis.

# Time Series Charts

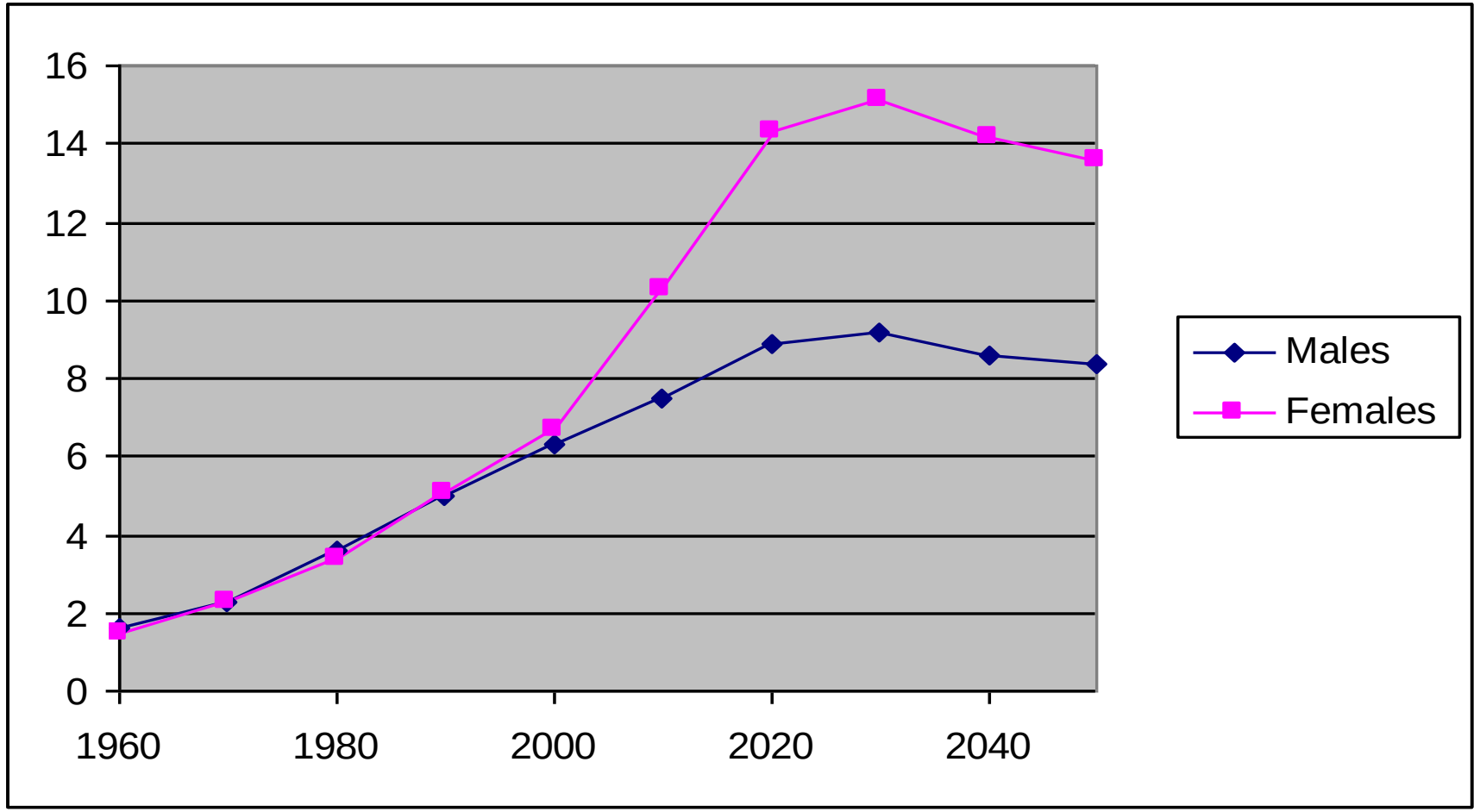
**Figure 3.12 Percentage of Total U. S. Population 65 Years and Over, 1900 to 2050**




Source: Federal Interagency Forum on Aging Related Statistics, *Older Americans 2004: Key Indicators of Well Being*, 2004.

# Time Series Charts

**Figure 3.13 Percentage Currently Divorced Among U.S. Population 65 Years and Over, by Gender, 1960 to 2040**




Source: U.S. Bureau of the Census, "65+ in America," Current Population Reports, 1996, Special Studies, P23-190, Table 6-1.

# Distortions in Graphs

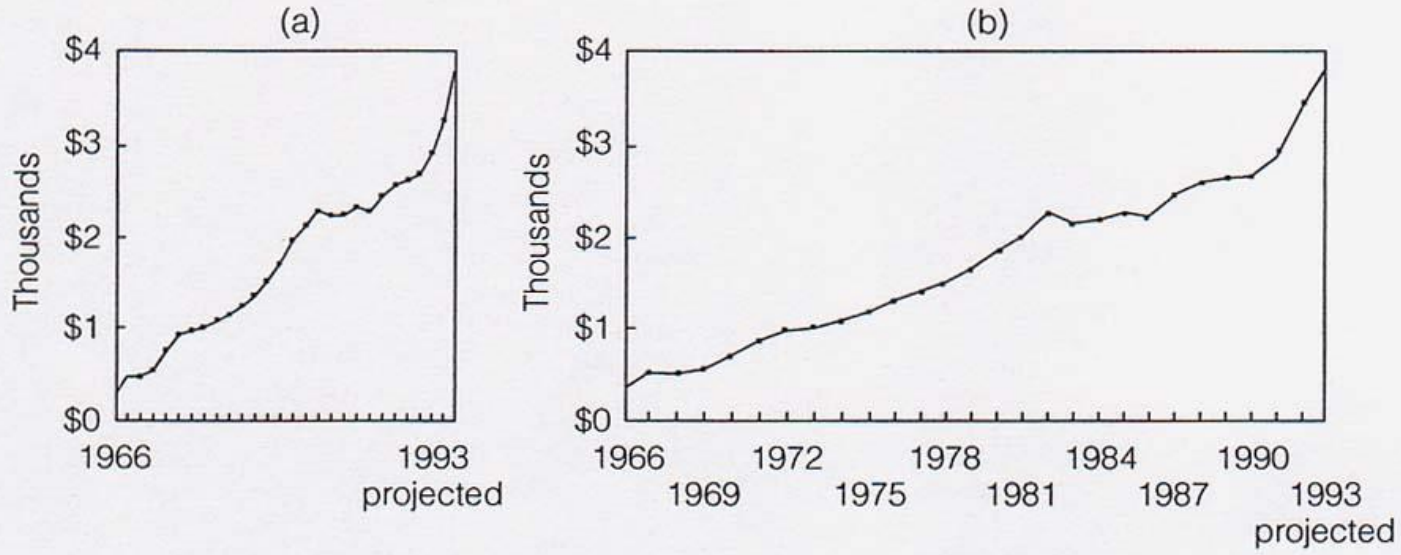
Graphs not only quickly inform us; they can quickly **deceive** us.

Because we are often more interested in general impressions than in detailed analyses of the numbers, we are **more vulnerable** to being swayed by **distorted graphs**.

# Distortions in Graphs

**Shrinking an Stretching the Axes: Visual Confusion**

Figure 3.16 **Cost per Child Enrolled in Head Start Program**




Cost per Child

Graph Showing
Skyrocketing Increase

Cost per Child

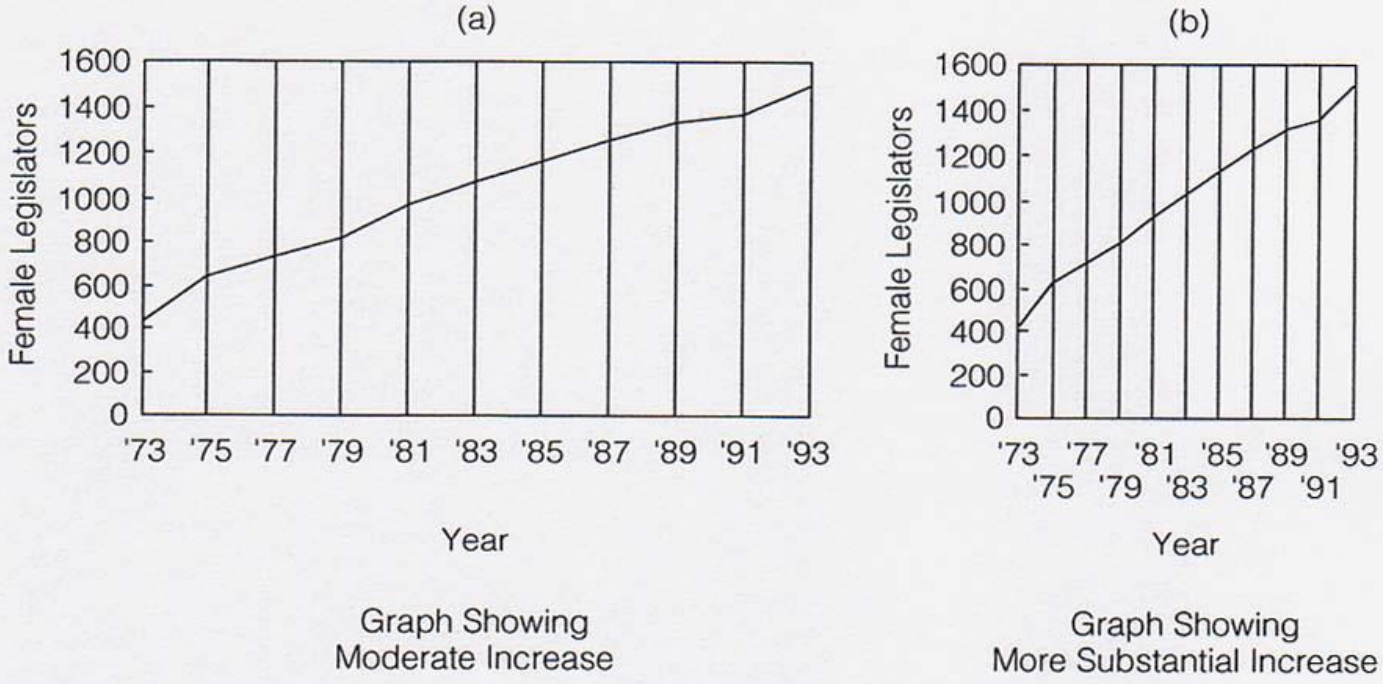
Graph Showing
More Moderate Increase

*Source:* Adapted from *USA TODAY*, March 15, 1993. Copyright 1993 USA TODAY. Reprinted with permission.

# Distortions in Graphs

**Shrinking an Stretching the Axes: Visual Confusion**

Figure 3.17 **Women in U.S. Legislatures, 1973 to 1993**




Graph Showing Moderate Increase

Graph Showing More Substantial Increase

*Source:* Adapted from Marty Baumann, *USA TODAY*, February 12, 1993. Copyright 1993 USA TODAY. Reprinted with permission.

# Why use charts and graphs?

- **What do you lose?**
  - –ability to examine numeric detail offered by a table
  - –potentially the ability to see **additional** relationships within the data
  - –potentially **time**: often we get caught up in selecting colors and formatting charts when a simply formatted table is sufficient

- **What do you gain?**
  - –ability to **direct readers' attention** to one aspect of the evidence
  - –ability to **reach readers** who might otherwise be intimidated by the same data in a tabular format
  - –ability to focus on **bigger picture** rather than perhaps minor technical details

# Shapes of distributions

The most common shapes of the distributions are:

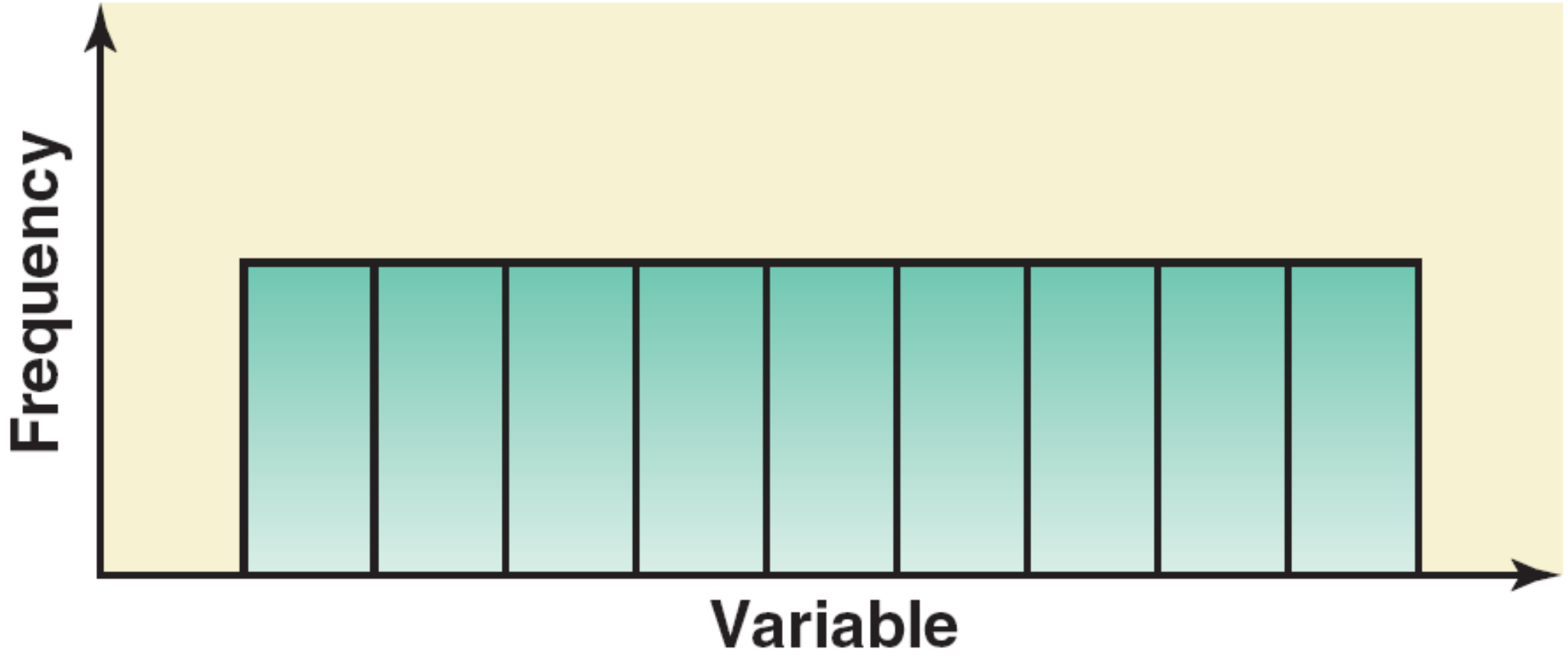
1) Uniform

2) Symmetric (but not uniform)

3) Skewed (to the left, to the right)

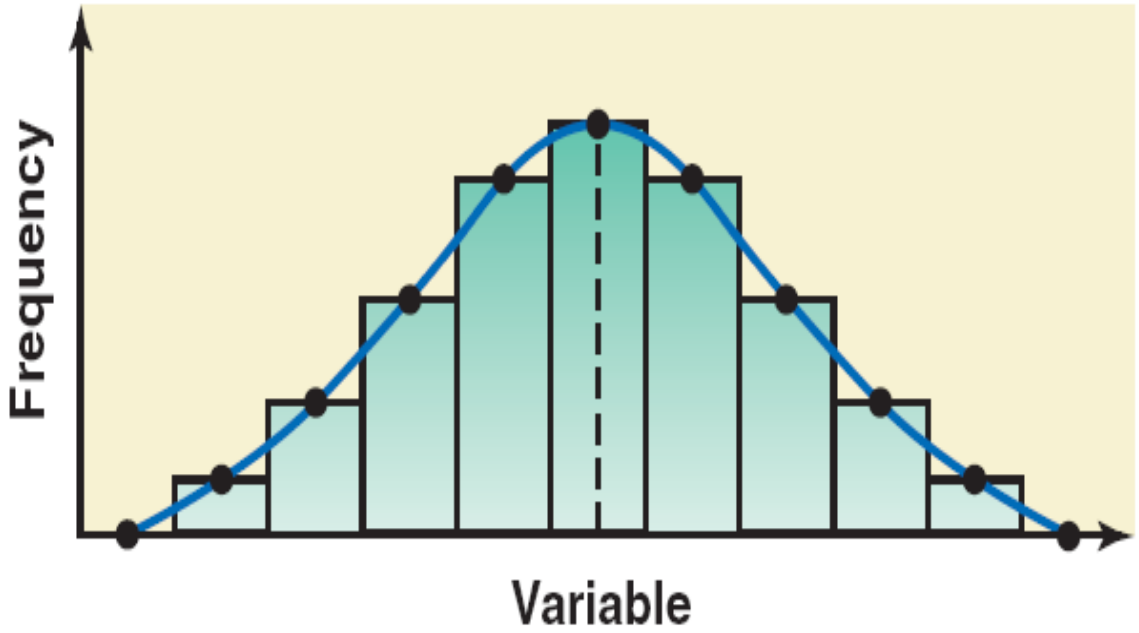
# Shapes of distributions: Uniform

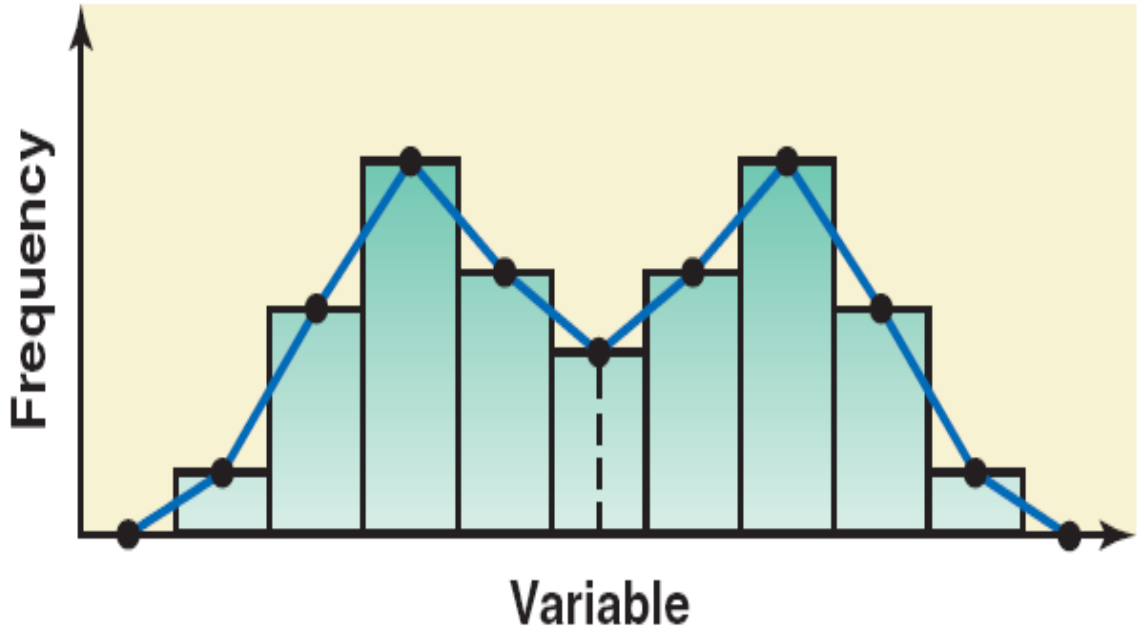
Uniform or Rectangular: same frequency for each category/value/class



# Shapes of distributions: Symmetric

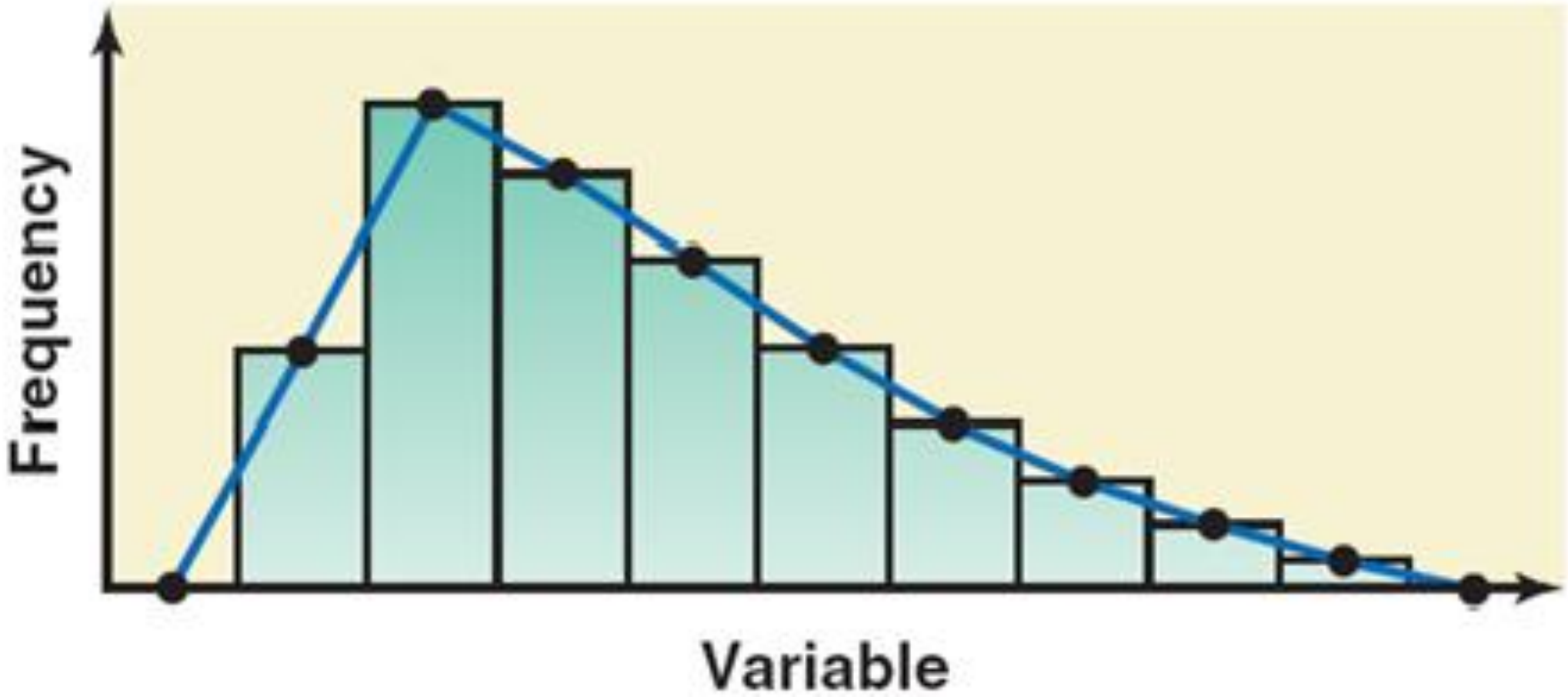
Identical on both sides of its central point



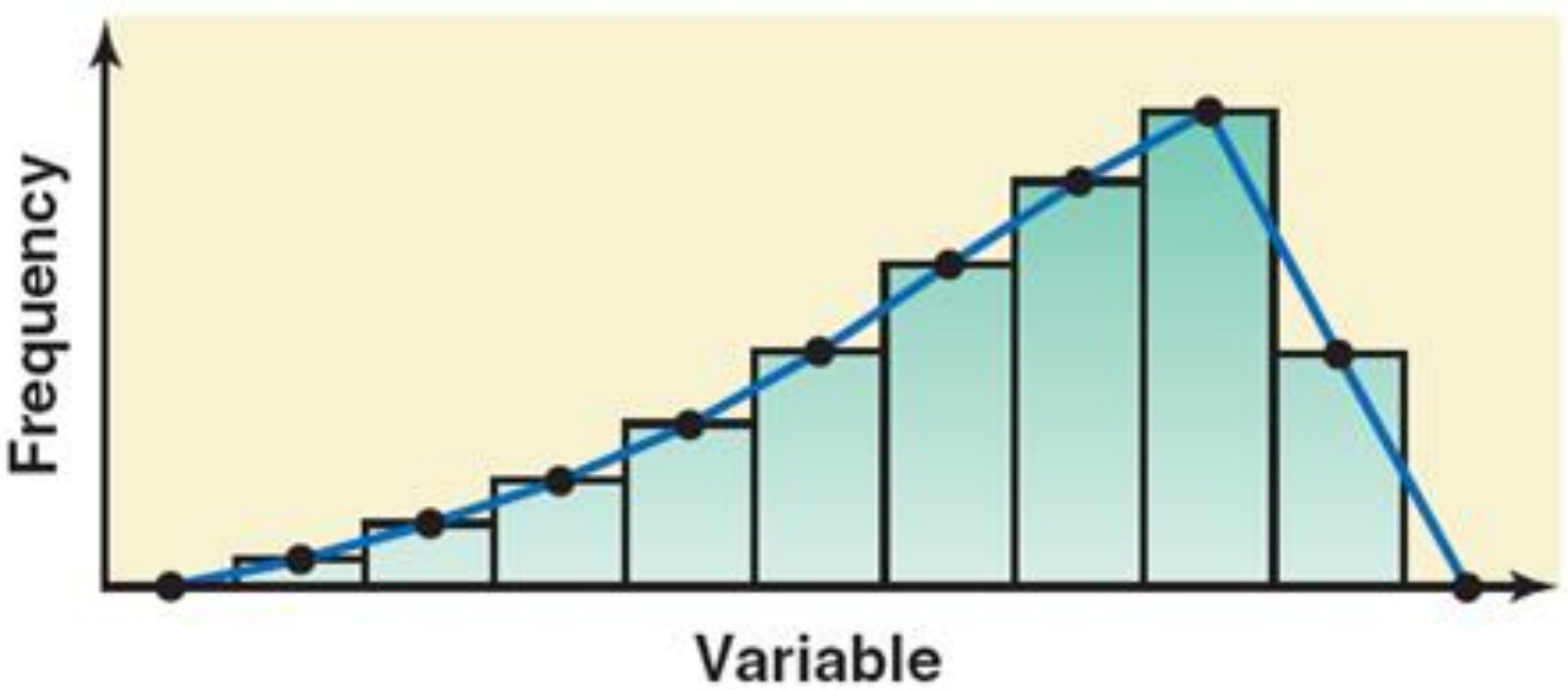
# Shapes of distributions: Skewed (right)

Non symmetric, with the right tail longer than the left one



# Shapes of distributions: Skewed (left)

Non symmetric, with the left tail longer than the right one



# How do you expect the distribution of income?

# How do you expect the distribution of income?