Quantitative Methods – I (Statistics)

A. Y. 2022-23

Prof. Lorenzo Cavallo

Chapter 6
Continuous Random Variables

Continuous Random Variables: Outline

1. Normal distribution $X \sim N(\mu, \sigma^2)$
2. Standard normal distribution $\mathrm{Z} \sim N(0,1)$
3. Chi-square distribution $X \sim \chi_n^2$
4. t-Student distribution $X \sim t_n$
5. F distribution $X \sim F_{n,m}$

Probability distributions of Continuous Variables

A continuous random variable X can assume any value over an interval.

2 characteristics (similar to discrete variables):

For any interval $[a, b]$,

$$0 \leq P(a < X < b) \leq 1$$

The total probability of all the intervals within which X can assume a value is 1

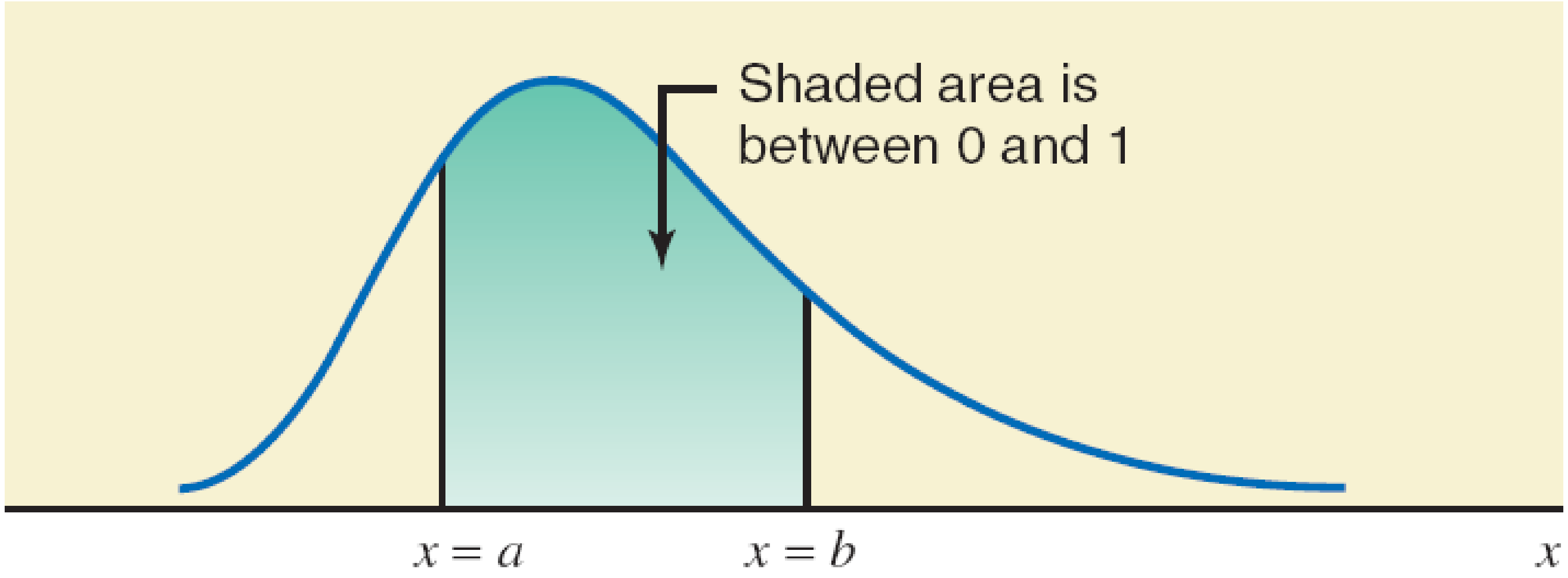
$$\sum P(-\infty < X < +\infty) = 1$$

Probability distributions of Continuous Variables

A ***continuous*** random variable (denoted by capital letter, e.g. X) can assume any value over an interval

2 characteristics:

1) For any interval $[a, b]$, $0 \leq P(a < X < b) \leq 1$

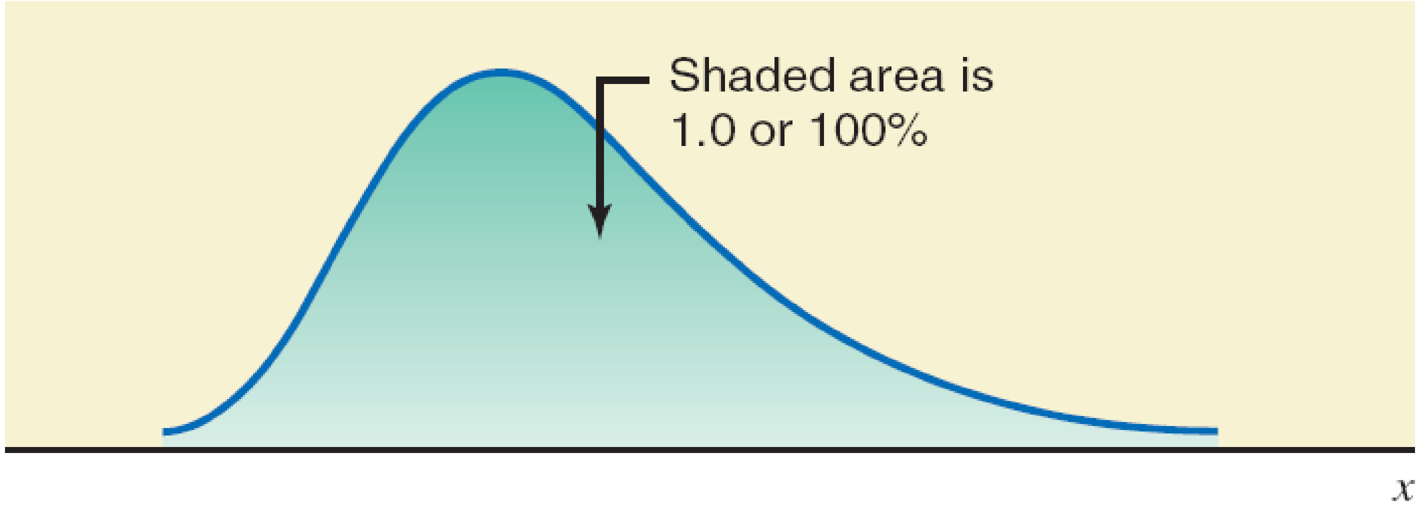


Probability distributions of Continuous Variables

A ***continuous*** random variable (denoted by capital letter, e.g. X) can assume any value over an interval

2 characteristics:

2) The total probability of all the (mutually exclusive) intervals within which X can assume a value is 1 ➔ $P(-\infty < X < \infty) = 1$



Normal distribution function

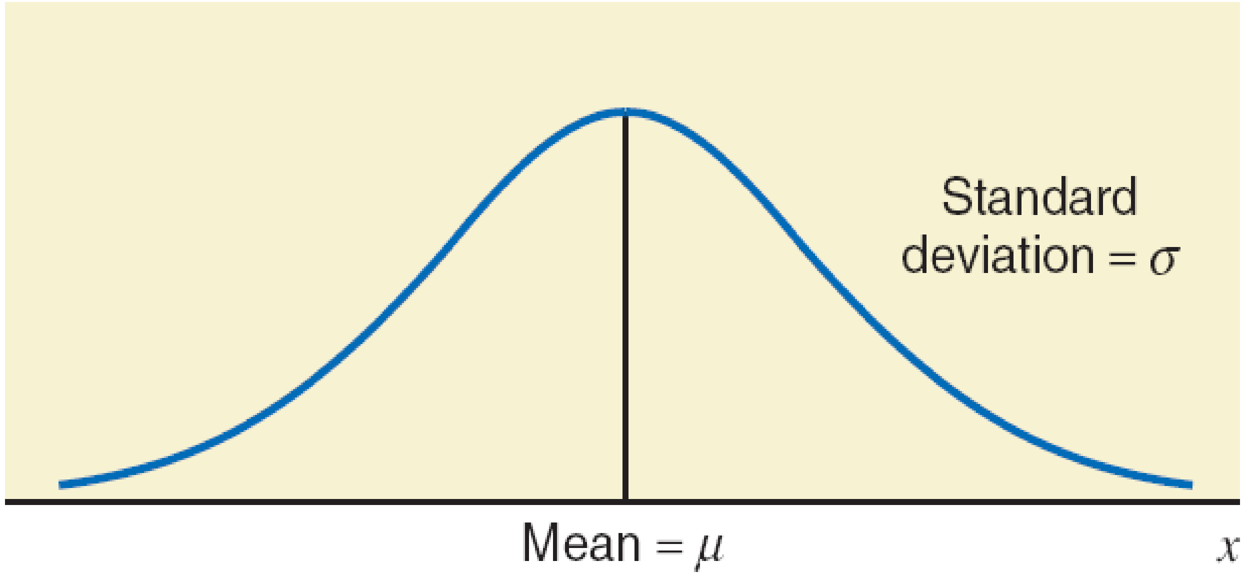
Most widely used continuous probability distribution

Density function is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

parameters: μ (mean), and

σ (the spread)



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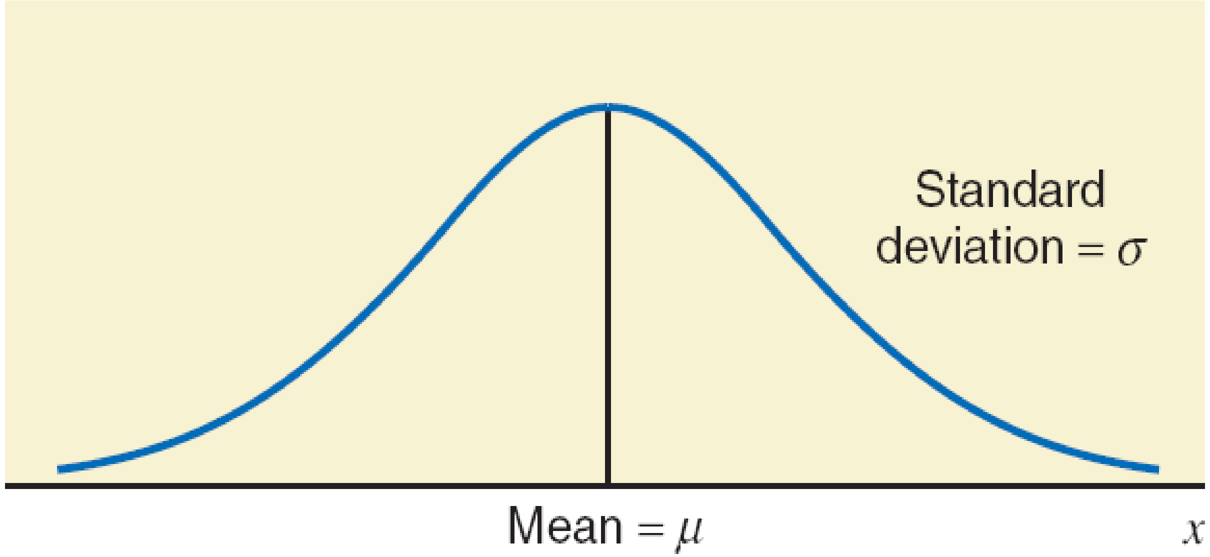
- Bell-shaped



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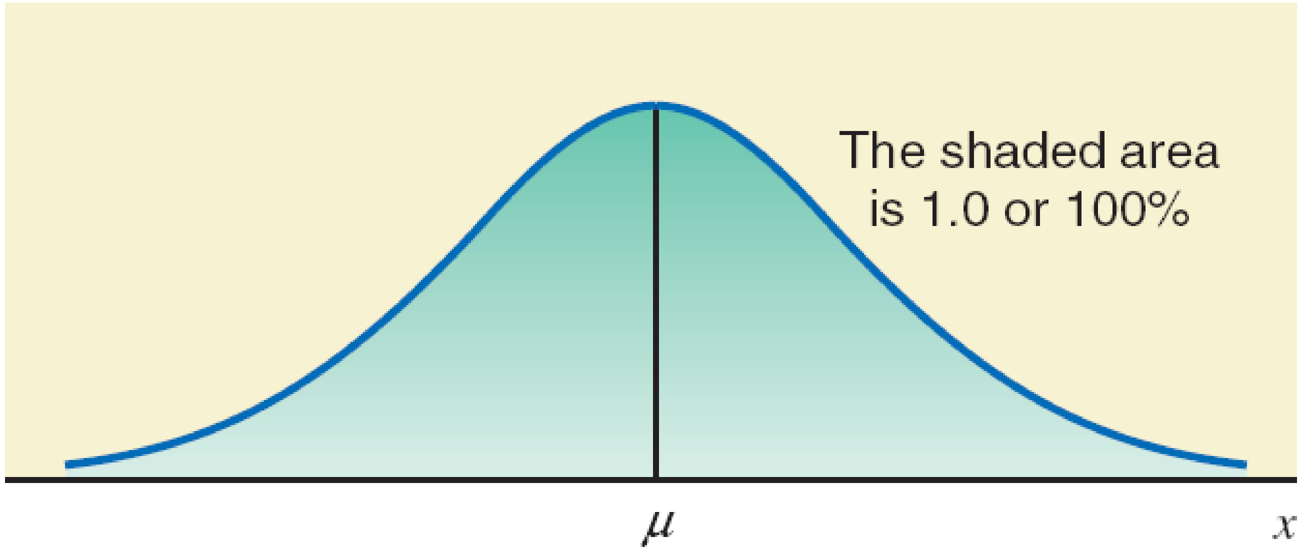
- Bell-shaped
- Total area under the curve =1



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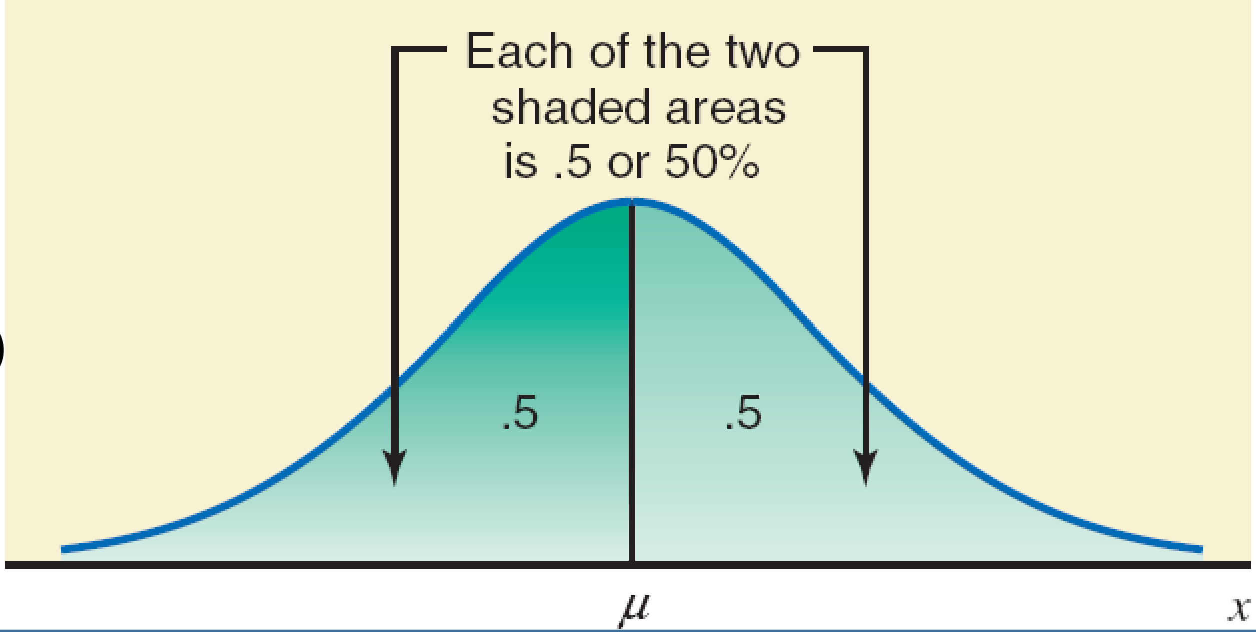
- Bell-shaped
- Total area under the curve =1
- Symmetric (Mode=Mean=Median)



Normal distribution function

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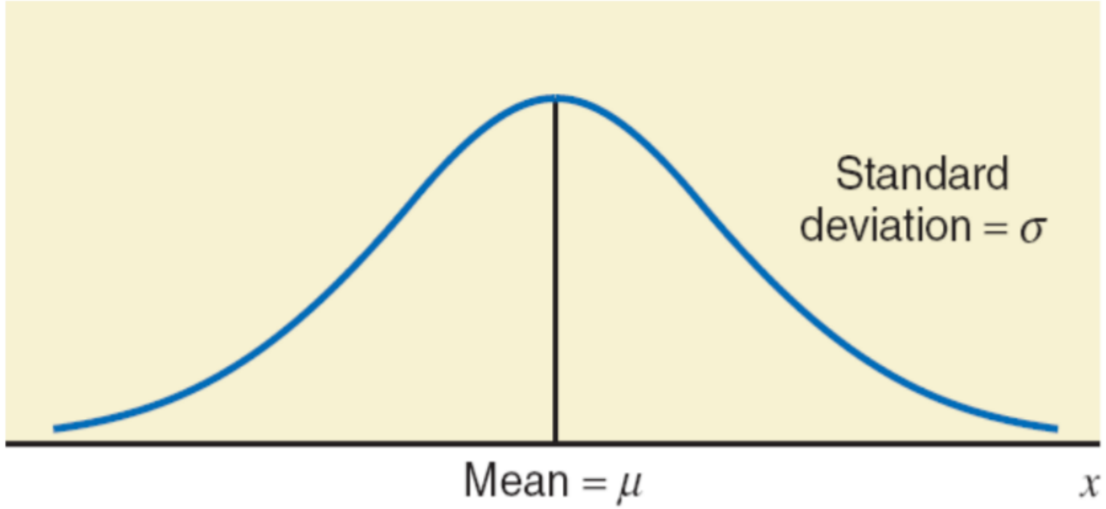
Density function is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

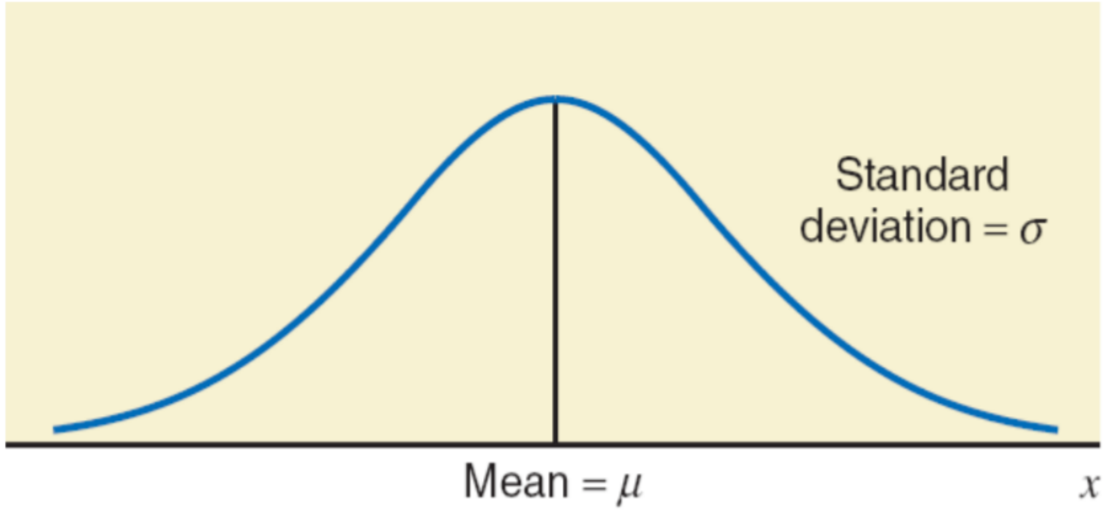
- Bell-shaped
- Total area under the curve =1
- Symmetric (Mode=Mean=Median)
- Tails extend indefinitely



Normal distribution function, $N(\mu, \sigma^2)$

Density function:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

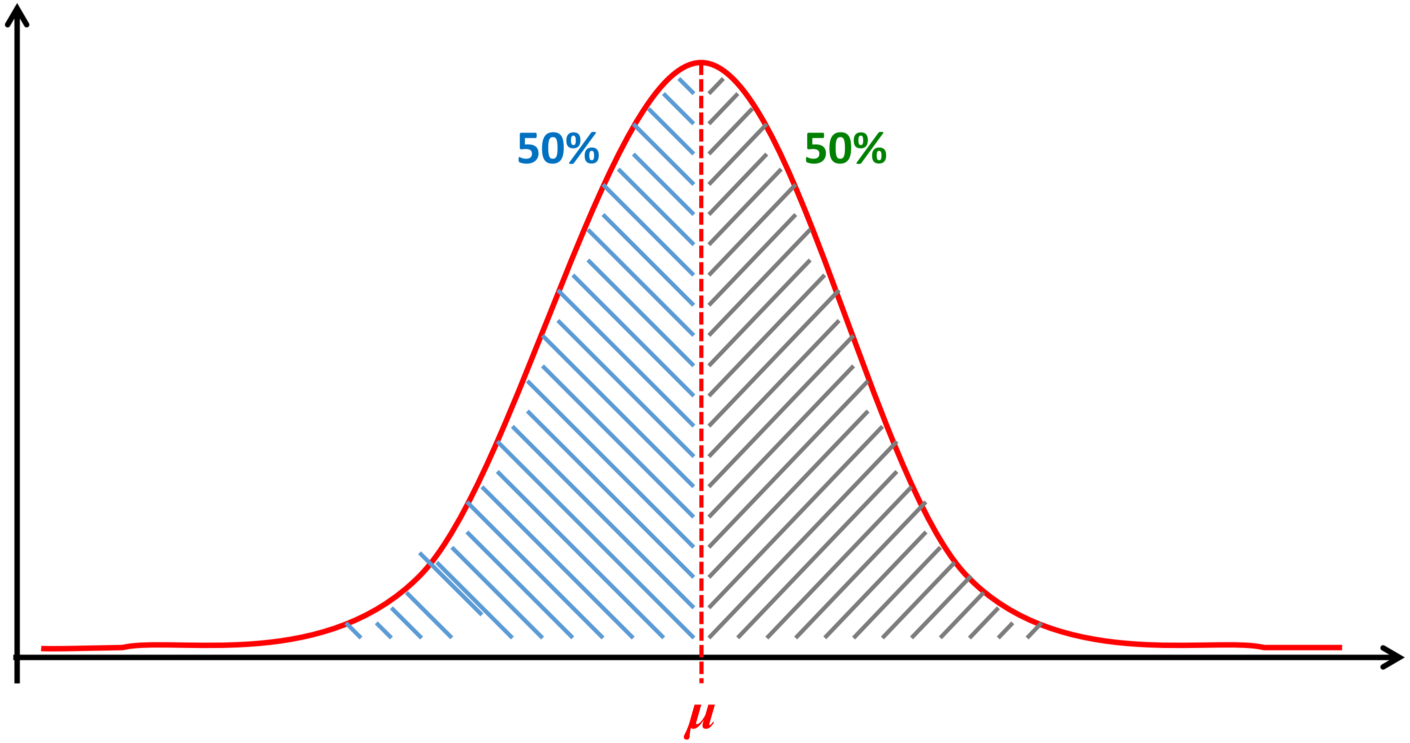


Some examples of things that follow a Normal Distribution:

- Heights of people
- Size of items produced by a machine
- Errors in measurements
- Blood Pressure
- Test Scores
- …

Normal distribution function, $N(\mu, \sigma^2)$

- It is a bell-shaped curve
- Symmetry about the mean μ (mean = mode = median)
- The total area under the curve is equal to 1 (or 100%)
- 50% of the area is to the left of the mean, and 50% to the right



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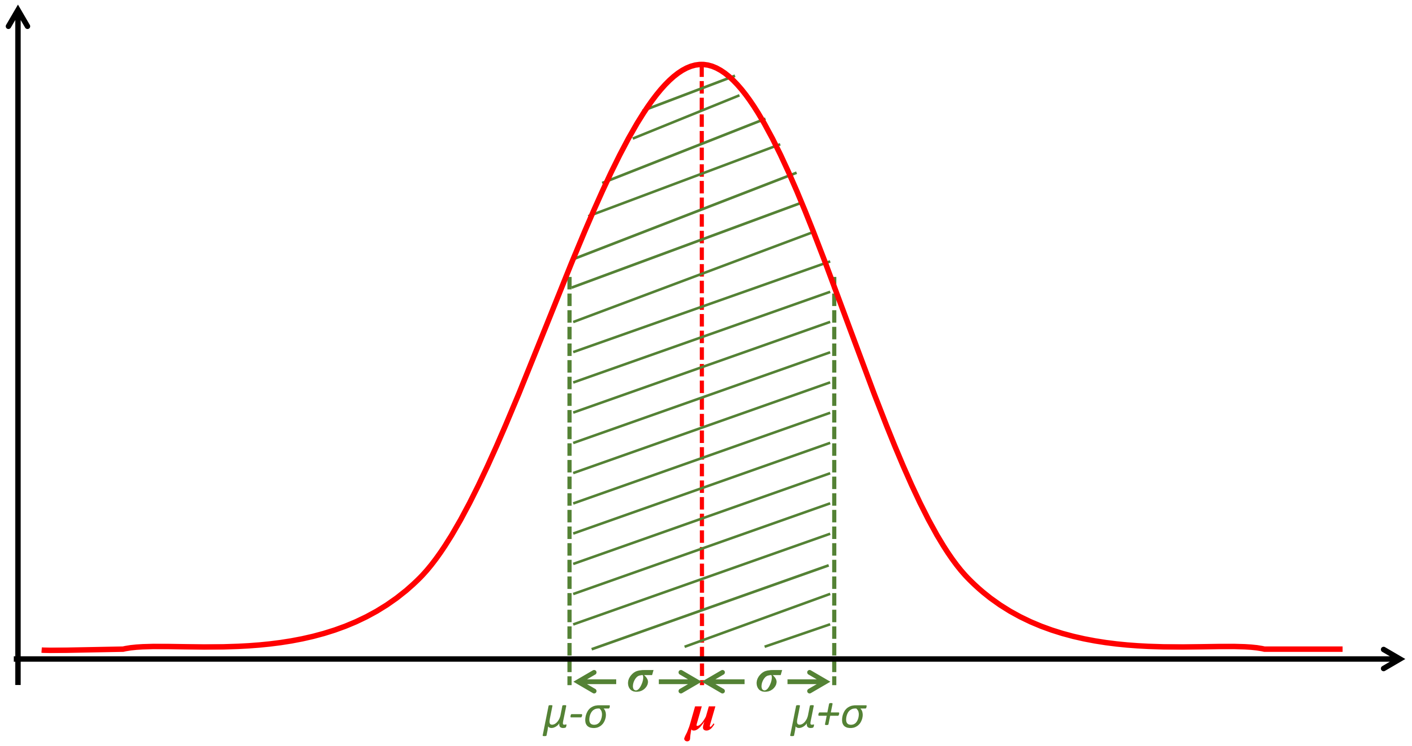

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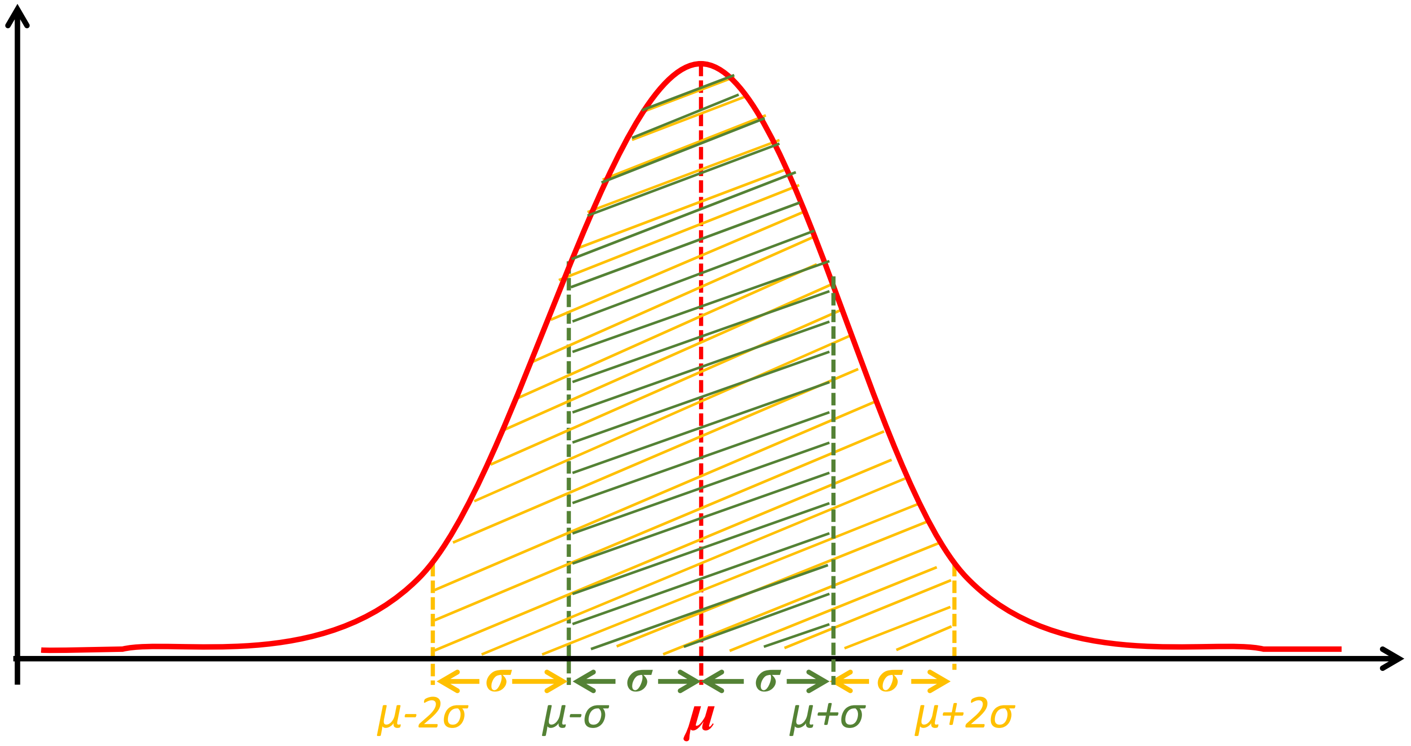

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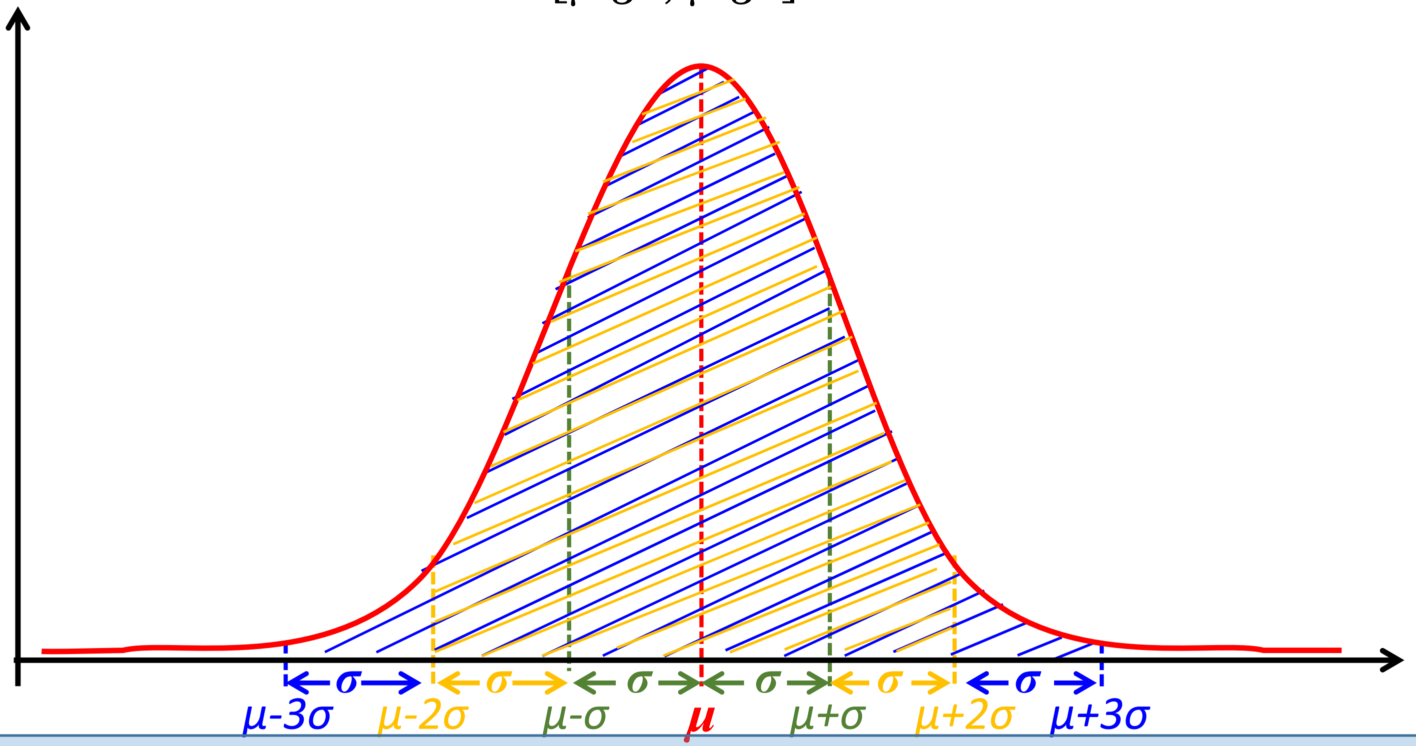
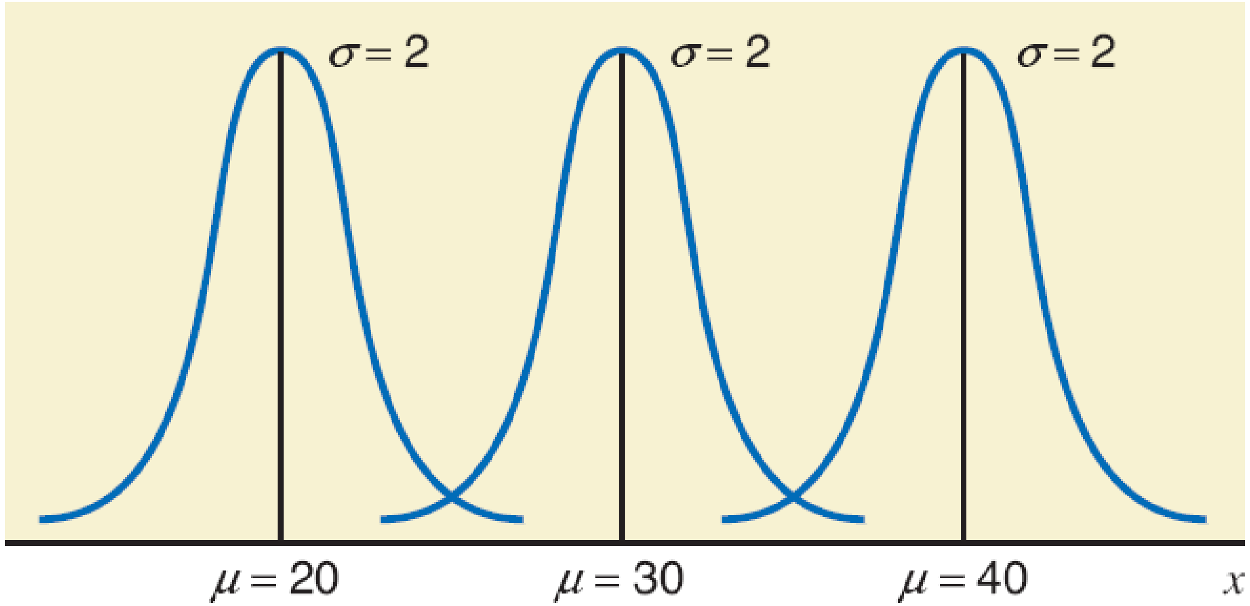
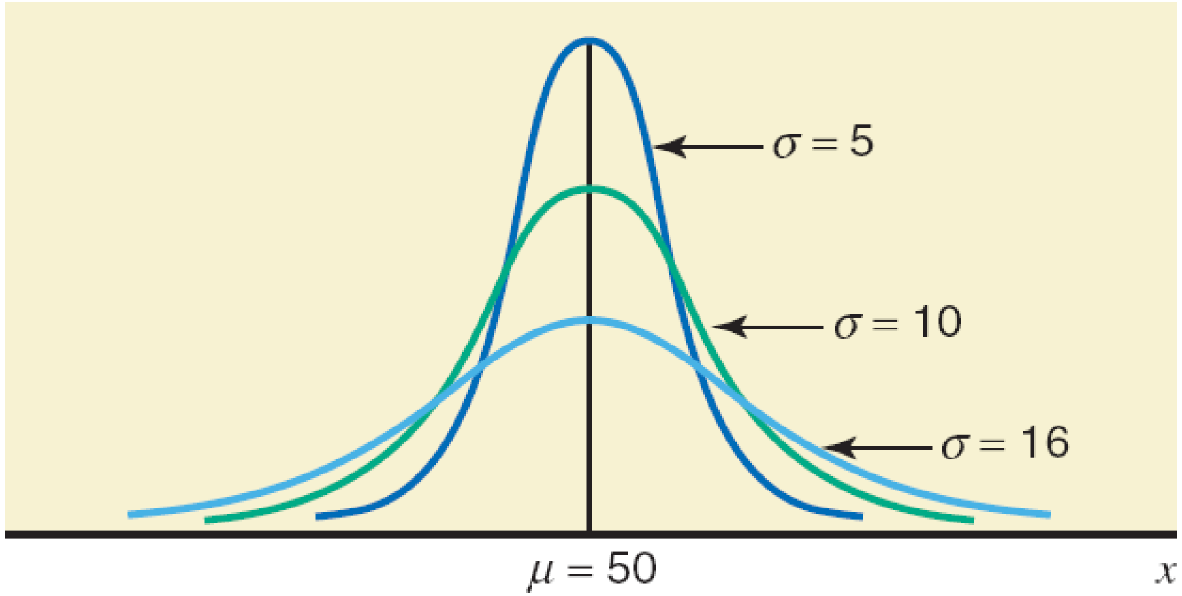


Normal distribution function: family

$x \sim N(\mu, \sigma^2)$ ➔ Density function: $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$

2 parameters: μ (mean, central value), and σ (std.dev., the spread)





Standard Normal distribution, z

The Standard Normal Distribution, $\boldsymbol{Z}$, is a Normal Distribution with mean equal to 0 and standard deviation equal to 1, $Z \sim N(0,1)$

$$Z \sim N(0,1)$$

Density function:

$$f(z) = \frac{1}{\sqrt{2\pi}} exp\left\{-\frac{1}{2} z^2\right\}$$

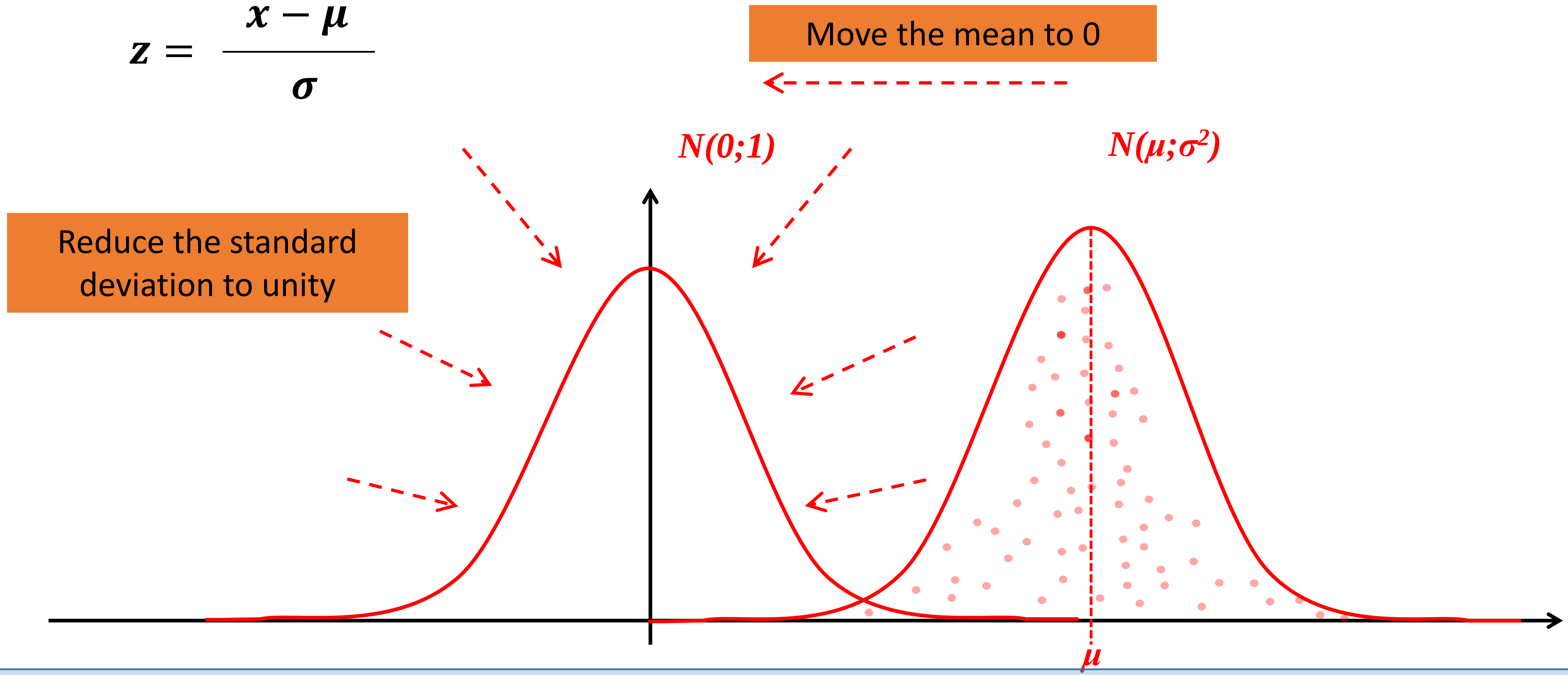
Standard Normal distribution, z

ANY Normal distribution $X \sim N(\mu, \sigma^2)$ can be reconverted into a Standard Normal $Z \sim N(0,1)$, by **standardization**

$$Z = \frac{X - \mu}{\sigma}$$

Standard Normal distribution, z

$$z = \frac{x - \mu}{\sigma}$$



Standard Normal distribution, z: examples

Example:

$X \sim N(50,100)$. Standardize the following values of x:

(a) $x = 55 \rightarrow z = \frac{55-50}{10} = 0.5$

(b) $x = 35 \rightarrow z = \frac{35-50}{10} = -1.5$

Standard Normal distribution, z

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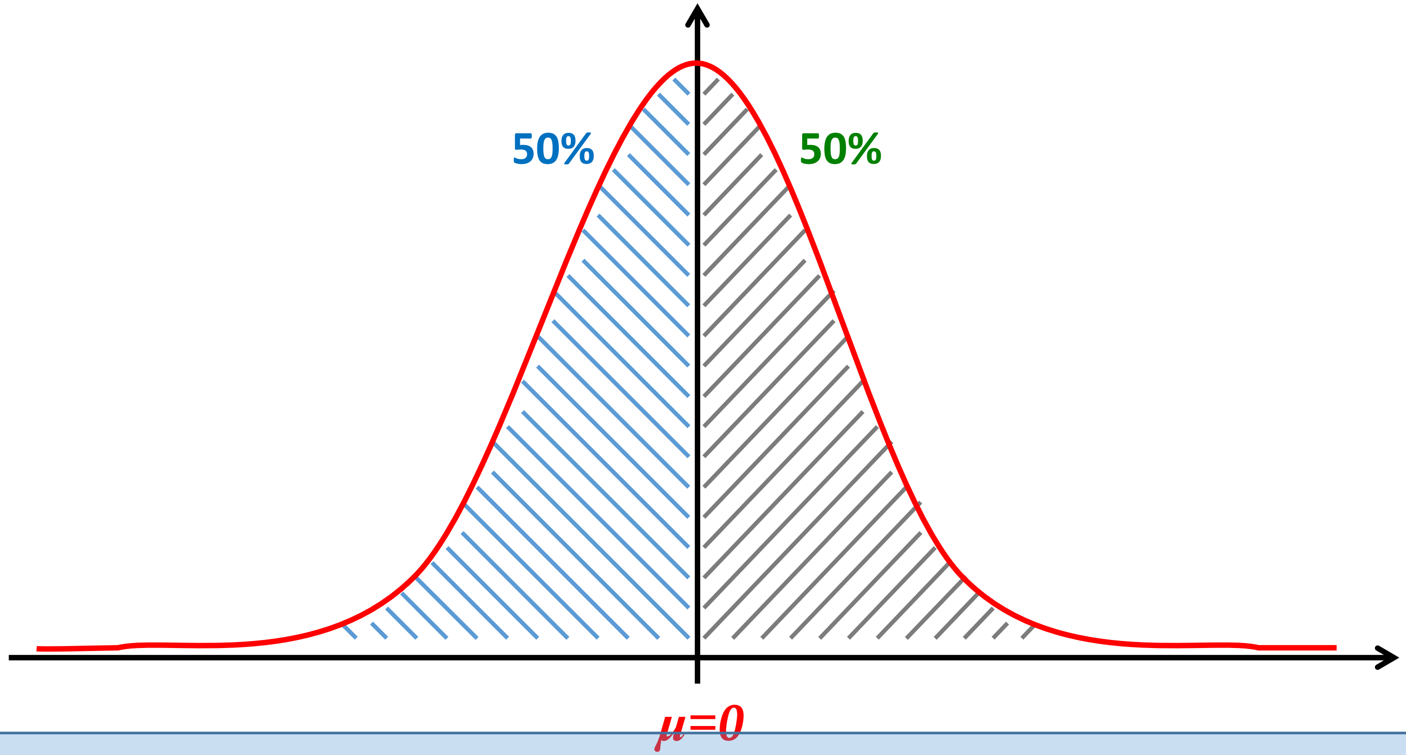

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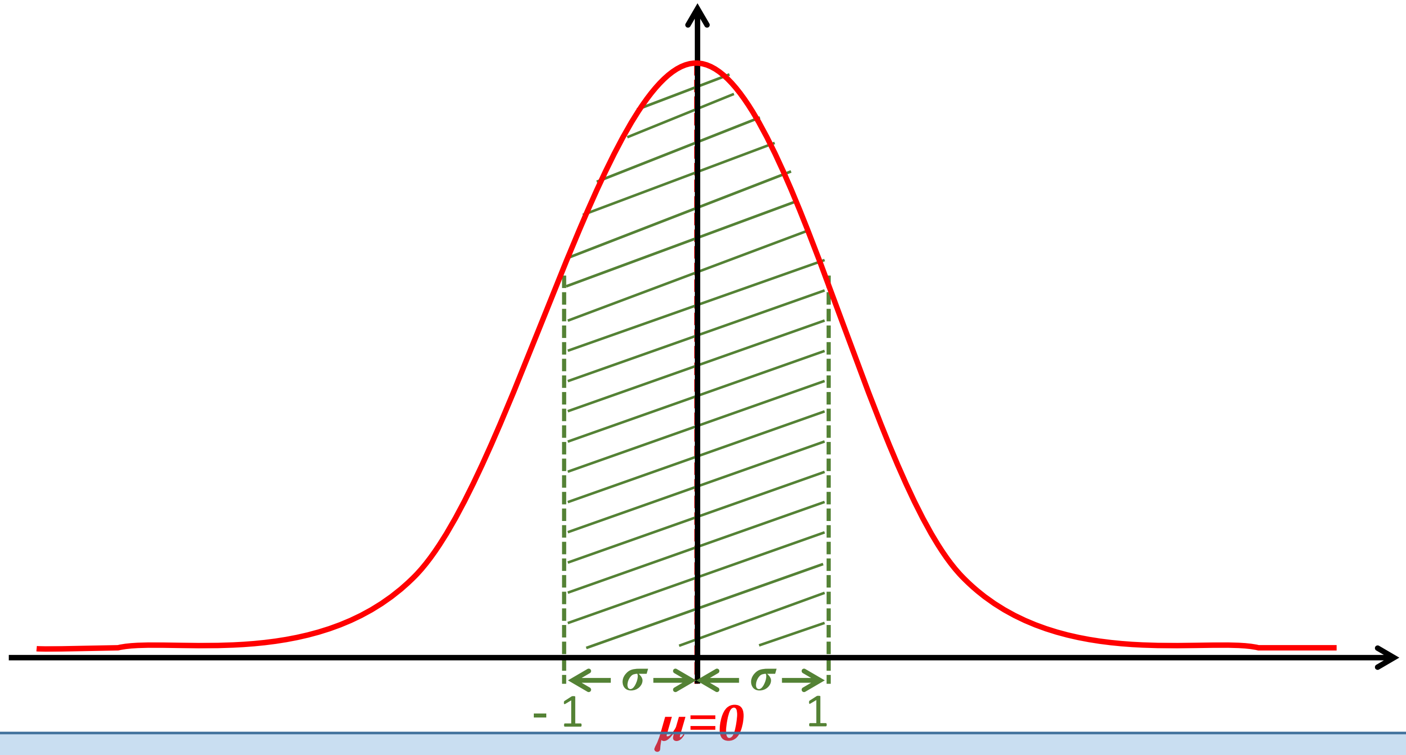

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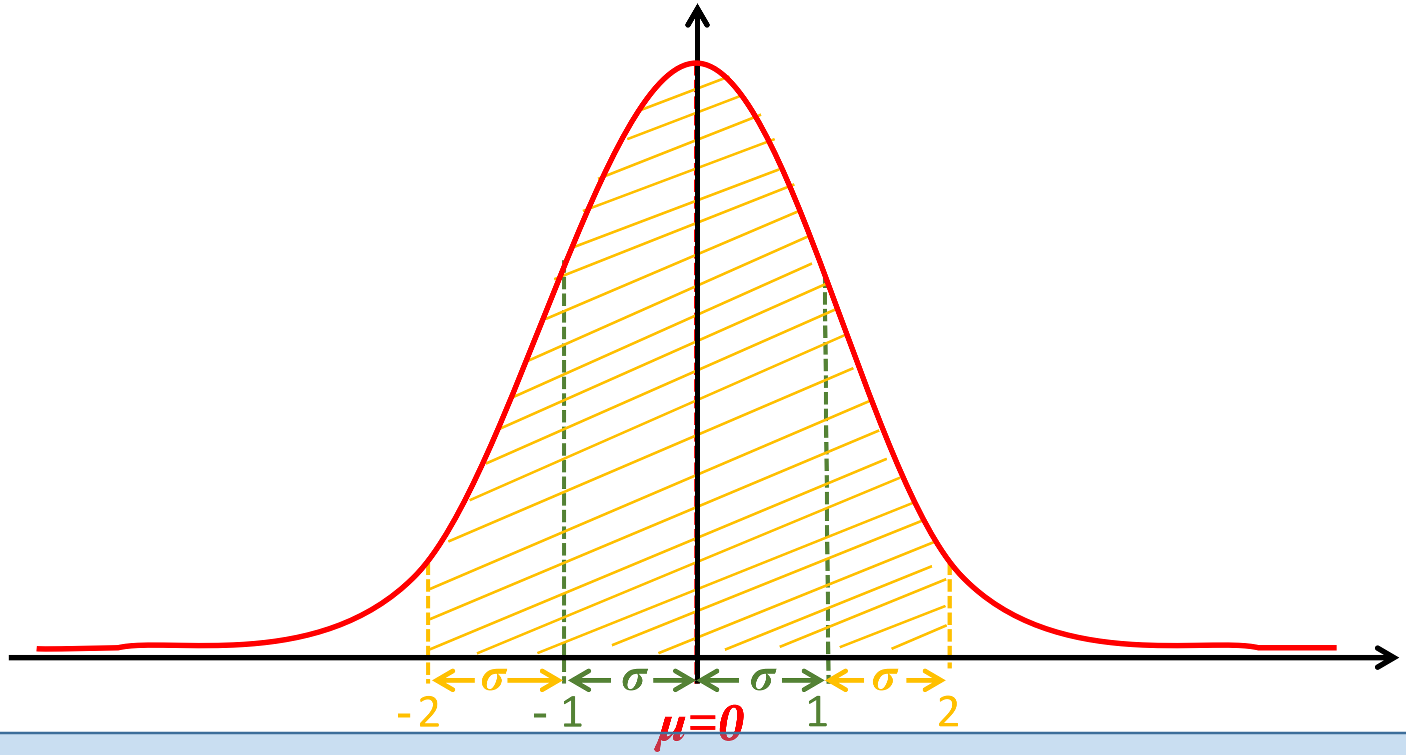

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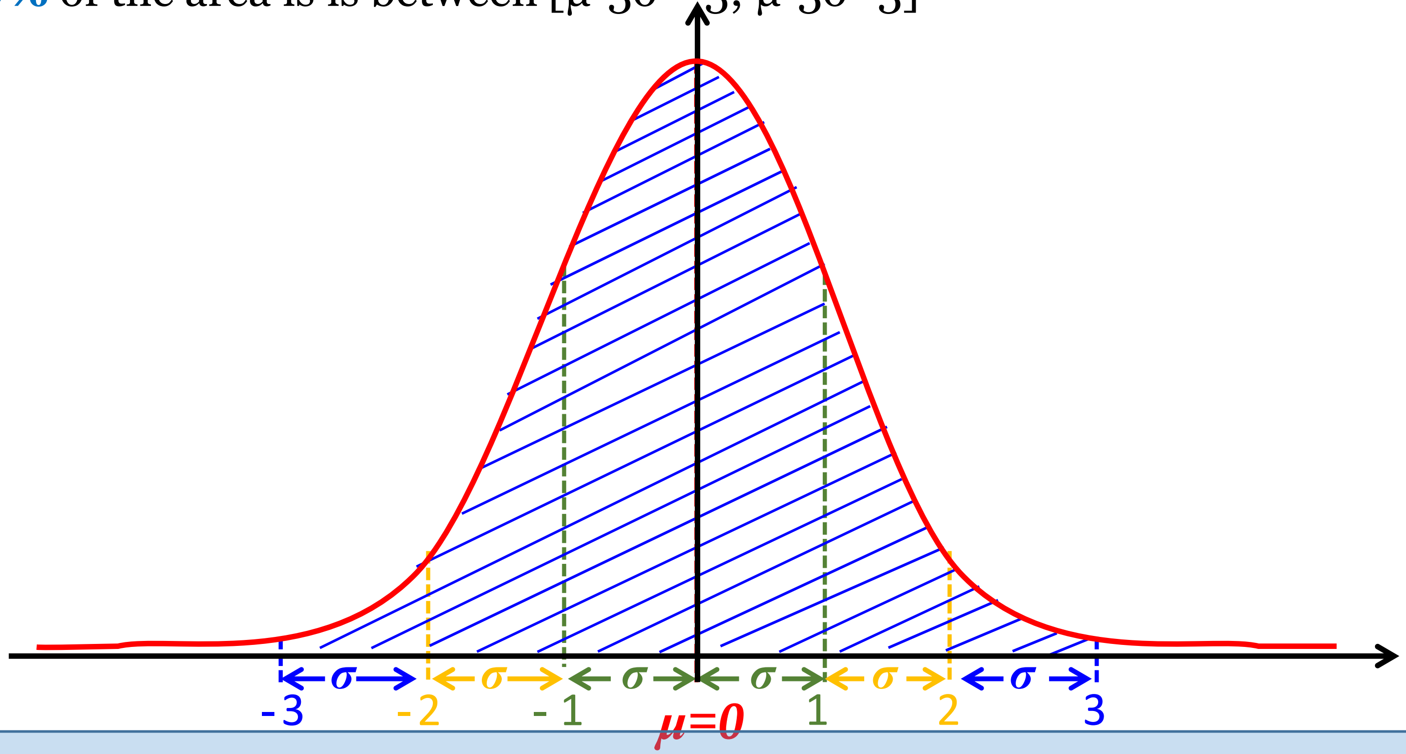
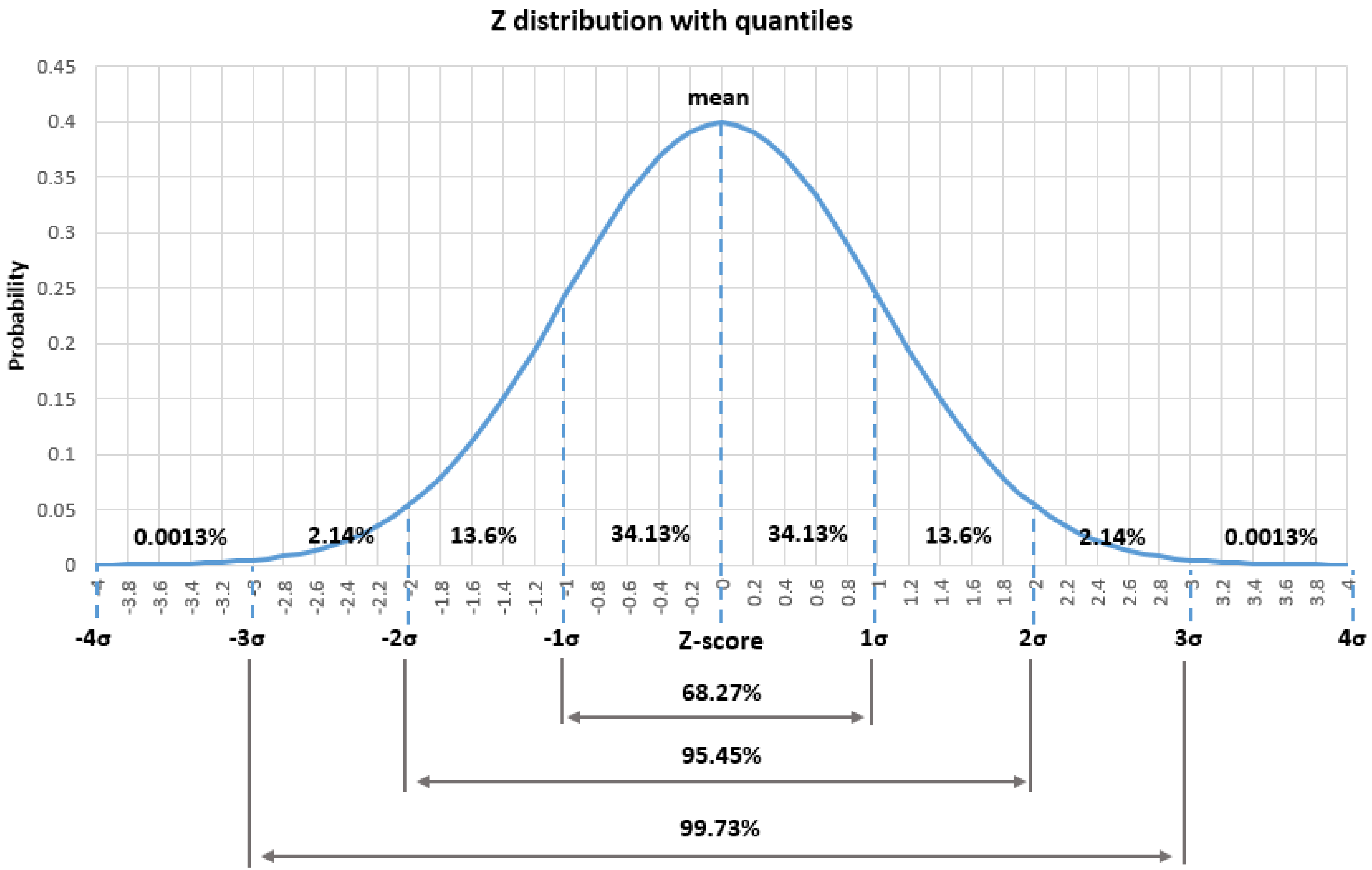


Standard Normal distribution, z

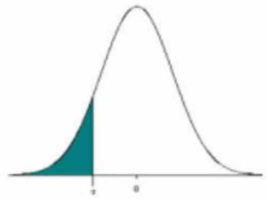


Statistical Table of Standard Normal distribution

Why it is so useful the Standard Normal Distribution $Z \sim N(0,1)$ and the standardization of a Normal distribution in a Standard Normal Distribution?

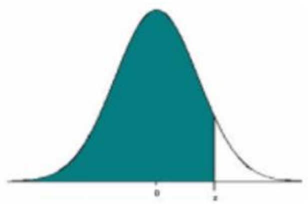
We have the tables of the Standard Normal Distribution $Z \sim N(0,1)$

Table of Standard Normal Probabilities for Negative Z-scores



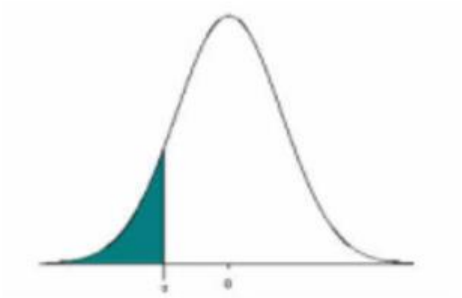
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



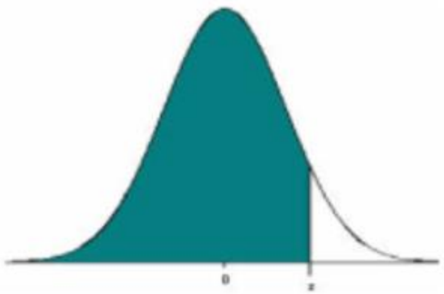
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table of Standard Normal Probabilities for Negative Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

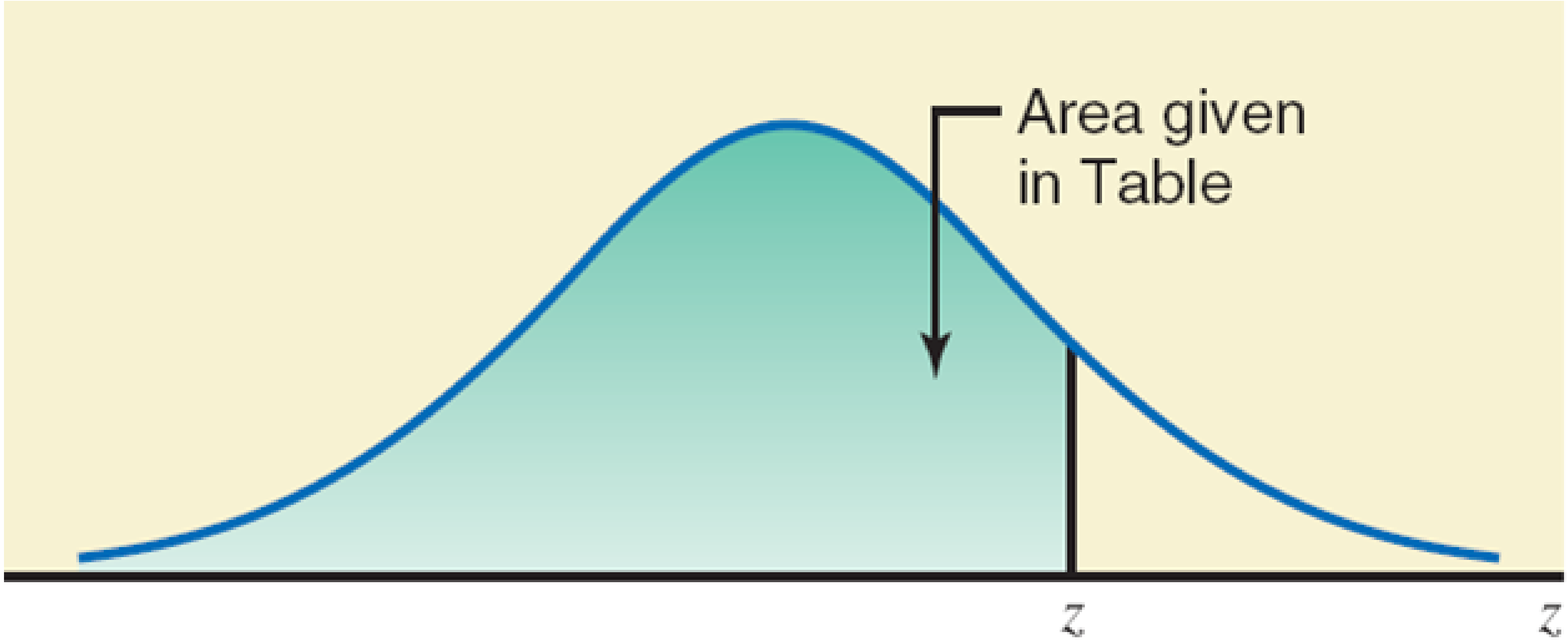
Table of Standard Normal Probabilities for Positive Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

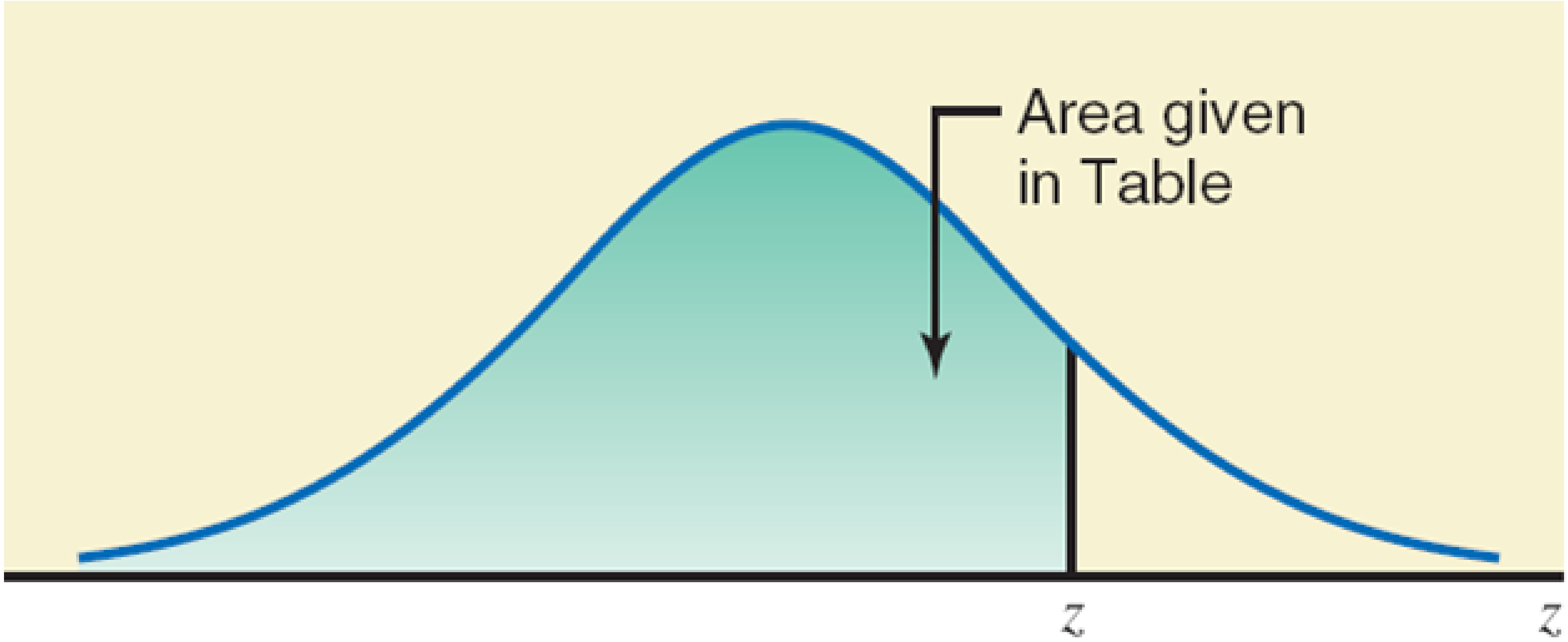
Statistical Table of Standard Normal distribution

The statistical table reports, for each value of z, the area under the curve at the left of z, denoted by $\Phi(z)$



Statistical Table of Standard Normal distribution

For each z-score, the tables give to us the value of the area between $-\infty$ to z.
The area in the tables is also the probability of having a value lower than z



Using the Standard Normal Table: example

Find the area under the standard normal curve to the left of z = 1.95.

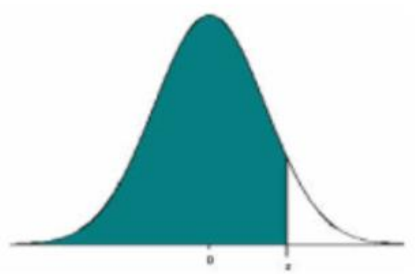
Area Under the Standard Normal Curve to the Left of $z = 1.95$

z	.00	.01	...	.05	...	.09
.	.	.	...	.	...	.
.	.	.	...	.	...	.
.	.	.	...	.	...	.
1.9	.9713	.9719	...	.9744 ← Required area	...	.9767
.	.	.	...	.	...	.
.	.	.	...	.	...	.
.	.	.	...	.	...	.
3.4	.9997	.9997	...	.9997	...	.9998

Using the Standard Normal Table: example

If we have only the table of Standard Normal Probabilities for Positive Z-scores we can find all the values (also for negative z)

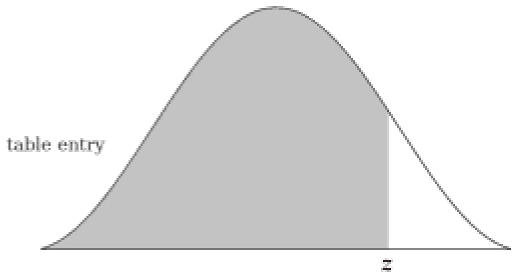
Table of Standard Normal Probabilities for Positive Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964

Using the Standard Normal Table: example

<u>Positive value of z</u>



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Find the value of z in the table.

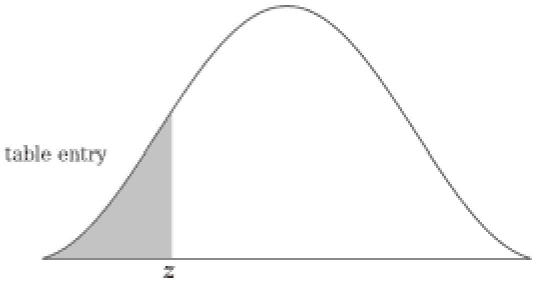
This is the area between [-∞;z] or the **probability lower than z**

Ex. $P(z < 2.04)$=?

$P(z < 2.04)=0.9793$

Using the Standard Normal Table: example

<u>Negative value of z</u>



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884

From the condition of probability distribution:

$\Sigma\, P(-\infty < Z < +\infty) = 1$

Z is also symmetric around the mean

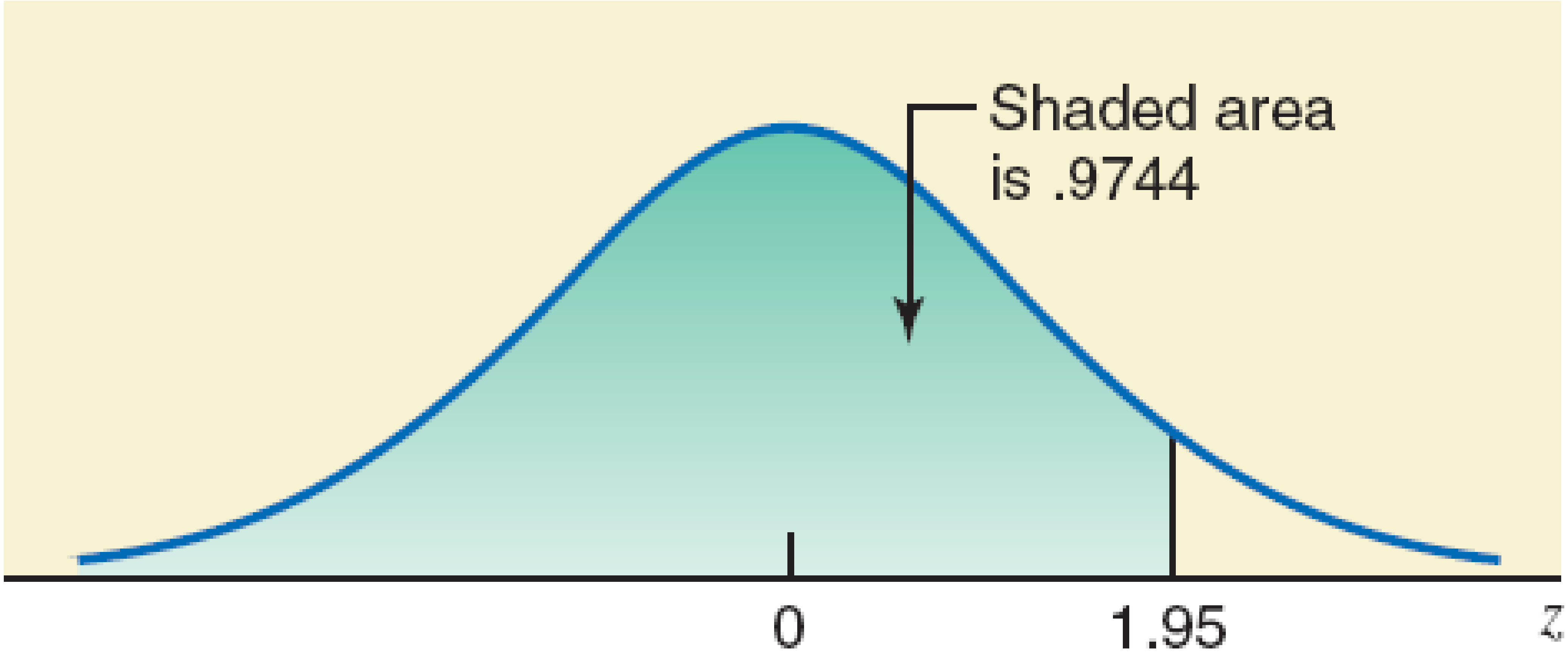
You can find the negative value of z taking the value of z in absolute value (or in modulus) doing $1 - |z|$

Ex. P(z < - 2.04)=?

P(z < - 2.04) =1 – P(z<2.04)= 1 - 0.9793 = 0.0207

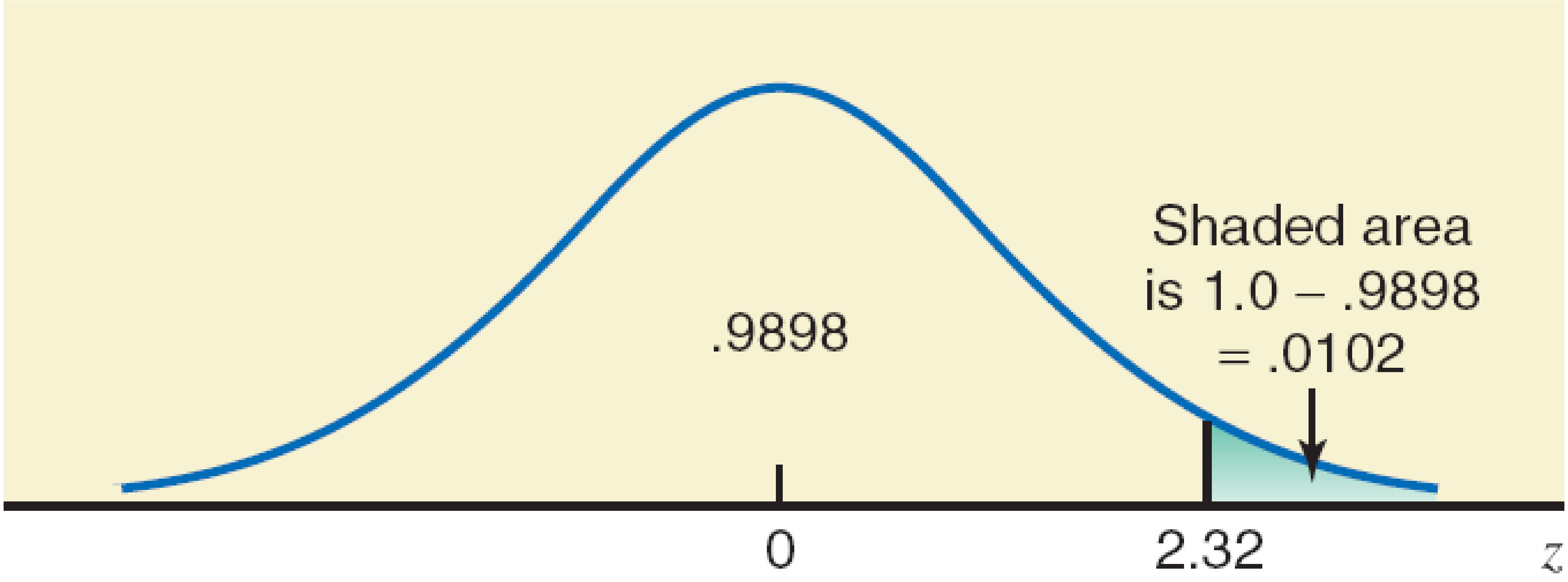
Using the Standard Normal Table: Ex 1

Find the area under the standard normal curve to the left of z = 1.95.



Using the Standard Normal Table: Ex 2

Find the area to the right of z = 2.32.



Using the Standard Normal Table: Ex 3

Find the area to the left of z = -1.54.

z is a negative value

You find the area taking the z in absolute value (or in modulus)

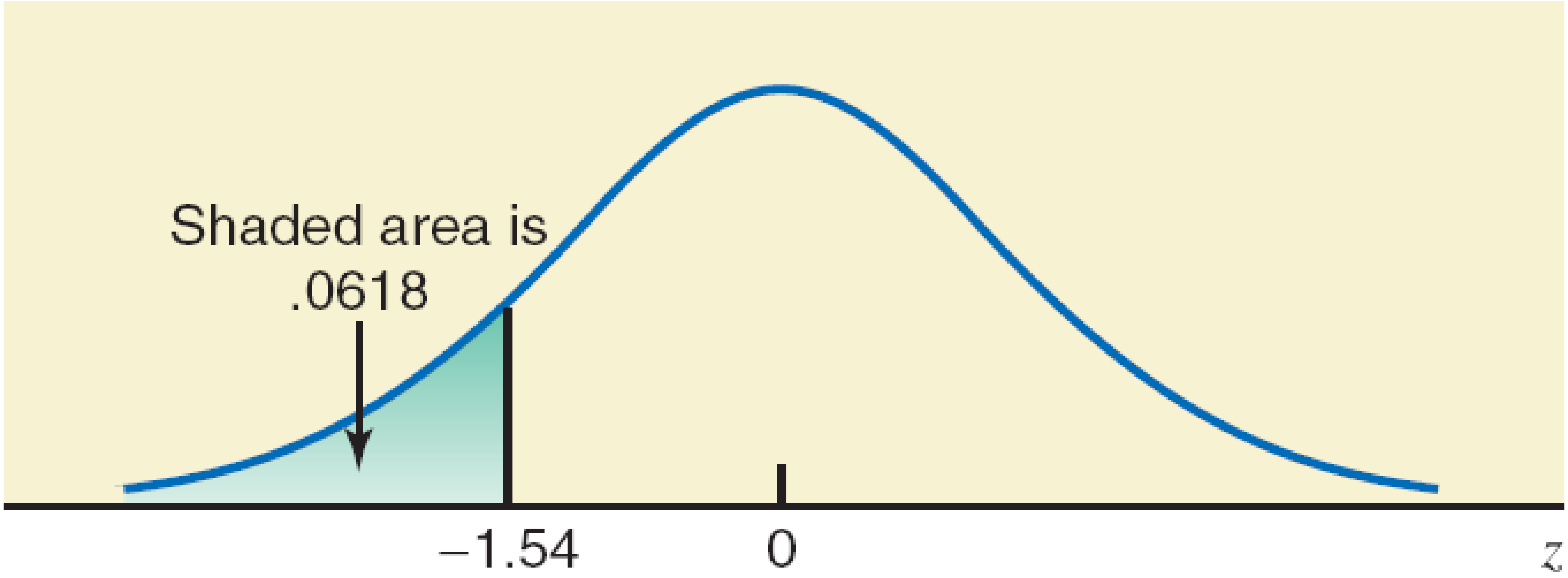
$|z| = 1.54$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545

The area will equal to be 1-P(|z|)=1-0.9382=0.0618

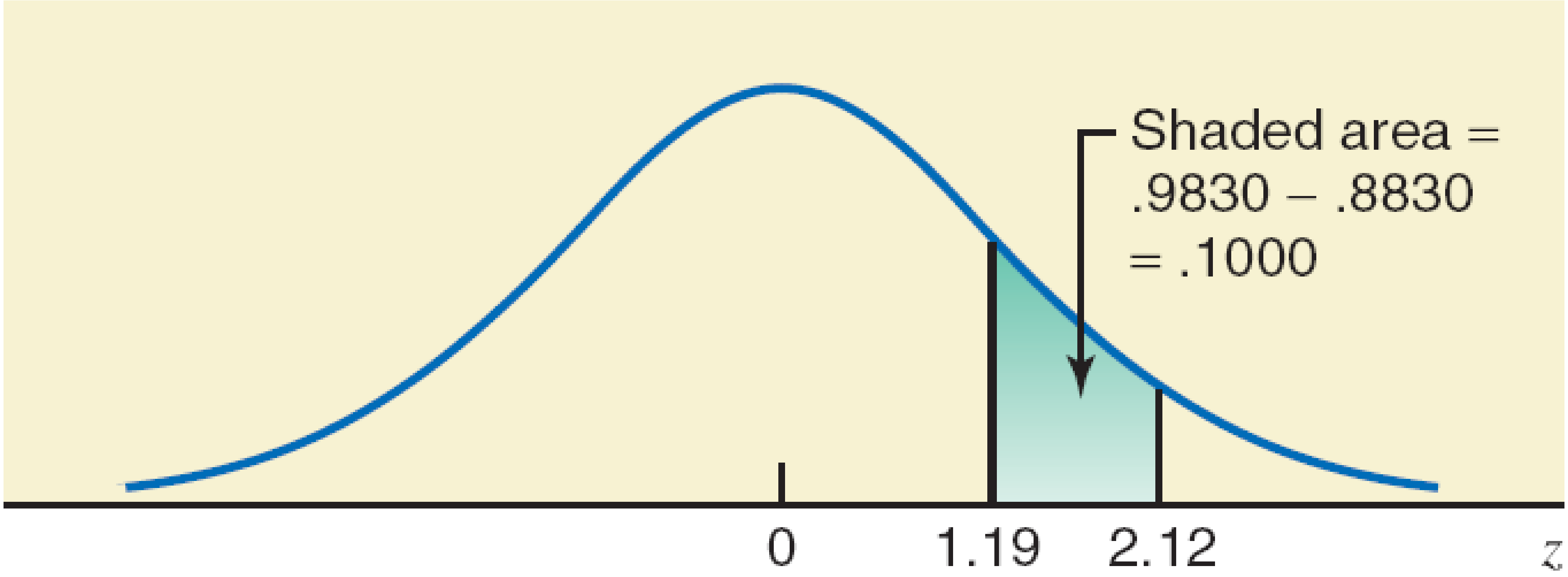
Using the Standard Normal Table: Ex 3

Find the area to the left of $z = -1.54$.



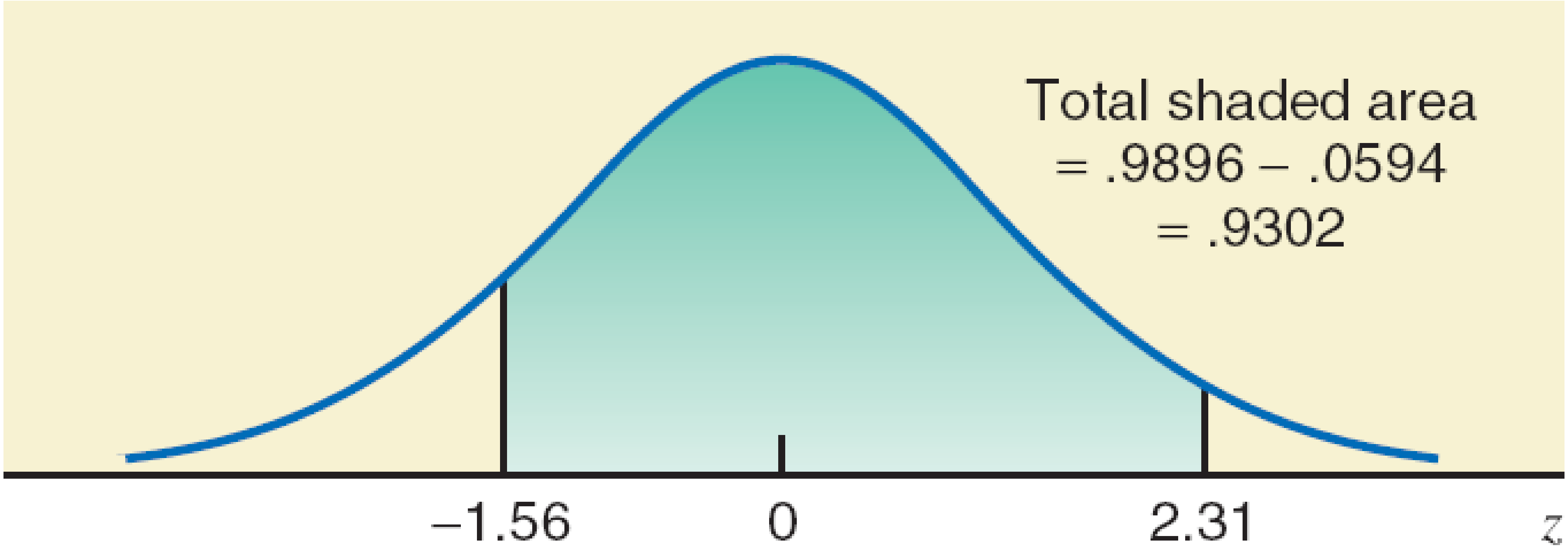
Using the Standard Normal Table: Ex 4

Find $P(1.19 < z < 2.12)$



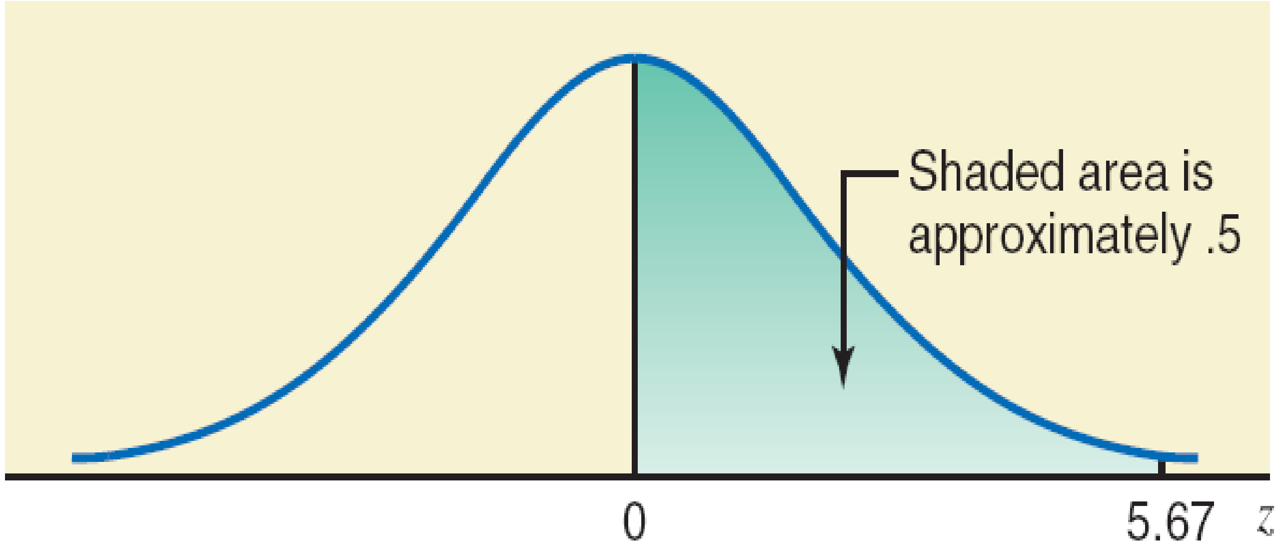
Using the Standard Normal Table: Ex 5

Find $P(-1.56 < z < 2.31)$

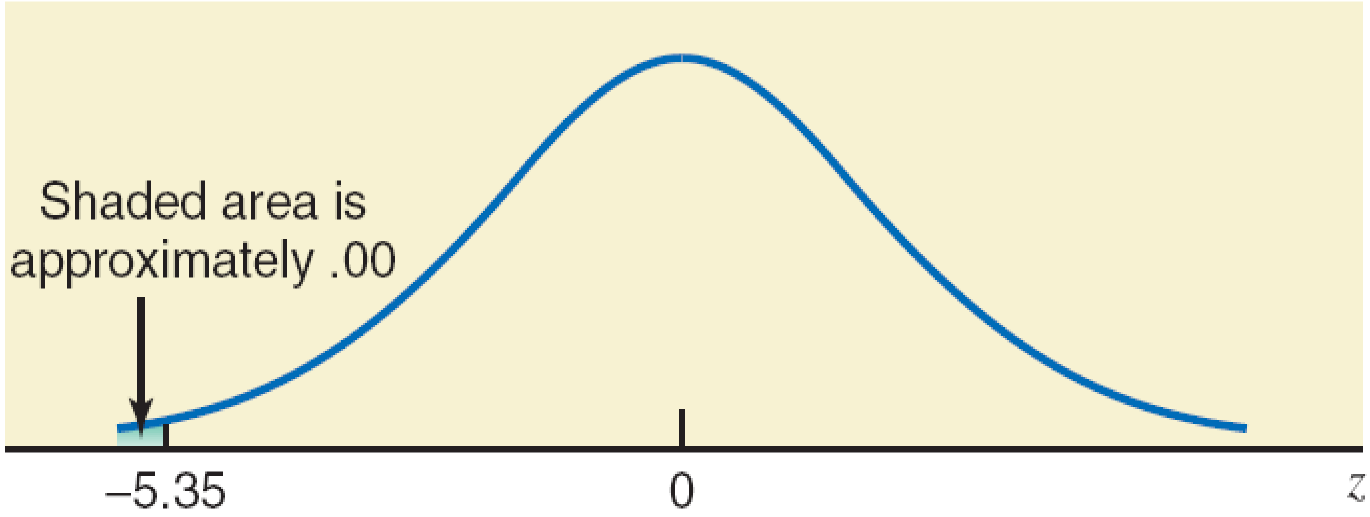


Using the Standard Normal Table: Ex 6

$P(0 < z < 5.67)$ ➔



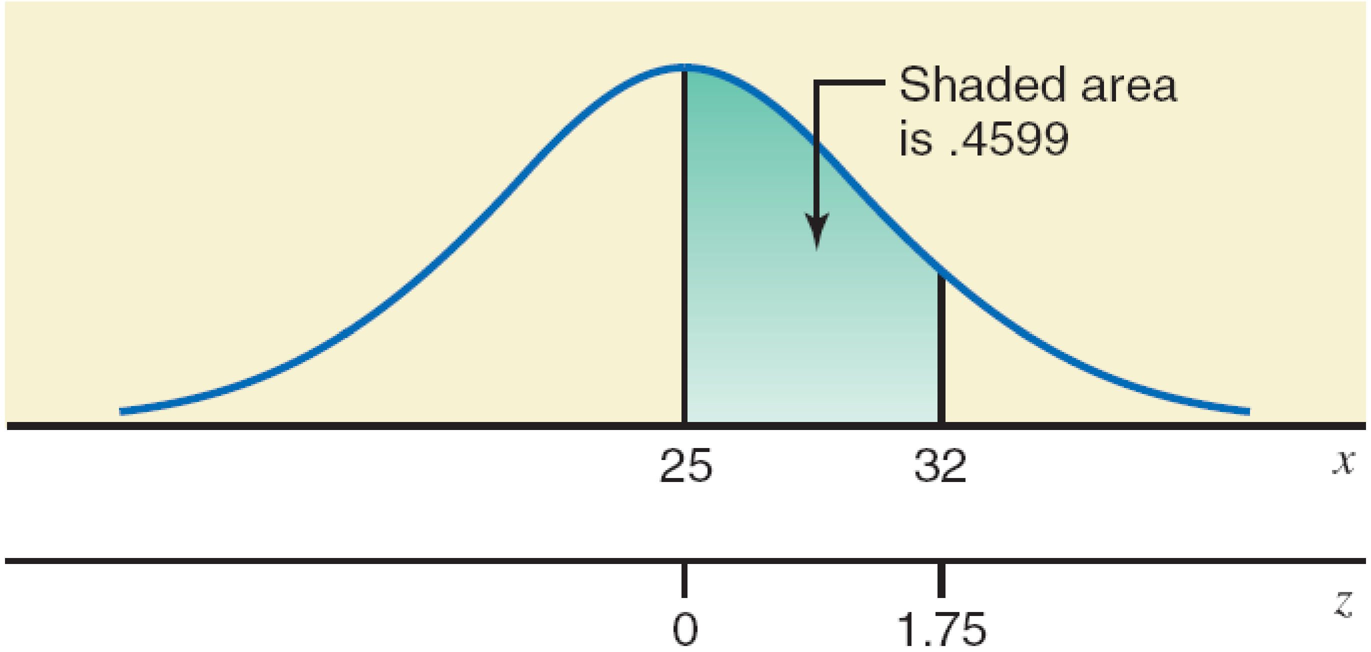
$P(z < -5.35)$ ➔



From x to z

Let $X \sim N(25,16)$. Find $P(25 < x < 32)$:

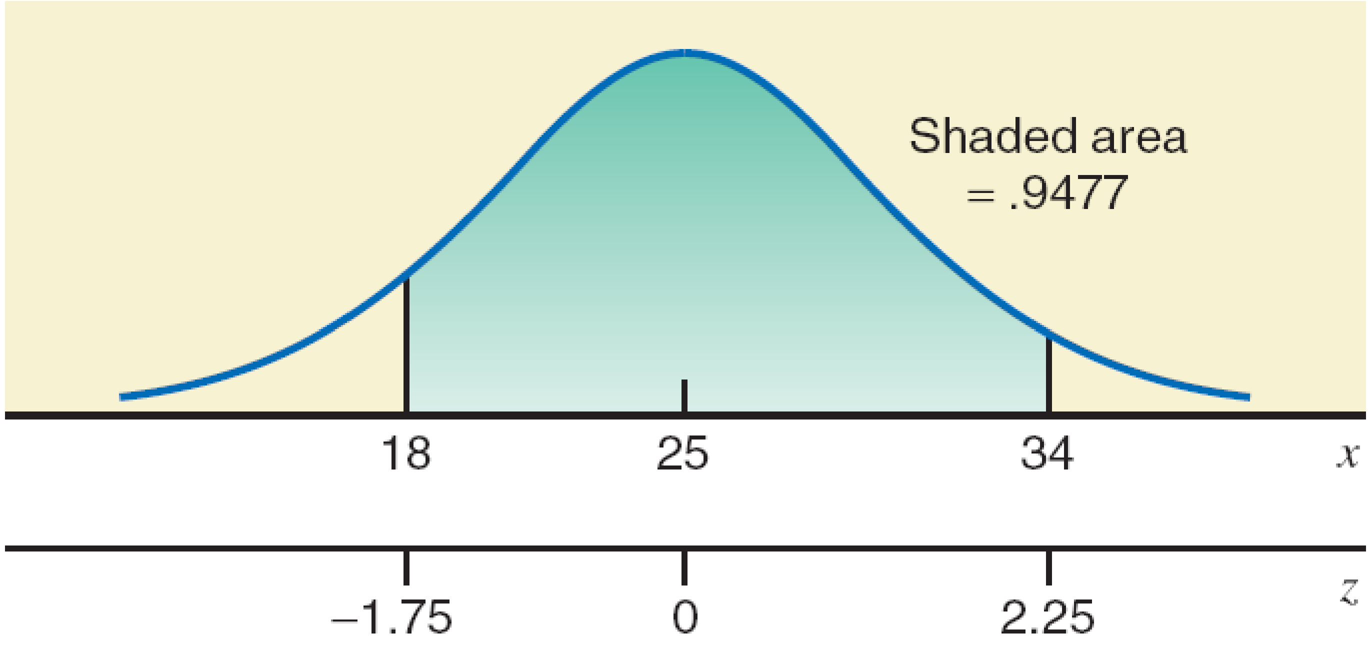
1) Standardize $\rightarrow z = \frac{25-25}{4} = 0$ and $z = \frac{32-25}{4} = 1.75$
2) Use the statistical table



From x to z

Let $X \sim N(25,16)$. Find $P(18 < x < 34)$

1) Standardize ➔ $z = \frac{18-25}{4} = -1.75$ and $z = \frac{34-25}{4} = 2.25$
2) Use the statistical table



Using the Standard Normal Table: Ex 7

Exercise 7

Z is a normally standard distributed variable with mean $\mu = 0$ and standard deviation $\sigma = 1$.

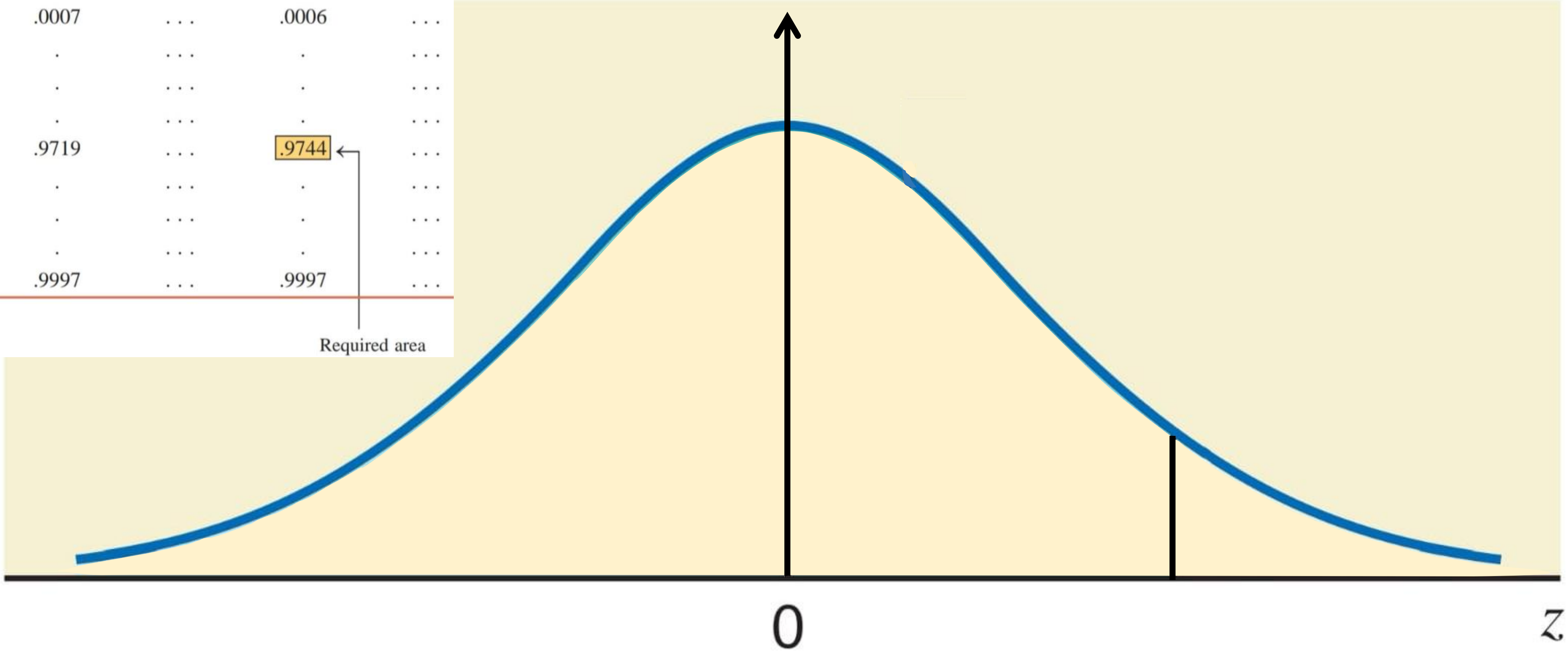
Find

a. $P(z < 1.95)$
b. $P(z > 2.32)$
c. $P(1.19 < z < 2.12)$

Using the Standard Normal Table: Ex 7

z	.00	.01	. . .	.05	. . .
−3.4	.0003	.0003	. . .	.0003	. . .
−3.3	.0005	.0005	. . .	.0004	. . .
−3.2	.0007	.0007	. . .	.0006	. . .
.	.	.	. . .	.	. . .
.	.	.	. . .	.	. . .
.	.	.	. . .	.	. . .
1.9	.9713	.9719	. . .	.9744	. . .
.	.	.	. . .	.	. . .
.	.	.	. . .	.	. . .
.	.	.	. . .	.	. . .
3.4	.9997	.9997	. . .	.9997	. . .

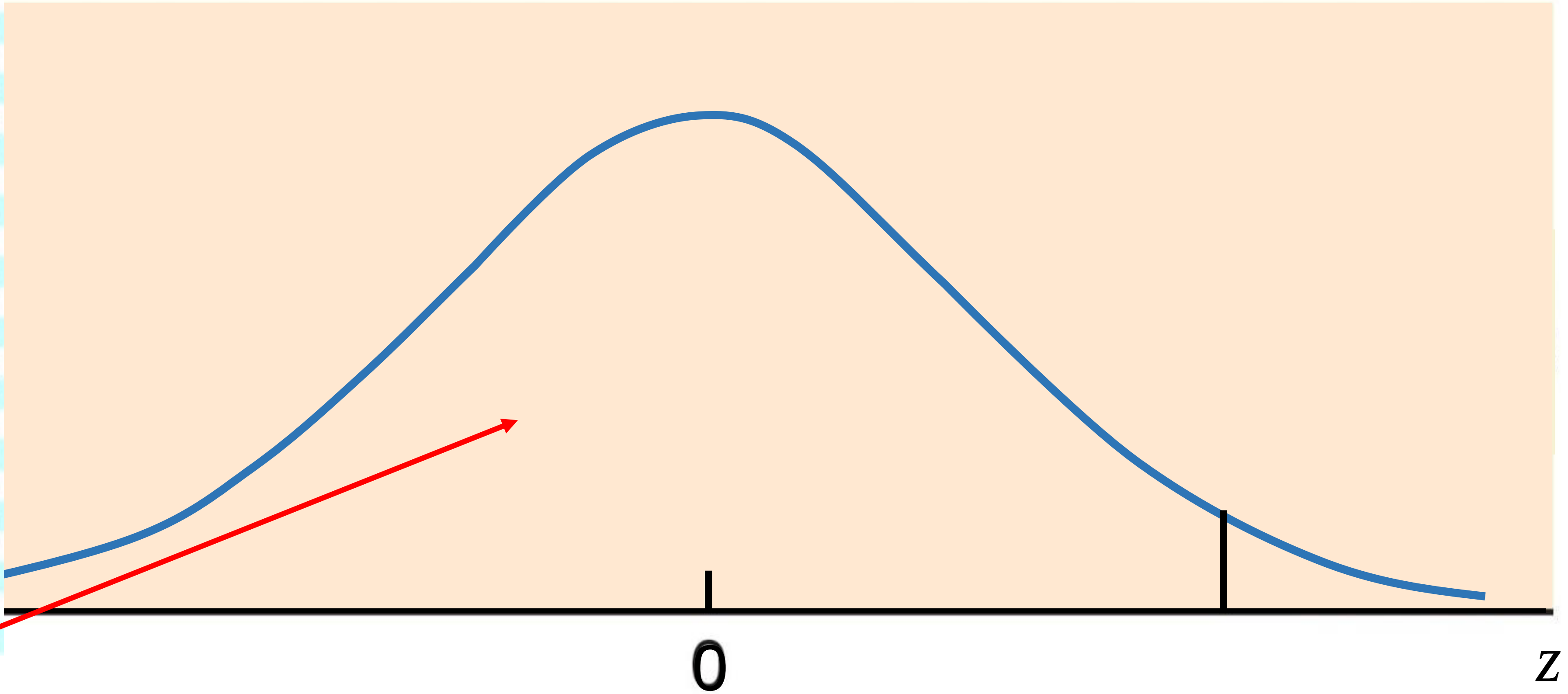
Required area



a. $P(z<1.95) = 0.9744 = 97.44\%$

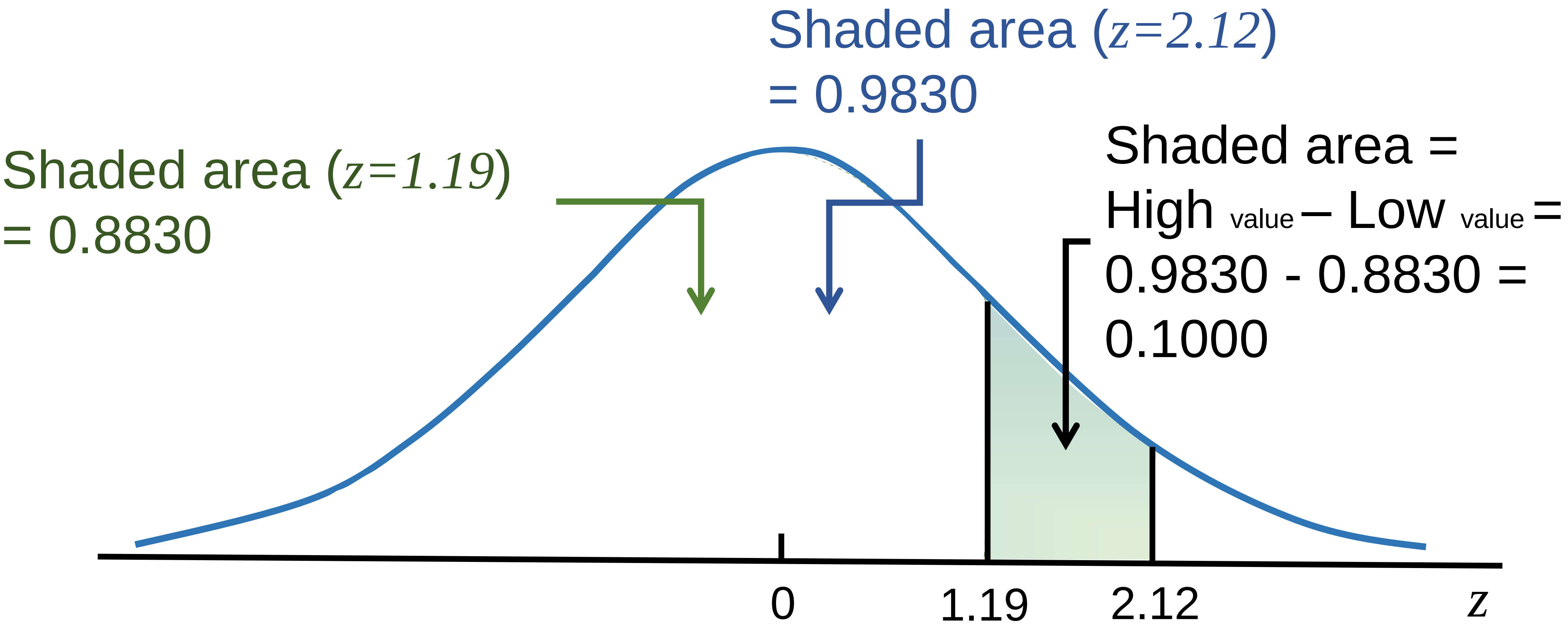
Using the Standard Normal Table: Ex 7

z	0.00	0.01	0.02
0.0	0.5000	0.5040	0.5080
0.1	0.5398	0.5438	0.5478
0.2	0.5793	0.5832	0.5871
0.3	0.6179	0.6217	0.6255
0.4	0.6554	0.6591	0.6628
0.5	0.6915	0.6950	0.6985
0.6	0.7257	0.7291	0.7324
0.7	0.7580	0.7611	0.7642
0.8	0.7881	0.7910	0.7939
0.9	0.8159	0.8186	0.8212
1.0	0.8413	0.8438	0.8461
1.1	0.8643	0.8665	0.8686
1.2	0.8849	0.8869	0.8888
1.3	0.9032	0.9049	0.9066
1.4	0.9192	0.9207	0.9222
1.5	0.9332	0.9345	0.9357
1.6	0.9452	0.9463	0.9474
1.7	0.9554	0.9564	0.9573
1.8	0.9641	0.9649	0.9656
1.9	0.9713	0.9719	0.9726
2.0	0.9772	0.9778	0.9783
2.1	0.9821	0.9826	0.9830
2.2	0.9861	0.9864	0.9868
2.3	0.9893	0.9896	$\boxed{0.9898}$



b. P(z>2.32) = 0.0102 = 1.02%

Using the Standard Normal Table: Ex 7



c. $P(1.19<z<2.12) = 0.1000 = 10\%$

Normal distribution

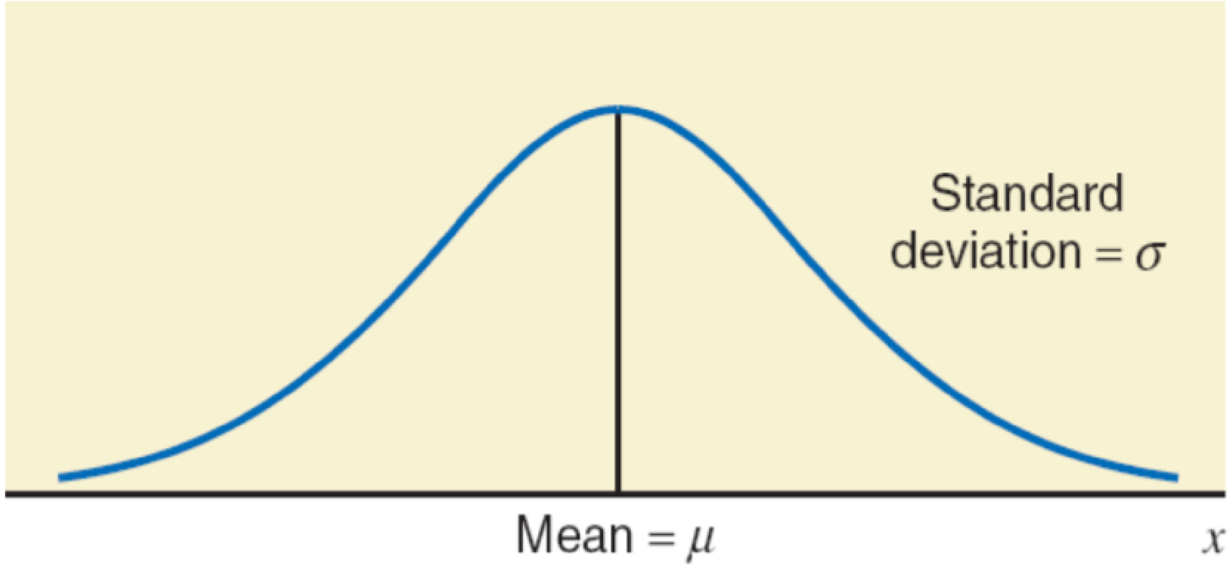
$$N(\mu, \sigma^2)$$

Density function is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

2 parameters:

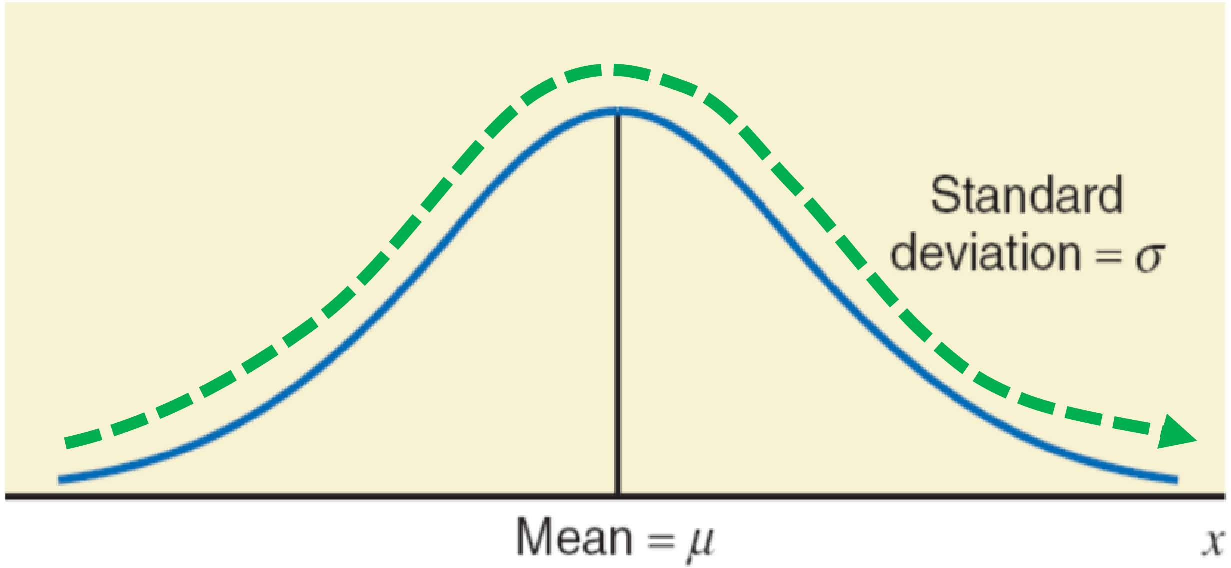
μ (mean) and σ (st. deviation)



Normal distribution

$$N(\mu, \sigma^2)$$

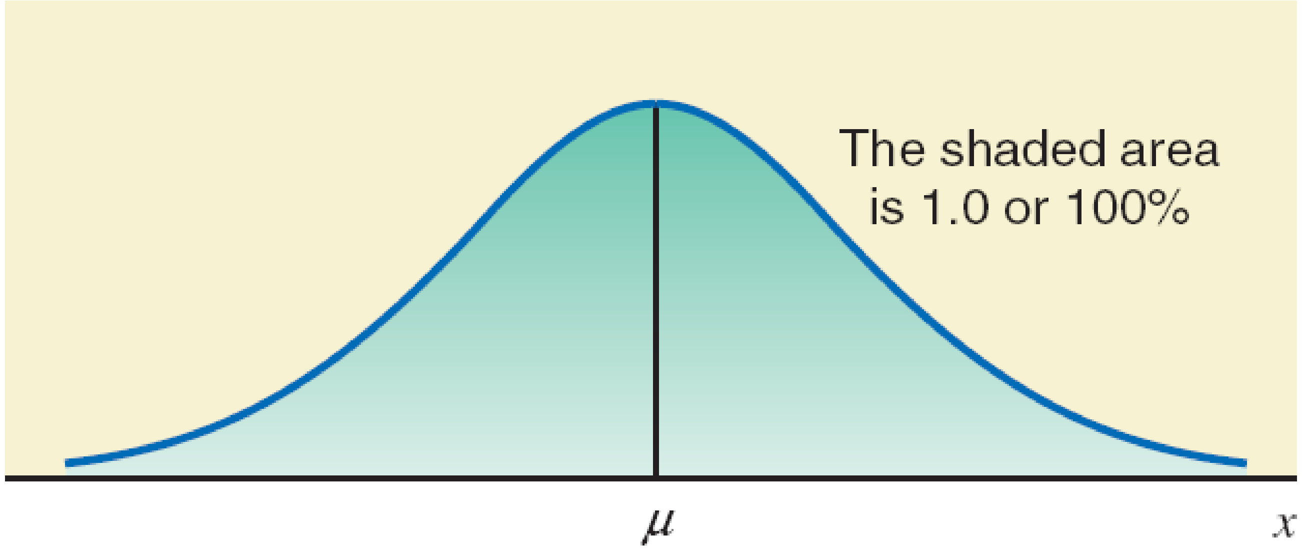
- Bell-shaped



Normal distribution

$$N(\mu, \sigma^2)$$

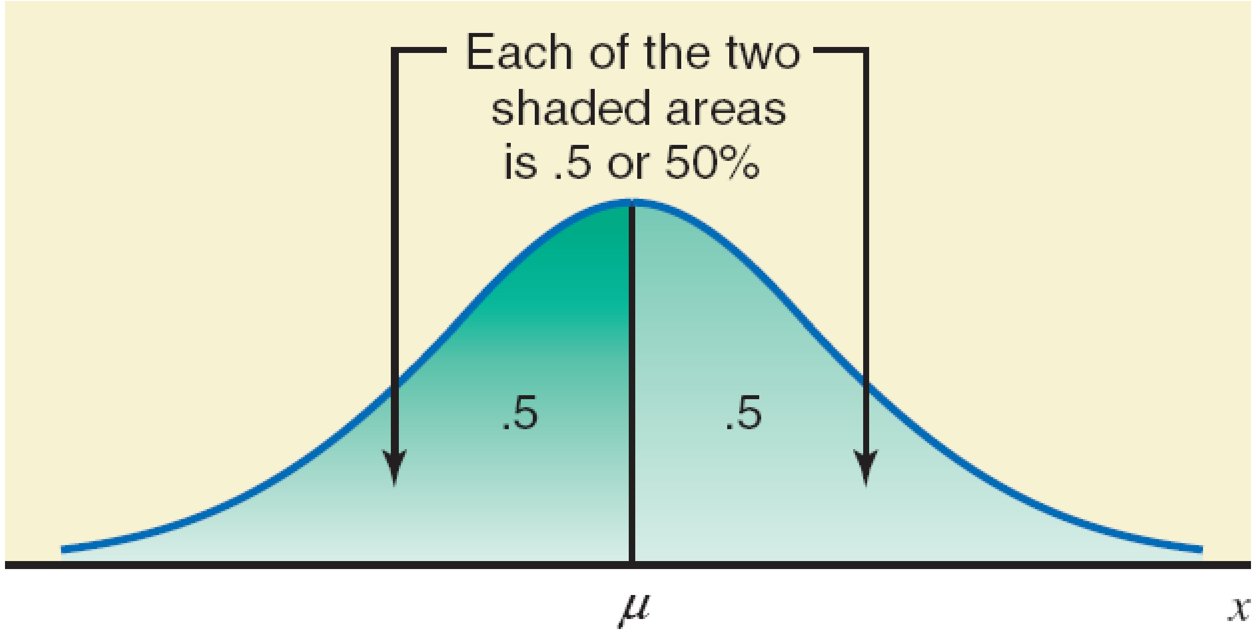
- Bell-shaped
- Total area under the curve =1



Normal distribution

$$N(\mu, \sigma^2)$$

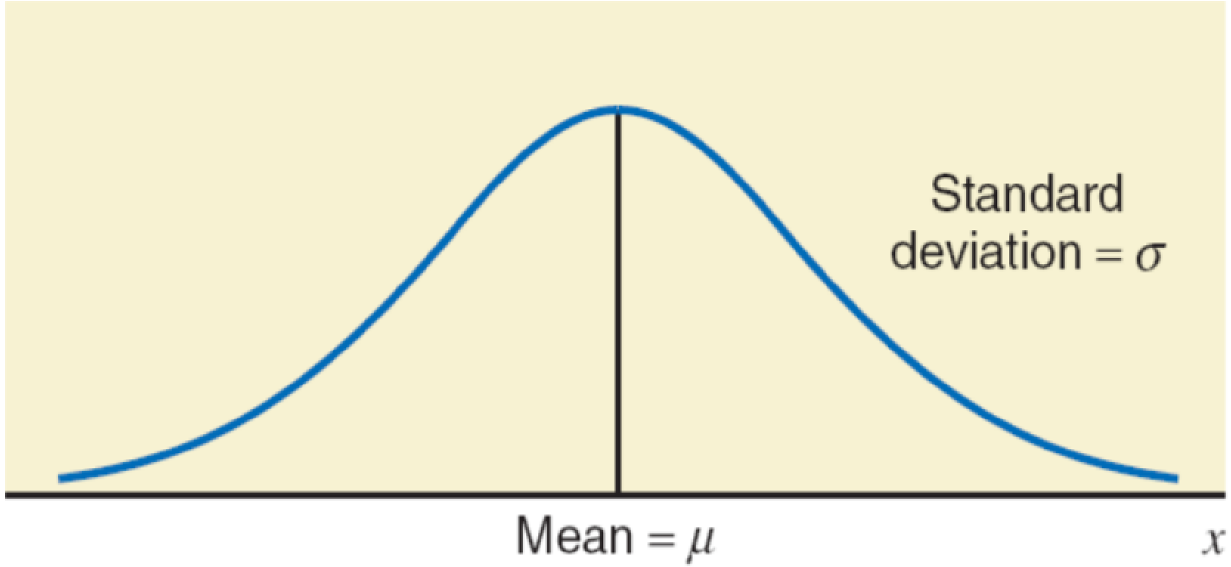
- Bell-shaped
- Total area under the curve =1
- Symmetric about the Mean



Normal distribution

$$N(\mu, \sigma^2)$$

- Bell-shaped
- Total area under the curve =1
- Symmetric about the Mean
- Mode=Mean=Median
- Tails extend indefinitely

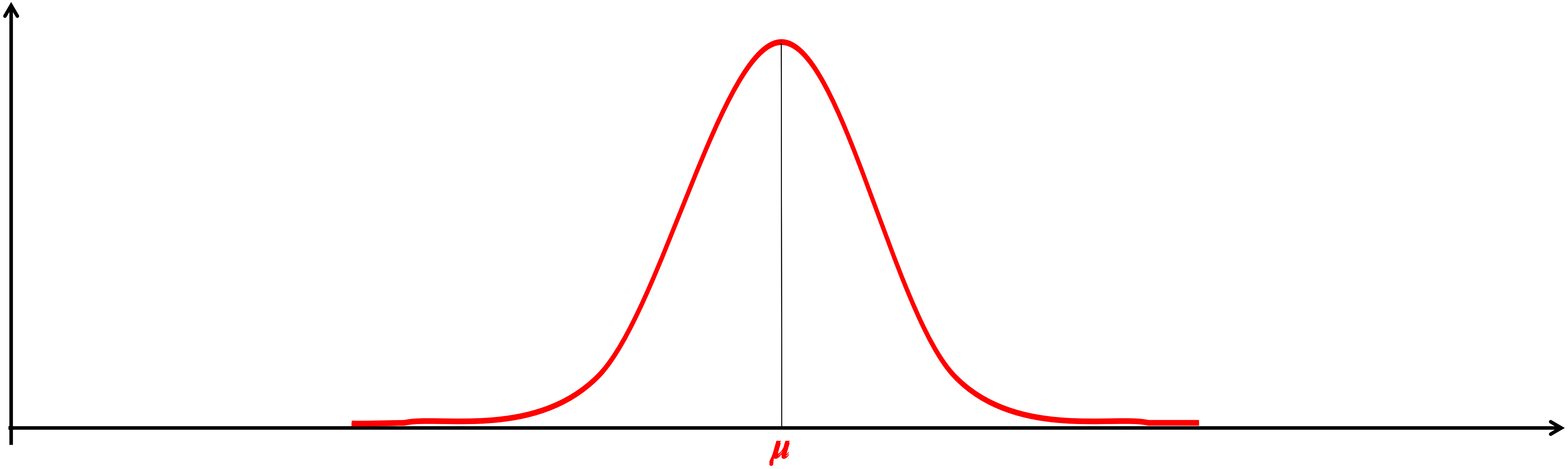


Normal distribution: the mean

$$x \sim N(\mu, \sigma^2)$$

2 parameters:

1. μ (mean)

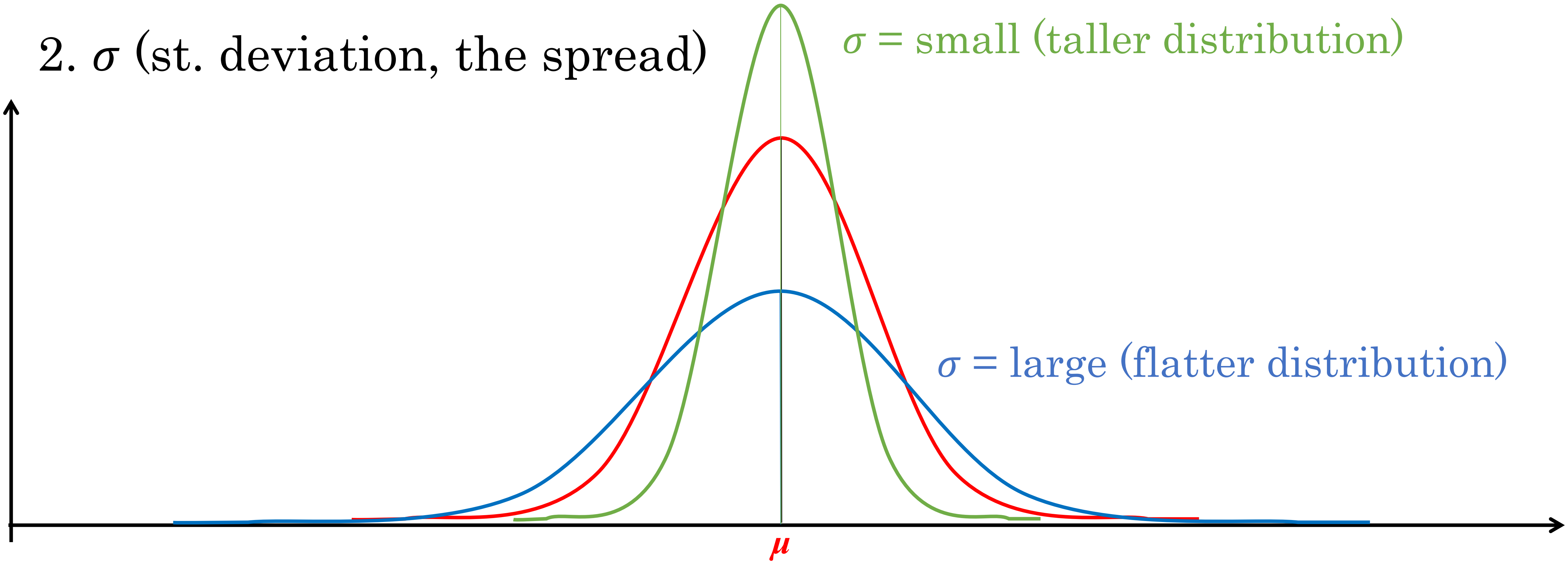


Normal distribution: the spread

$$x \sim N(\mu, \sigma^2)$$

2 parameters:

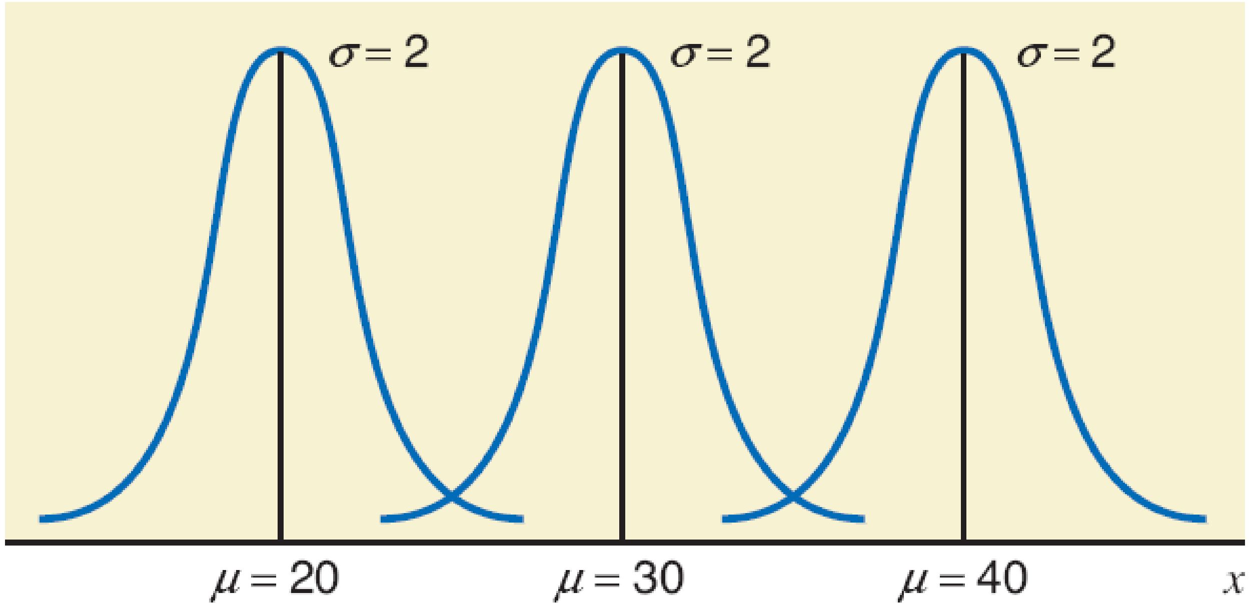
2. σ (st. deviation, the spread)

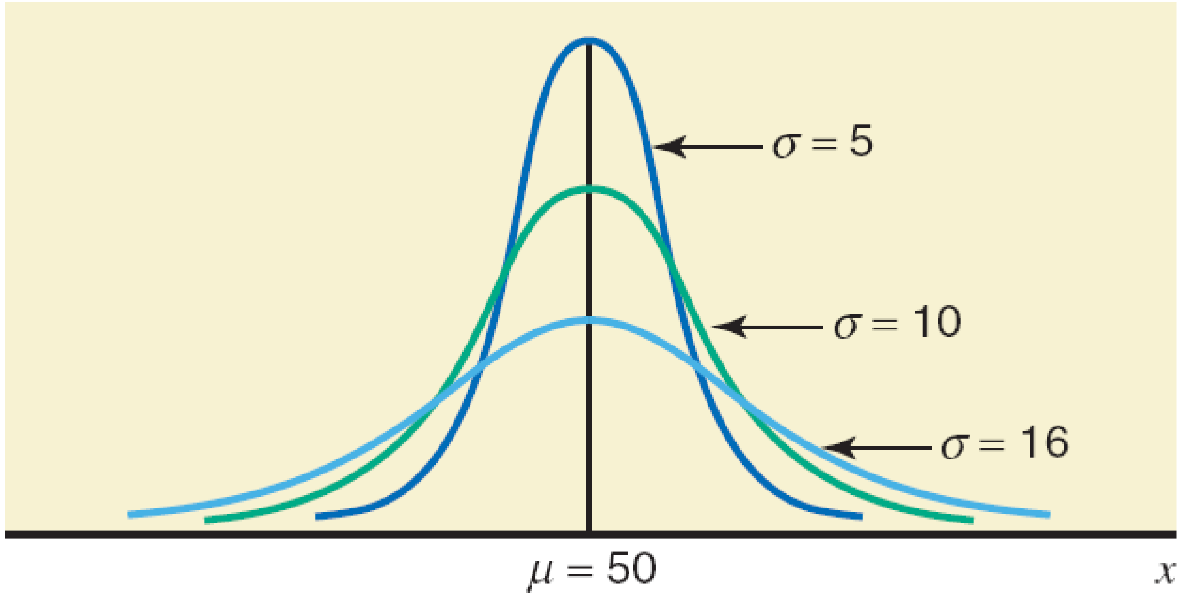


Normal distribution function: family

$x \sim N(\mu, \sigma^2)$ ➔ Density function: $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$

2 parameters: μ (mean, central value), and σ (std.dev., the spread)





Standard Normal distribution, z

The Standard Normal Distribution, *Z*, is a Normal Distribution with mean equal to 0 and standard deviation equal to 1,

$$Z \sim N(0,1)$$

ANY Normal distribution $X \sim N(\mu, \sigma^2)$ can be reconverted into a Standard Normal $Z \sim N(0,1)$, by **standardization**

$$Z = \frac{X - \mu}{\sigma}$$

Applications – 1

According to the Physician Compensation Report, American Internal medicine physicians earned an average of \$196,000 in 2014. Suppose that these earnings are normally distributed with $\sigma = \$20{,}000$, find the probability that a randomly selected American internal medicine physician earned in 2014 between \$169,400 and \$206,800.

Applications – 1

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$$P(x_1 < X < x_2) = P(z_1 = \frac{x_1 - \mu}{\sigma} < Z < z_2 = \frac{x_2 - \mu}{\sigma})$$

$$P(169{,}400 < X < 206{,}800) = P\left(\frac{169{,}400 - 196{,}000}{20{,}000} < Z < \frac{206{,}800 - 196{,}000}{20{,}000}\right) =$$

$$= P(-1.33 < Z < 0.54) = P(Z < 0.54) - P(Z < -1.33) =$$

$$= P(Z < 0.54) - (1 - P(Z < 1.33)) =$$

For negative z we find the probability with $1 - P(Z < |z|)$

Applications – 1

According to the Physician Compensation Report, American Internal medicine physicians earned an average of $196,000 in 2014. Suppose that these earnings are normally distributed with $\sigma = \$20{,}000$, find the probability that a randomly selected American internal medicine physician earned in 2014 between $169,400 and $206,800.

$$P(Z < 0.54) - (1 - P(Z < 1.33) =$$

$$= 0.7054 - (1 - 0.9082) = 0.6136$$

For negative z we find the probability with $\mathbf{1 - P(Z < |z|)}$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

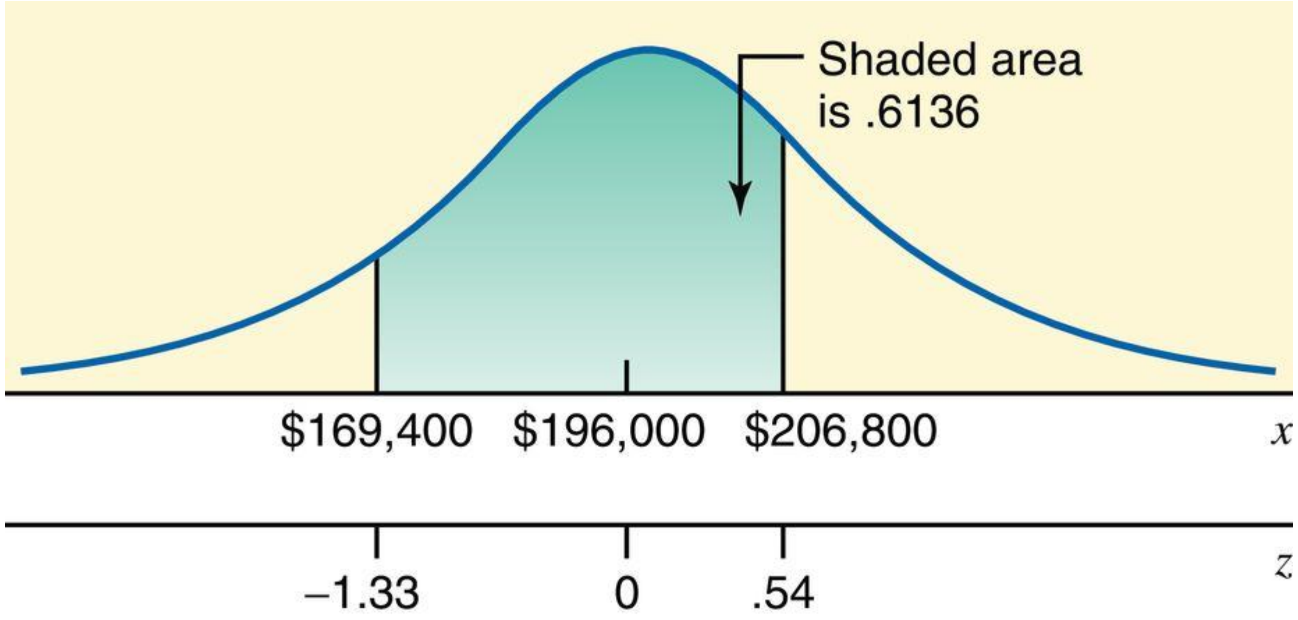
Applications – 1

According to the Physician Compensation Report, American Internal medicine physicians earned an average of \$196,000 in 2014. Suppose that these earnings are normally distributed with $\sigma = \$20{,}000$, find the probability that a randomly selected American internal medicine physician earned in 2014 between \$169,400 and \$206,800.

$$P(169{,}400 < X < 206{,}800) =$$

$$P(-1.33 < Z < 0.54) =$$

$$= 0.6136 = 61.36\%$$



Applications – 2

The assembly time for a toy follows a normal distribution with a mean of 55 minutes and a standard deviation of 4 minutes. The company closes at 5 p.m. every day. If one worker starts to assemble a toy at 4 p.m., what is the probability that he/she will finish this job before the company closes for the day?

Applications – 2

The assembly time for a toy follows a normal distribution with a mean of 55 minutes and a standard deviation of 4 minutes. The company closes at 5 p.m. every day. If one worker starts to assemble a toy at 4 p.m., what is the probability that he/she will finish this job before the company closes for the day?

$$P(X < x) = P(X < 60) = P(Z < z) =$$

$$= P\left(Z < \frac{x-\mu}{\sigma}\right) = P\left(Z < \frac{60-55}{4}\right) =$$

$$= P(Z < 1.25) =$$

$$= 0.8944 = 89.44\%$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

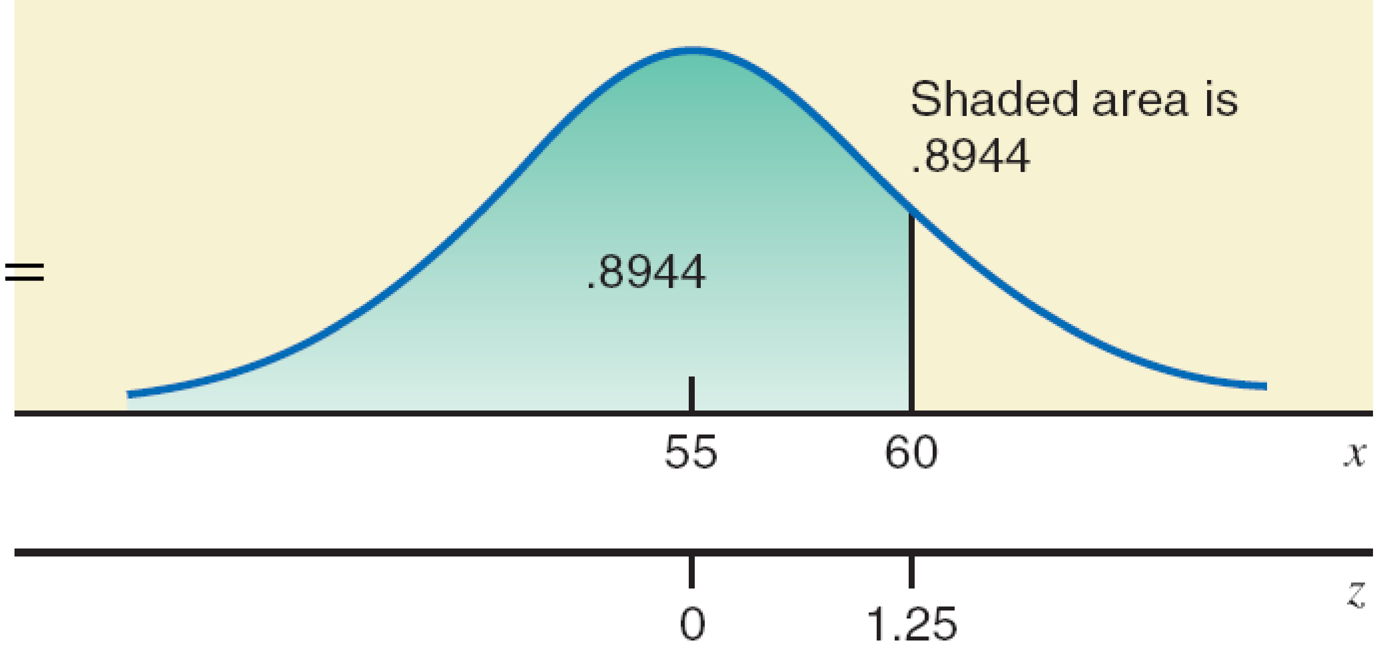
Applications – 2

The assembly time for a toy follows a normal distribution with a mean of 55 minutes and a standard deviation of 4 minutes. The company closes at 5 p.m. every day. If one worker starts to assemble a toy at 4 p.m., what is the probability that he/she will finish this job before the company closes for the day?

$$P(X < 60) = P(Z < z) =$$

$$= P\left(Z < \frac{x-\mu}{\sigma}\right) = P\left(Z < \frac{60-55}{4}\right) =$$

$$= P(Z < 1.25) = 0.8944 = 89.44\%$$



Applications – 3

It has been observed that the net amount of soda in such a can has a normal distribution with a mean of 12 ounces and a standard deviation of .015 ounce. What percentage of the Orange Cola cans contain 12.02 to 12.07 ounces of soda?

Applications – 3

It has been observed that the net amount of soda in such a can has a normal distribution with a mean of 12 ounces and a standard deviation of .015 ounce. What percentage of the Orange Cola cans contain 12.02 to 12.07 ounces of soda?

$$\mu = 12, \qquad \sigma = 0.015, \qquad x_1 = 12.02, \qquad x_2 = 12.07$$

$$P(x_1 < X < x_2) = P(z_1 = \frac{x_1 - \mu}{\sigma} < Z < z_2 = \frac{x_2 - \mu}{\sigma})$$

$$P(12.02 < X < 12.07) = P\left(\frac{12.02 - 12}{0.015} < Z < \frac{12.07 - 12}{0.015}\right) =$$

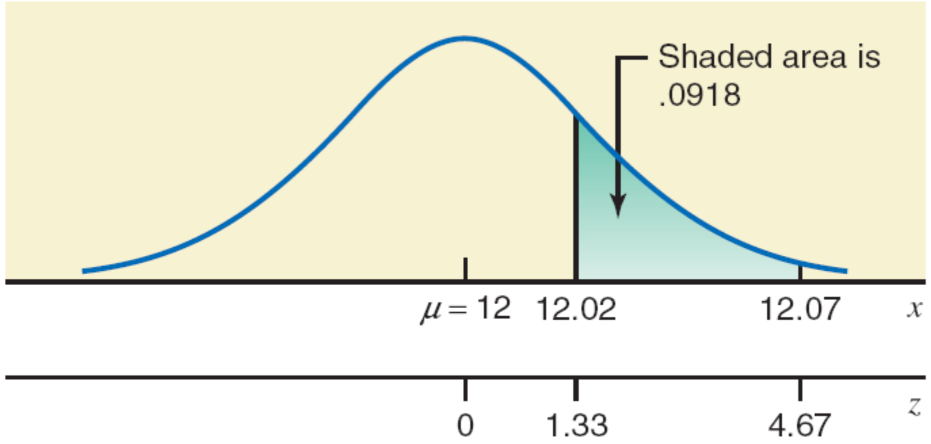
$$= P(1.33 < Z < 4.67) = P(Z < 4.67) - P(Z < 1.33)$$

Applications – 3

It has been observed that the net amount of soda in such a can has a normal distribution with a mean of 12 ounces and a standard deviation of .015 ounce. What percentage of the Orange Cola cans contain 12.02 to 12.07 ounces of soda?

$$P(Z < 4.67) - P(Z < 1.33) =$$

$$\cong 1 - 0.9082 = 0.0918 = 9.18\%$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
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0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
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1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

For value of z>3.6 we have $P(Z < z) \cong 1$

Applications – 4

Suppose the life span of a calculator has a normal distribution with a mean of 54 months and a standard deviation of 8 months. The company guarantees the calculator for 36 months: in case of malfunctioning the purchase will be replaced by a new one. About what percentage of calculators made by this company are expected to be replaced?

Applications – 4

Suppose the life span of a calculator has a normal distribution with a mean of 54 months and a standard deviation of 8 months. The company guarantees the calculator for 36 months: in case of malfunctioning the purchase will be replaced by a new one. About what percentage of calculators made by this company are expected to be replaced?

$$P(X < x) = P(X < 36) = P(Z < z) =$$

$$= P\left(Z < \frac{x-\mu}{\sigma}\right) = P\left(Z < \frac{36-54}{8}\right) =$$

$$= P(Z < -2.25) =$$

$$= 1 - P(Z < 2.25) = 1 - 0.9878 = 0.0122 = 1.22\%$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
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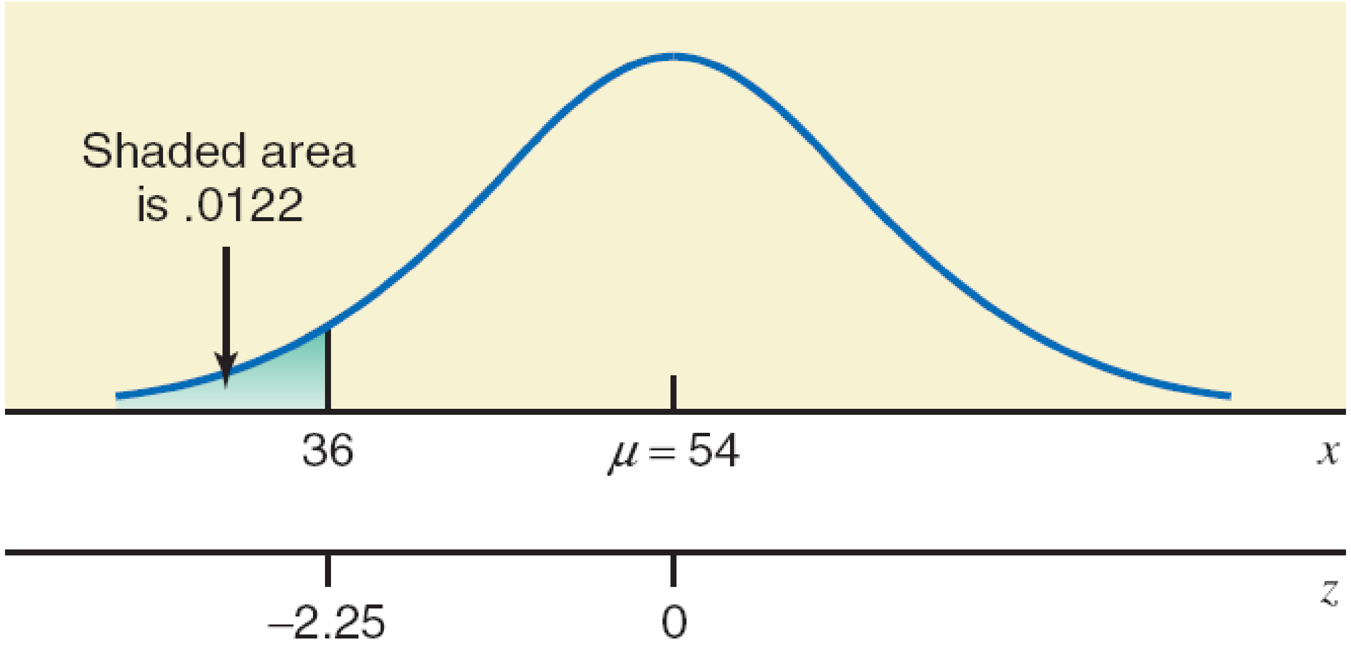
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890

Applications – 4

Suppose the life span of a calculator has a normal distribution with a mean of 54 months and a standard deviation of 8 months. The company guarantees the calculator for 36 months: in case of malfunctioning the purchase will be replaced by a new one. About what percentage of calculators made by this company are expected to be replaced?

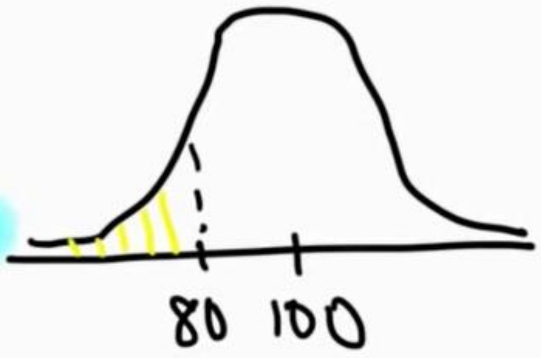


Applications – 5

3. Normally distributed IQ scores have a mean of 100 and a standard deviation of 15. Use the standard z-table to answer the following questions: What is the probability of randomly selecting someone with an IQ score that is (a) less than 80? (b) greater than 136? (c) between 95 and 110?

$\mu = 100 \quad \sigma = 15$

$$z = \frac{x - \mu}{\sigma} = \frac{80 - 100}{15} = -1.33$$



$P(x < 80)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	$\boxed{0.0918}$	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823

$$P(X < 2.12) = P(Z < -1.33) = 0.0918 = 9.18\%$$

Applications – 5

3. Normally distributed IQ scores have a mean of 100 and a standard deviation of 15. Use the standard z-table to answer the following questions: What is the probability of randomly selecting someone with an IQ score that is (a) less than 80? (b) greater than 136? (c) between 95 and 110?

$\mu = 100 \quad \sigma = 15$

$$z = \frac{x-\mu}{\sigma} = \frac{136-100}{15} = 2.4$$

$z = 2.4 \rightarrow A_L = \boxed{0.9918}$

$$P(x > 136) = 1 - P(x \le 136)$$

$$= 1 - 0.9918 = 0.0082$$

$$\simeq \boxed{.82\%}$$

Applications – 5

3. Normally distributed IQ scores have a mean of 100 and a standard deviation of 15. Use the standard z-table to answer the following questions: What is the probability of randomly selecting someone with an IQ score that is (a) less than 80? (b) greater than 136? (c) between 95 and 110?

$\mu = 100 \quad \sigma = 15$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{110 - 100}{15} = 0.67 \rightarrow A_L = \boxed{0.74857}$$

$$z = \frac{95 - 100}{15} = -0.33 \rightarrow A_L = \boxed{0.37070}$$

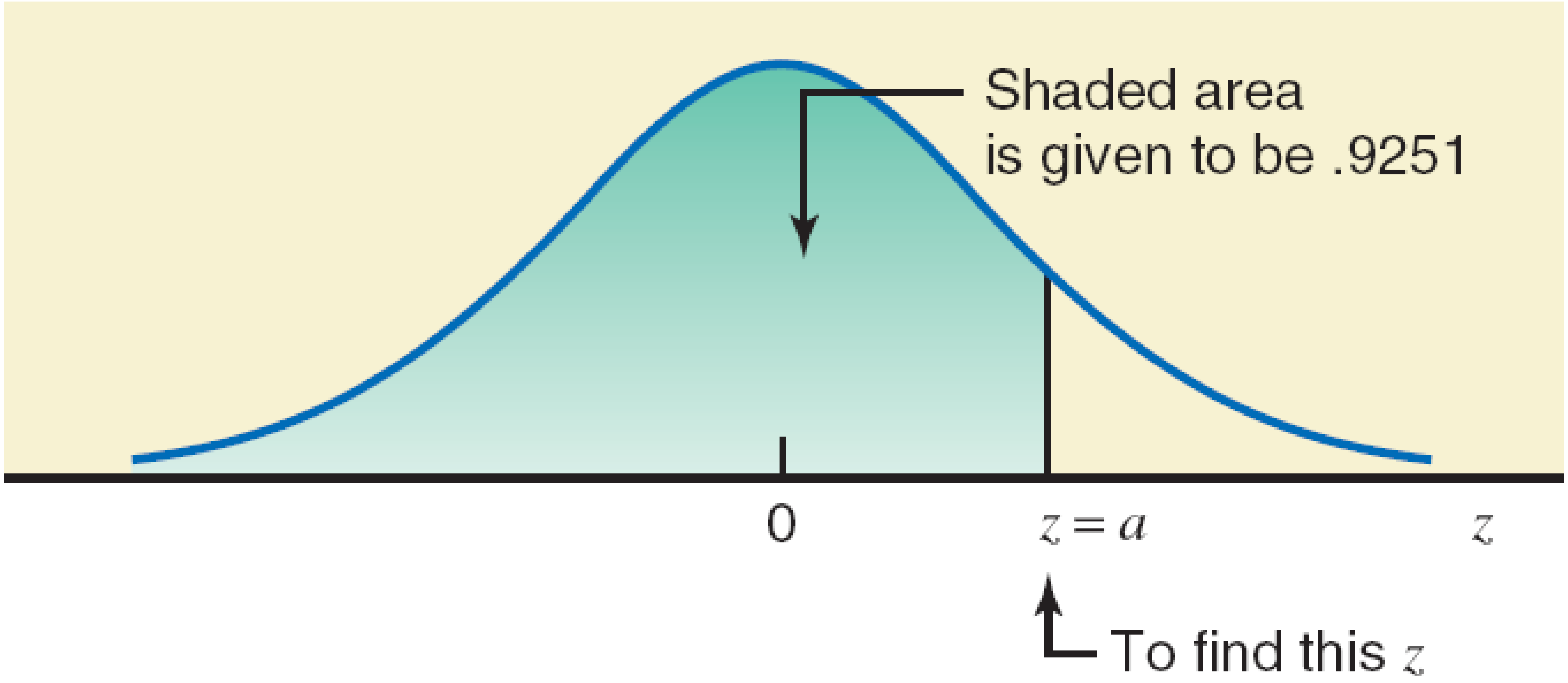
$$P(95 < x < 110) = P(x < 110) - P(x < 95)$$

$$= 0.74857 - 0.37070$$

$$= 0.37787$$

Using the Standard Normal Table…again

Find z s.t. the area under the standard normal to its left is 0.9251.



Using the Standard Normal Table…again

Find z s.t. the area under the standard normal to its left is 0.9251.

z	.00	.01	…	.04	…	.09
·	·	·	…		…	·
·	·	·	…		…	·
·	·	·	…		…	·
1.4				.9251	…	…
·	·	·	…	·	…	·
·	·	·	…	·	…	·
·	·	·	…	·	…	·
3.4	.9997	.9997	…	.9997	…	.9998

1. Find this value (or closest to) ***inside*** the table
2. z given by its “coordinates”

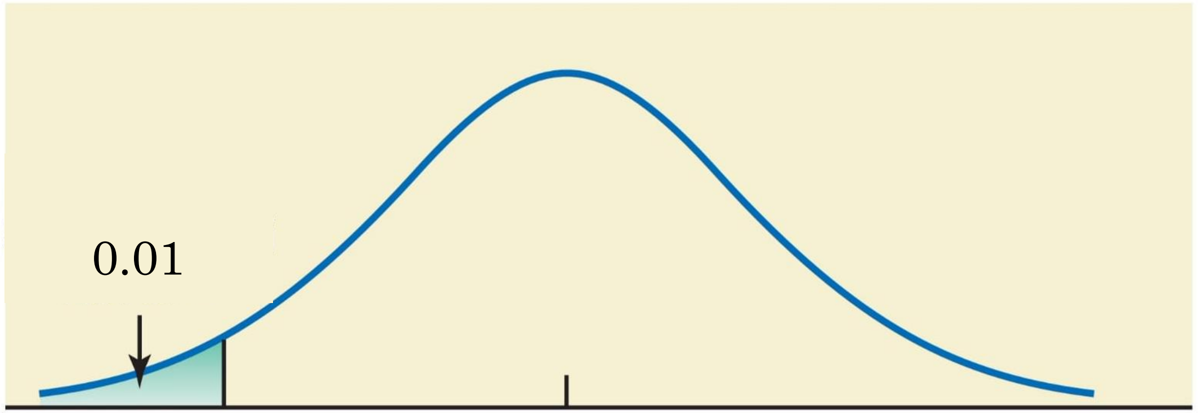
Applications – 4 Revisited

The life span of a calculator, X, has a normal distribution with a mean of 54 months and a standard deviation of 8 months. What should the warranty period if the company does not want to replace more than 1% of the calculators sold?

Applications – 4 Revisited

The life span of a calculator, X, has a normal distribution with a mean of 54 months and a standard deviation of 8 months. What should the warranty period if the company does not want to replace more than 1% of the calculators sold?

We have to find the value that leaves to its left 0.01 (negative z)

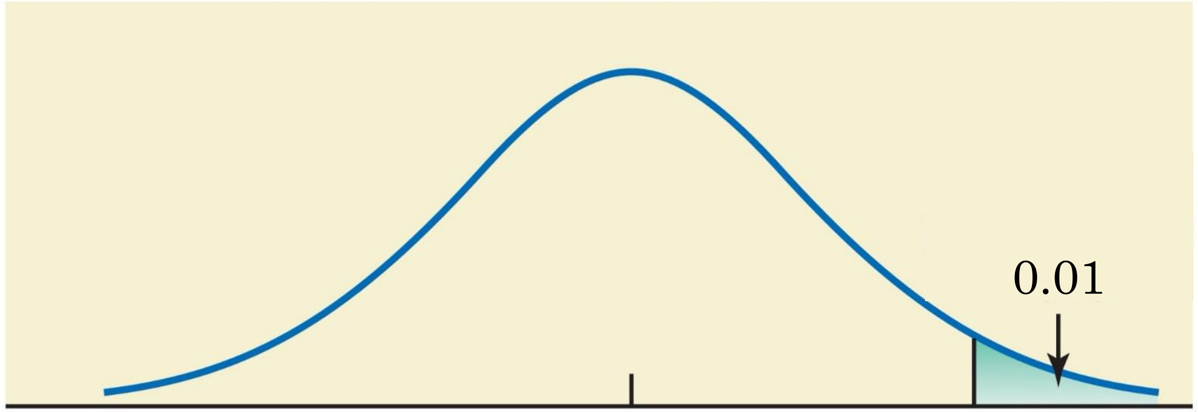


Z is a symmetric distribution

$P(Z < -z) = P(Z > z) = 0.01$

Negative z

Positive z



We have to find this |z| value

Applications – 4 Revisited

The life span of a calculator, X, has a normal distribution with a mean of 54 months and a standard deviation of 8 months. What should the warranty period if the company does not want to replace more than 1% of the calculators sold?

The z-value that leaves to its right 0.01:

$\mathrm{P}(Z > z) = 0.01 \quad \rightarrow \quad 1 - \mathrm{P}(Z < z) = 0.01$

$1 - \mathrm{P}(Z < z) = 0.01$

$\rightarrow P(Z < z) = 1 - 0.01 = 0.99$

The nearest value to $\mathrm{P}(Z < z) = 0.99$ is for

$z = 2.33$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936

$$\mathrm{P}(Z > 2.33) = 0.01 \; and \; \mathrm{P}(Z < -2.33) = 0.01$$

Applications – 4 Revisited

The life span of a calculator, X, has a normal distribution with a mean of 54 months and a standard deviation of 8 months. What should the warranty period if the company does not want to replace more than 1% of the calculators sold?

$$z = -2.33$$

We have to derive x from z

$$z = \frac{x-\mu}{\sigma} \rightarrow x = \mu + z\,\sigma$$

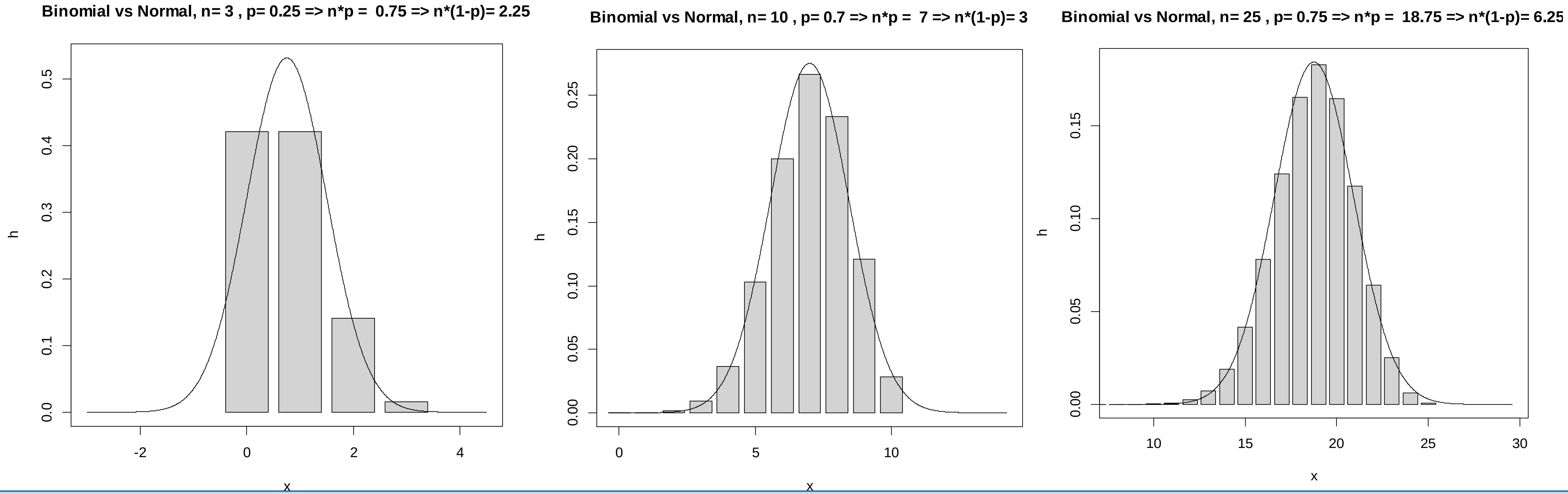
$$x = \mu + z\,\sigma = 54 - 2.33 \cdot 8 = 35.36$$

The warranty period should be of approximately 36 months (i.e. 3 years)

Normal approximating Binomial

The normal distribution is a good approximation of the binomial distribution when BOTH:

$$np > 5 \quad \text{AND} \quad n(1-p) > 5$$



Normal approximating Binomial: how

1. Check if both $np > 5$ AND $n(1-p) > 5$
2. Compute E(X) and V(X) of binomial distribution.
3. Convert the discrete random variable into a continuous one, using the correction for continuity➔ add and subtract 0.5 from the value of interest
4. Compute the required probability using the normal distribution.

Normal approximating Binomial: example

It is known that 50% of people in the United States have at least one credit card. If a random sample of 30 people is selected, what is the probability that 19 of them will have at least one credit card?

Exact Solution: $P(X = 19) = \binom{30}{19} 0.5^{19}(1 - 0.5)^{30-19} = 0.0509 \cong 5\%$

Normal approximating Binomial: example

It is known that 50% of people in the United States have at least one credit card. If a random sample of 30 people is selected, what is the probability that 19 of them will have at least one credit card?

1. $np = 15$ AND $n(1-p) = 15$ ➔ approximation

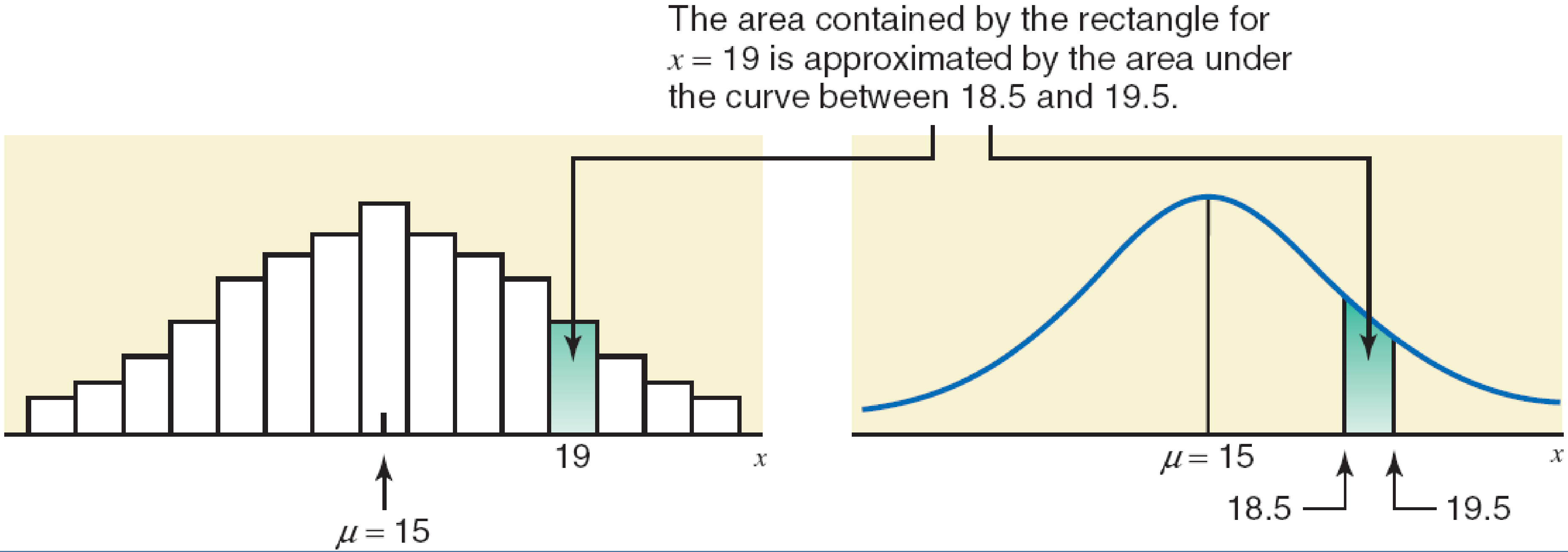
2. Compute E(X) and V(X) of binomial distribution:

$$E(X) = 30 * 0.5 = 15$$
$$\mathrm{V}(X) = 30 * 0.5 * (1 - 0.5) = 7.5$$

Normal approximating Binomial: example

It is known that 50% of people in the United States have at least one credit card. If a random sample of 30 people is selected, what is the probability that 19 of them will have at least one credit card?

3. Correction for continuity: $P(X = 19) \cong P(18.5 \le X \le 19.5)$



Normal approximating Binomial: example

It is known that 50% of people in the United States have at least one credit card. If a random sample of 30 people is selected, what is the probability that 19 of them will have at least one credit card?

4. Compute $P(X = 19) \cong P(18.5 \leq X \leq 19.5)$

$$= P\left(\frac{18.5 - 15}{\sqrt{7.5}} \leq X \leq \frac{19.5 - 15}{\sqrt{7.5}}\right)$$

$$= P(1.28 \leq Z \leq 1.64)$$

$$= 0.9495 - 0.8997 = 0.0498 \cong 5\%$$

Normal approximating Binomial: example II

According to a survey, 32% of people working would prefer to work from home since there is no commute. Suppose that this result is true for the whole population. What is the probability that in a random sample of 400 workers 108 to 122 will prefer working from home?

Normal approximating Binomial: example II

According to a survey, 32% of people working would prefer to work from home since there is no commute. Suppose that this result is true for the whole population. What is the probability that in a random sample of 400 workers 108 to 122 will prefer working from home?

1. $np = 128$ AND $n(1-p) = 272$ ➔ approximation

2. $E(X) = 400 * 0.32 = 128$ and $\mathrm{V}(X) = 400 * 0.32 * (1 - 0.68) = 87.04$

3. Correction for continuity and standardization:

$$= P\left(\frac{107.5 - 128}{\sqrt{87.04}} \le X \le \frac{122.5 - 128}{\sqrt{87.04}}\right) = P(-2.20 \le Z \le -0.59)$$

4. Compute the probability ➔ $0.2776 - 0.0139 = 0.2637$

t-Student distribution, $X \sim t_n$

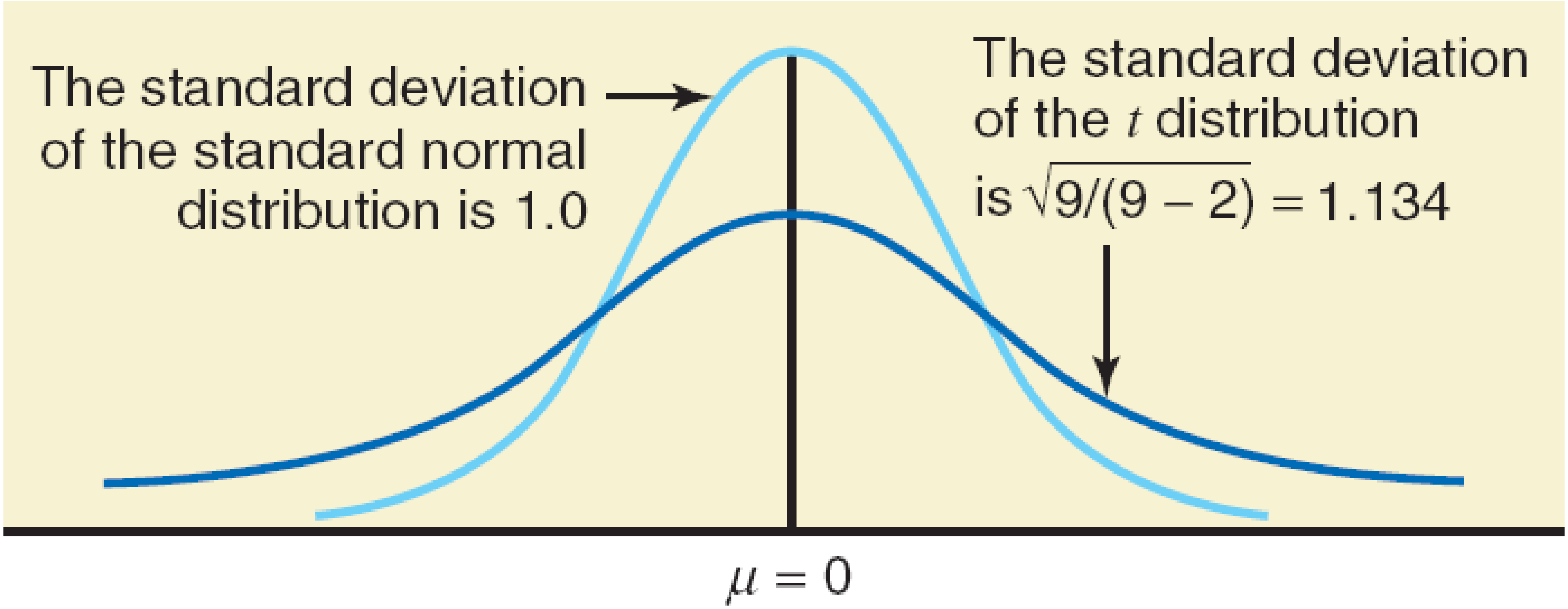
Let $Z \sim N(0,1)$ and $Y \sim \chi^2_n$,

$$X = \frac{Z}{\sqrt{Y/n}} \sim t_n$$

Where *n=degrees of freedom* (only parameter)

- The total area under a t-student distribution curve is 1
- Distribution is bell-shaped and symmetric around $E(X) = 0$
- $V(X) = \frac{n}{n-2}$, for $n > 2$
- Fatter tails w.r.t. Normal distribution
- For $n > 60$, X converges to $Z \sim N(0,1)$

t-Student distribution, $X \sim t_n$



t-Student distribution, $X \sim t_n$ – Example

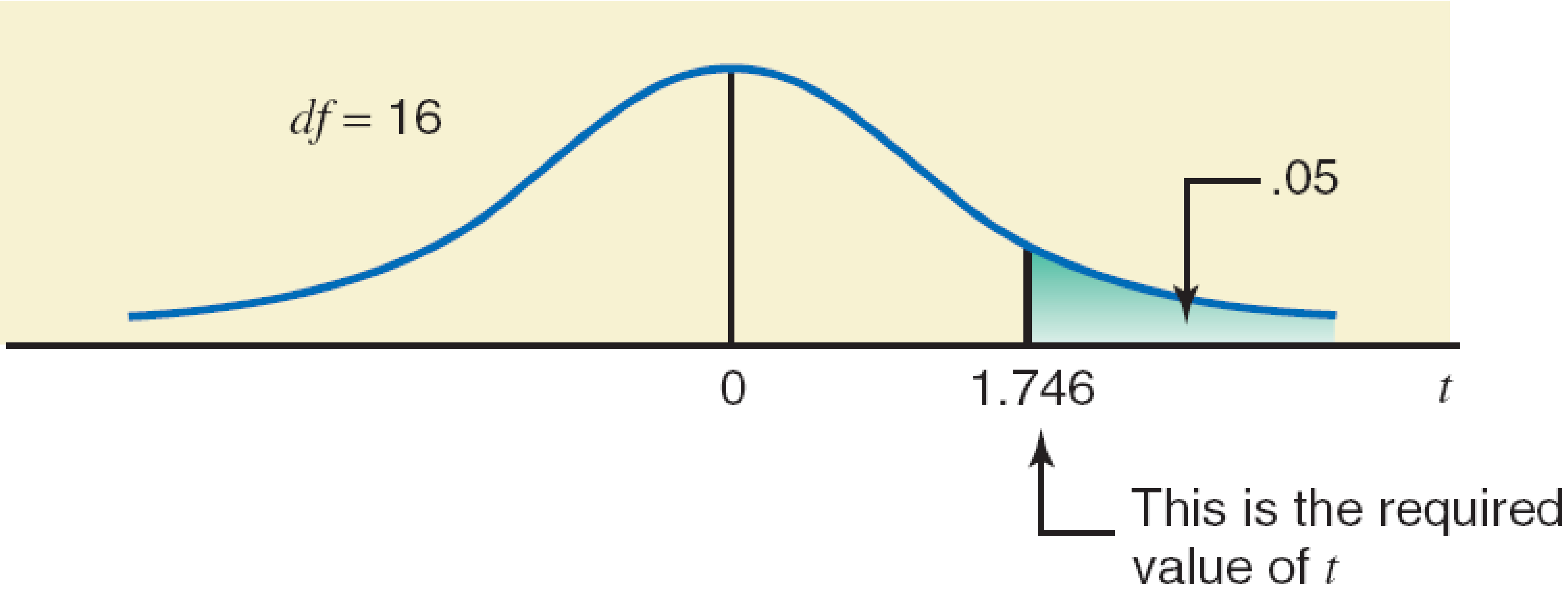
Find the value of t for 16 df and .05 area in the right tail

	Area in the Right Tail Under the t Distribution Curve				
df	.10	**.05**	.025	. . .	.001
1	3.078	6.314	12.706	. . .	318.309
2	1.886	2.920	4.303	. . .	22.327
3	1.638	2.353	3.182	. . .	10.215
.	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .
16	1.337	**1.746**	2.120	. . .	3.686
.	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .
75	1.293	1.665	1.992	. . .	3.202
∞	1.282	1.645	1.960	. . .	3.090

The required value of t for 16 *df* and .05 area in the right tail

t-Student distribution, $X \sim t_n$ – Example

Find the value of t for 16 df and .05 area in the right tail



Chi square distribution, $X \sim \chi^2$

Let $Z_1, Z_2, Z_3, \dots, Z_n$ be n i.i.d. random variables $Z_i \sim N(0,1)$

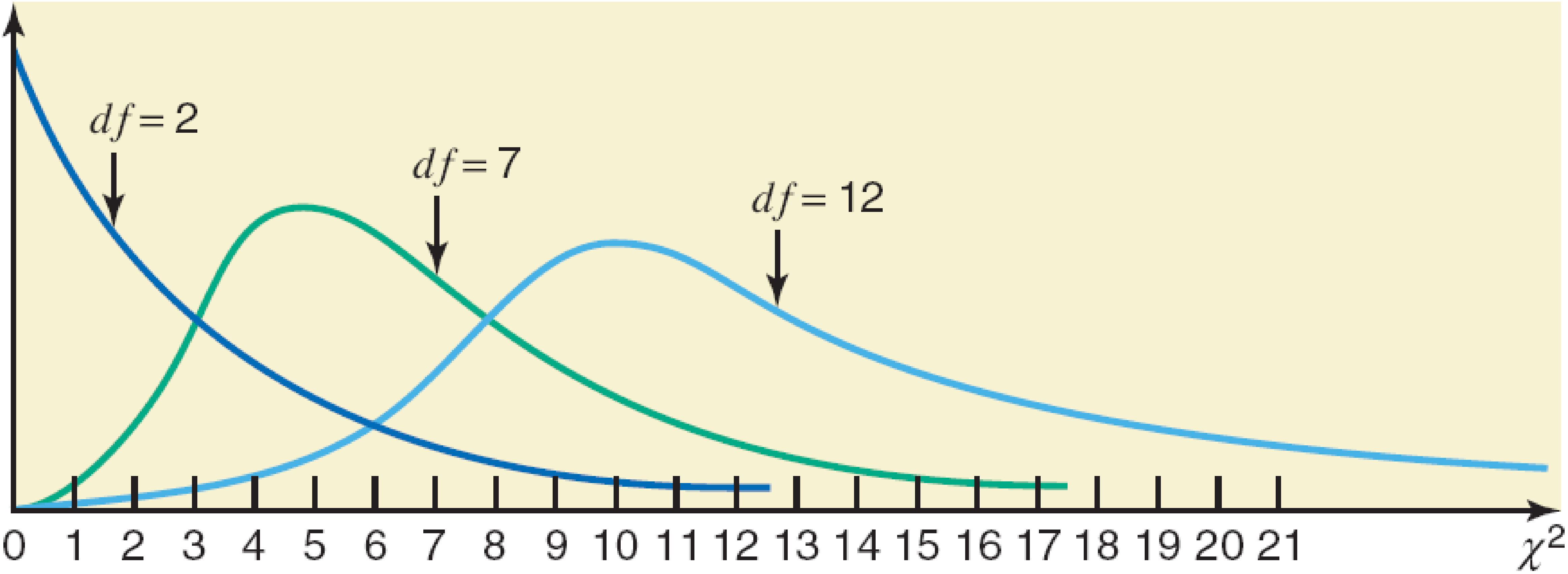
$$X = \sum_{i=1}^{n} Z_i^2 \sim \chi_n^2$$

Where *n=degrees of freedom* (only parameter)

- The chi-square distribution lies to the right of the vertical axis.
- The total area under a chi-square distribution curve is 1.
- $E(X) = n$
- $V(X) = 2n$

Chi square distribution, $X \sim \chi^2_n$

The shape of a chi-square distribution curve is skewed to the right for small *n* and becomes symmetric for large *n*.



Chi square distribution – Example

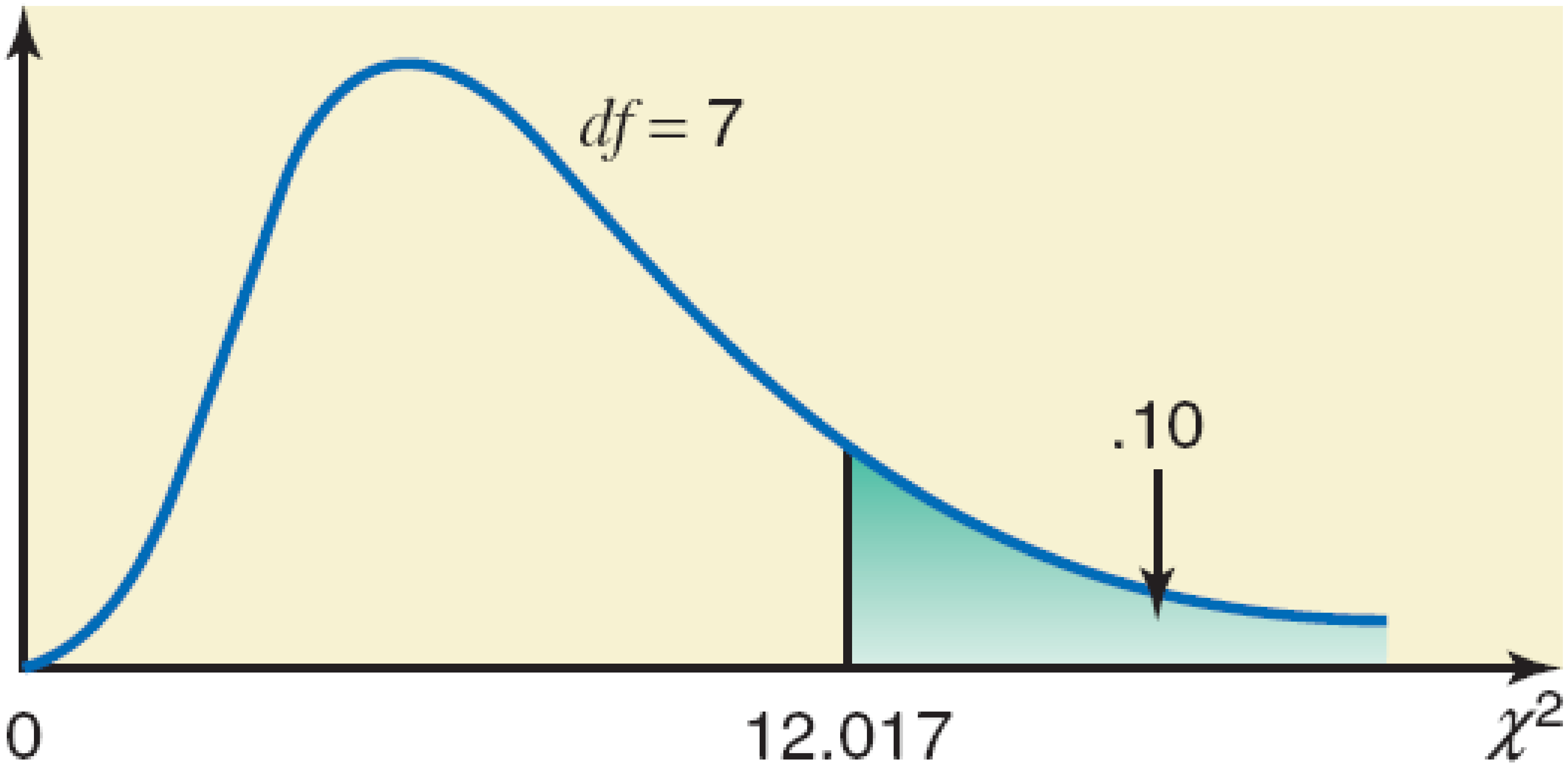
Find the value of χ^2 for 7 *df* that leaves an area of .10 in the right tail.

	Area in the Right Tail Under the Chi-Square Distribution Curve				
df	**.995**	**. . .**	**.100**	**. . .**	**.005**
1	0.000	. . .	2.706	. . .	7.879
2	0.010	. . .	4.605	. . .	10.597
.	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .
7	0.989	. . .	**12.017**	. . .	20.278
.	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .
100	67.328	. . .	118.498	. . .	140.169

Required value of χ^2

Chi square distribution – Example

Find the value of χ^2 for 7 *df* that leaves an area of .10 in the right tail.



F distribution, $F \sim F_{n,m}$

Let $X \sim \chi_m^2$ and $Y \sim \chi_n^2$,

$$F = \frac{X/m}{Y/n} \sim F_{m,n}$$

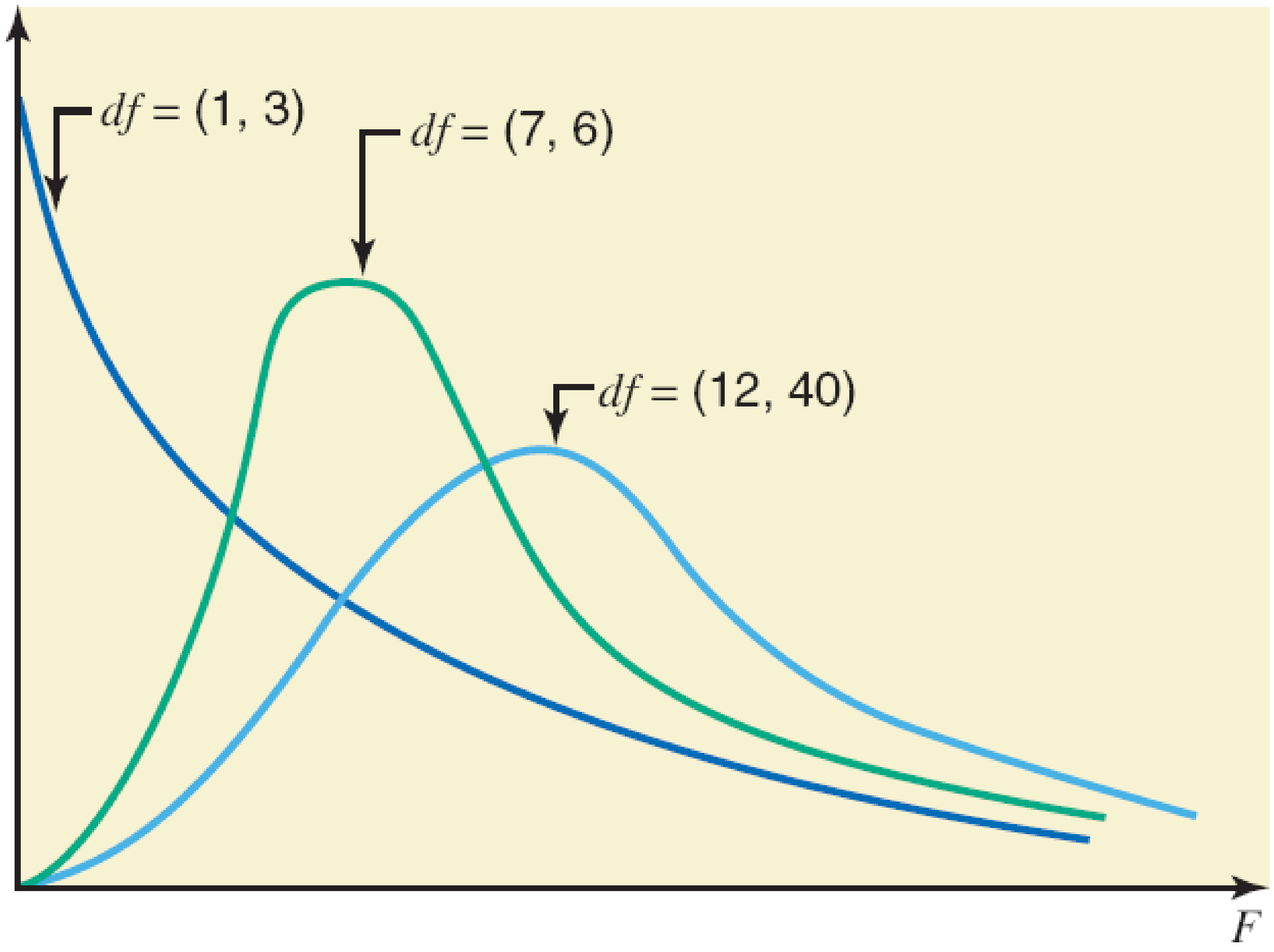
Where *m=numerator's df, n= denominator's df* (2 parameters)

- The curve lies to the right of the vertical axis and the total area under F distribution curve is 1

F distribution, F$\sim F_{n,m}$

Let X$\sim\chi_m^2$ and Y$\sim\chi_n^2$,

$$F = \frac{X/m}{Y/n} \sim F_{m,n}$$



F distribution, F~$F_{n,m}$ – Example

Find the value $F_{n=8,m=14}$ and .05 area in the right tail

Degrees of Freedom for the Denominator	Degrees of Freedom for the Numerator					
	1	2	. . .	8	. . .	100
1	161.5	199.5	. . .	238.9	. . .	253.0
2	18.51	19.00	. . .	19.37	. . .	19.49
.	. . .	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .	. . .
14	4.60	3.74	. . .	2.70	. . .	2.19
.	. . .	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .	. . .
100	3.94	3.09	. . .	2.03	. . .	1.39

F distribution, F$\sim F_{n,m}$ – Example

Find the value $F_{n=8,m=14}$ and .05 area in the right tail