TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

FACOLTA' DI ECONOMIA

Bachelor in in Business Administration and Economics

Quantitative Methods

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Mock Exam

1. A group of 10 employees of a company have the following annual incomes:

72577 €	80171 €	49942 €	84568 €	78920 €
65551 €	71895 €	69576 €	42090 €	64120 €
19795 €	92764 €	35049 €	64731 €	91004 €
70514 €	74065 €	17330 €	20293 €	75652 €

a. Find the mode.
b. Calculate the median.
c. Calculate the mean.
d. The company grants a 10% increase in annual incomes. Calculate the new mean.
e. Convert previous data into grouped frequency distribution using the following classes: 0 € ⊣ 30000 €, 30000 € ⊣ 50000 €, 50000 € ⊣ 80000 €, 80000 € ⊣ 100000 €. Calculate the mean and the standard deviation.
f. Draw the histogram.

2. Solve the following problems.

a. 55.6% of those born in 1930 lived over 80 years and 72.3% lived over 60 years. Determine the probability that by randomly choosing a person born in 1930 he has lived more than 80 years.

b. In a town there are 4 crossroads with traffic lights. Each traffic light opens or closes the traffic with the same probability of 0.5. Determine the probability of:

a) a car crossing the first crossroad without stopping

b) a car crossing first two crossroads without stopping

c) a car crossing all the crossroads (4) without stopping

c. Pedro observed what customers ordered at his ice cream shop and found that the 30% of customers selected a vanilla ice cream, the 20% selected a sundae and the 15% selected a vanilla ice cream and a sundae. What is the probability that a customer ordered vanilla ice cream given they ordered a sundae.

d. Researchers surveyed recent graduates of two different universities about their annual incomes. The following two-way table displays data for the 300 graduates who responded to the survey.

Annual income	University A	University B	TOTAL
Under $20,000	36	24	**60**
$20,000 to 39,999	109	56	**165**
$40,000 and over	35	40	**75**
TOTAL	**180**	**120**	**300**

Suppose we choose a random graduate from this data.

Are the events "income is $40,000 and over" and "attended University B" independent?

e. In a company with 360 employees, 288 of workers commute using the car, while the rest use the public transports. Those commuting by car arrive on time 70% of the times, while those commuting with public transports arrive on time 80% of the times. If an employee arrives late, what is the probability that he/she used the car?

3. Let X the random variable with the following probability distribution:

X	$P(X=x)$
0	0.15
1	0.20
2	0.18
3	0.30
4	0.10
?	0.07
Tot	**1.00**

a. If $E(X) = 2.35$, what is the missing value of the X.
b. Calculate the $V(X)$.
c. Y is a variable that has a "success" for $X = 3$, "failure" otherwise, what kind of random variable is Y?
d. In 5 different trials of the variable Y what is the probability to have less than 2 "success"?
e. Calculate the expected value and the variance from the previous point.

4.

a. Find the probabilities indicated.

1. $P(Z > 1.60)$
2. $P(Z > -1.02)$
3. $P(0.5 < Z < 1.57)$
4. $P(-2.55 < Z < 0.09)$

b. The lifetimes of the tread of a certain automobile tire are normally distributed with mean 37,500 miles and standard deviation 4,500 miles. Find the probability that the tread life of a randomly selected tire will be between 30,000 and 40,000 miles.

c. Scores on a standardized college entrance examination (CEE) are normally distributed with mean 510 and standard deviation 60. A selective university considers for admission only applicants with CEE scores over 650. Find percentage of all individuals who took the CEE who meet the university's CEE requirement for consideration for admission.

Solutions:

1.

a. No mode

b. Median=70045 €

c. μ=62030.35 €

d. From the **Linearity** of the mean if Y=1.1 · X → μ_Y=1.1 · μ_X = 68233.39 €

e. μ=61250

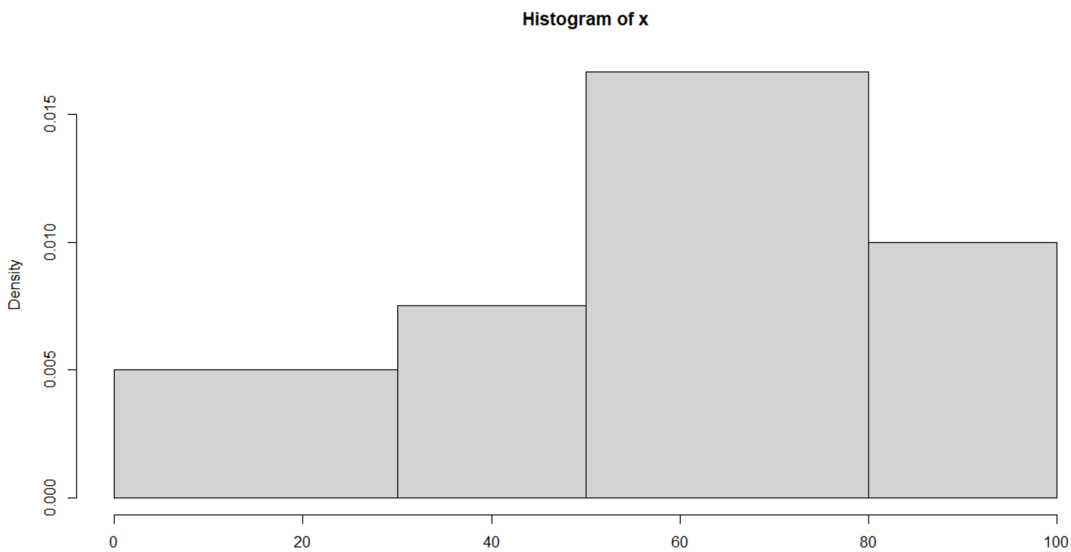
$$\sigma^2 = \sum \frac{x^2}{n} - \mu^2 = \frac{80125}{20} - 61.25^2 = 4006{,}25 - 3751{,}5625 = 254.6875\,,$$

$$\sigma = 15.959$$

X (in thousands)	f_i	m_i	$m_i \cdot f_i$	$m_i^2 \cdot f_i$
0 ⊣ 30	3	15	45	675
30 ⊣ 50	3	40	120	4800
50 ⊣ 80	10	65	650	42250
80 ⊣ 100	4	90	360	32400
Tot	**20**		1225	80125

f.

X (in thousands)	f_i	fr_i	Width	h_i
0 ⊣ 30	3	0,15	30	0,0050
30 ⊣ 50	3	0,15	20	0,0075
50 ⊣ 80	10	0,50	30	0,0167
80 ⊣ 100	4	0,20	20	0,0100
Tot	**20**	1,00		



2.

a. A={ more than 80 years} and B={more than 60 years}

P(A)=0.556 and P(B)=0.723

$A \subset B$ so $P(A \cap B)=P(A)=0.556$

b. a) P(A)=0.5=50%

b) $P(A \cap B)=P(A)P(B)=0.5 \cdot 0.5=0.25=25\%$

c) $P(A \cap B \cap C \cap D)=P(A)P(B)P(C)P(D)=0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5=0.0625=6.25\%$

c. P(vanilla | sundae)=0.75=75%

d. No, $P(\geq \$40{,}000)=\frac{75}{300}=0.25 \neq P(\geq \$40{,}000 \mid \text{Uni. B})=\frac{40}{120}=1/3 \approx 0.333$

e. $P(C) = 0.8\,,\ P(\bar{C}) = 1 - 0.8 = 0.2$

$P(L \mid C) = (1 - 0.7) = 0.3, \qquad P(\bar{L} \mid \bar{C}) = 0.8 \rightarrow P(L \mid \bar{C}) = 0.2$

$$P(C \mid L) = \frac{P(L|C) \cdot P(C)}{P(L)} = \frac{P(L|C) \cdot P(C)}{P(L|C)P(C) + P(L|\bar{C})P(\bar{C})} =$$

$$= \frac{0.3 \cdot 0.8}{0.3 \cdot 0.8 + 0.2 \cdot 0.2} = \frac{0.24}{0.24 + 0.04} = 0.8571 = 85.71\%$$

3.

a. The missing value of the X is 7

b. V(X)=3.1275

c. Bernoullian

d. P(X<2)=P(X=0)+P(X=1)=0.16807+0.36015=0.52822

e. $E(X)=np=5 \cdot 0.3=0.15$, $V(X)=np(1-p)=5 \cdot 0.3 \cdot 0.7=1.05$

4.

a. 1) 0.0548

2) 0.8461

3) 0.2503

4) 0.5305

b. 0.6648

c. 0.0099