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Quantitative Methods III - Time Series: Complete practice

Exercise All of the following questions refer to the models (whose R output is reported below) and the time series of the Italian inflation rate, denoted by y_t, and the Italian unemployment rate, denoted by x_t. The models were estimated using data from 1970 to 2019.

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.17784    0.54516   2.161   0.0359 *
y_lag1       0.87943    0.05692  15.450   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.7547 on 48 degrees of freedom
Multiple R-squared:  0.8355, Adjusted R-squared:  0.832
F-statistic: 238.7 on 1 and 48 DF,  p-value: < 2.2e-16
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.61398    0.50728   3.182  0.00265 **
y_lag1       1.09880    0.13543   8.114 2.36e-10 ***
y_lag2       0.04824    0.21103   0.229  0.82022
y_lag3      -0.31720    0.13733  -2.310  0.02554 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.6803 on 46 degrees of freedom
Multiple R-squared:  0.872, Adjusted R-squared:  0.8635
F-statistic: 102.2 on 3 and 46 DF,  p-value: < 2.2e-16
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.19219    0.72356   1.648   0.1065
y_lag1       1.13685    0.14503   7.838 6.89e-10 ***
y_lag2      -0.26843    0.14565  -1.843   0.0721 .
x_lag1      -0.04550    0.04931  -0.923   0.3612
x_lag2       0.05824    0.04809   1.211   0.2324
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.7146 on 45 degrees of freedom
Multiple R-squared:  0.8619, Adjusted R-squared:  0.8494
F-statistic: 68.67 on 4 and 45 DF,  p-value: < 2.2e-16
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.3756     0.5196   2.648   0.0111 *
y_lag1        1.2009     0.1339   8.967 1.16e-11 ***
y_lag2       -0.3438     0.1312  -2.620   0.0119 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.7116 on 47 degrees of freedom
Multiple R-squared:  0.8569, Adjusted R-squared:  0.8506
F-statistic: 137.7 on 2 and 47 DF,  p-value: < 2.2e-16
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.180648   0.727233   1.623   0.1115
y_lag1       1.202443   0.135232   8.892  1.8e-11 ***
y_lag2      -0.329857   0.137232  -2.404   0.0204 *
x_lag1       0.008311   0.021477   0.387   0.7006
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.7183 on 46 degrees of freedom
Multiple R-squared:  0.8573, Adjusted R-squared:  0.8478
F-statistic: 90.14 on 3 and 46 DF,  p-value: < 2.2e-16
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  10.1910     0.3305  30.839   <2e-16 ***
x            -0.2266     0.1191  -1.903   0.0636 .
x_lag1       -0.0777     0.1490  -0.522   0.6046
x_lag2        0.0293     0.1535   0.191   0.8495
x_lag3        0.1355     0.1052   1.287   0.2047
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.57 on 45 degrees of freedom
Multiple R-squared:  0.3333, Adjusted R-squared:  0.2727
F-statistic: 5.498 on 4 and 45 DF,  p-value: 0.001116
```

The values of the two series in the last 5 years are shown below:

Year (t)	2019	2020	2021	2022	2023
Unemployment (x_t)	9.95	9.16	9.50	8.07	7.63
Inflation (y_t)	0.61	-0.14	1.87	8.20	5.62

1. Indicate the type of model estimated.

2. Calculate the AIC and the BIC of the models. Which model would you choose based on Akaike's and Bayes' criteria?

3. Obtain the $RM\hat{S}FE$ of the Model (1) using the SER.

4. Derive the Out-Of-Sample (OOS) forecasts one step ahead of inflation from 2020 to 2023 using the Model (1) and obtain the $RM\hat{S}FE_{OOS}$.

5. Provide an estimate of the forecast interval at 95% for 2023 using both the $RM\hat{S}FE$ calculated.

6. The process y_t in Model (3) is suspected to exhibit a unit root. To test this hypothesis, the first difference of the AR(3) process is computed (standard errors are shown in parentheses):

$$\Delta\hat{y}_t = \underset{(0.012)}{0.019} + \underset{(0.0002)}{0.00005}t - \underset{(0.2403)}{0.901}y_{t-1} + \underset{(0.0382)}{0.048}\Delta y_{t-1} - \underset{(0.042)}{0.317}\Delta y_{t-2}$$

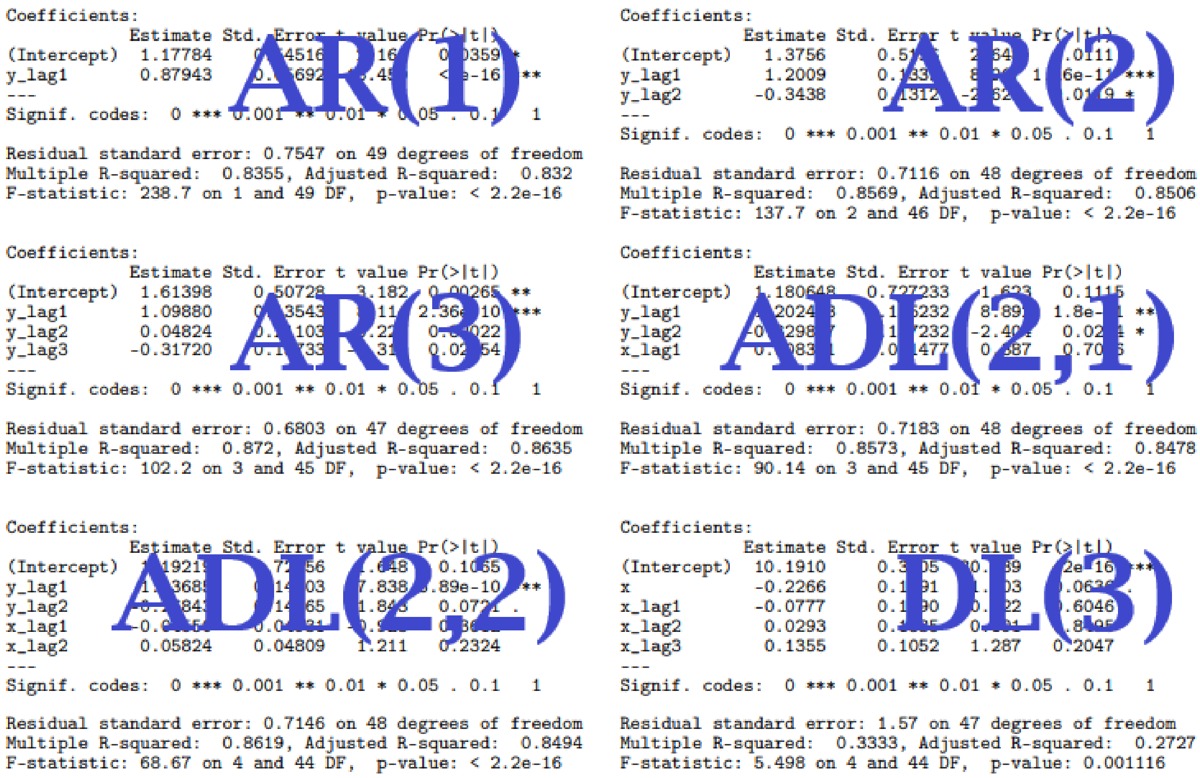
   Perform the appropriate test to determine whether a stochastic trend is present.

7. Assuming that Model (3) may undergo a structural break at observation $t^* = 24$, a Chow test is performed, yielding the result: $F = 1.39$.
   Can we conclude whether there is a structural break at observation $t^* = 24$ or not?

8. Concerned about a potential break, a QLR test (with 15% trimming) is performed on Model (3). The resulting QLR statistic is 5.21. Is there evidence of a break? Explain.

9. Consider the Model (6), which coefficient in the model corresponds to the impact factor? How can its statistical significance be tested?

10. Given the estimated coefficients of Model (6), compute the cumulative multipliers up to $t + 3$.What does it suggest about the overall effect of x_t on y_t?



Solutions

1. The six regression models in the R output are:

$$
\begin{aligned}
&\text{Model 1:} \quad \hat{y}_t = 1.178 + 0.879y_{t-1} \\
&\text{Model 2:} \quad \hat{y}_t = 1.376 + 1.201y_{t-1} - 0.344y_{t-2} \\
&\text{Model 3:} \quad \hat{y}_t = 1.614 + 1.099y_{t-1} + 0.048y_{t-2} - 0.317y_{t-3} \\
&\text{Model 4:} \quad \hat{y}_t = 1.181 + 1.202y_{t-1} - 0.330y_{t-2} + 0.008x_{t-1} \\
&\text{Model 5:} \quad \hat{y}_t = 1.192 + 1.137y_{t-1} - 0.268y_{t-2} - 0.046x_{t-1} + 0.058x_{t-2} \\
&\text{Model 6:} \quad \hat{y}_t = 10.191 - 0.227x_t - 0.078x_{t-1} + 0.029x_{t-2} + 0.136x_{t-3}
\end{aligned}
$$

   In the first model, y_t is regressed on a constant and on its first lag. The estimated regression equation is:

$$\hat{y}_t = 1.178 + 0.879y_{t-1}$$

   This is an AutoRegressive model of order 1, AR(1).

   The second and third models extend this structure to include additional lags of y_t.

Specifically:

- Model (2) is an AutoRegressive model of order 2, AR(2), as it includes y_{t-1} and y_{t-2}.
- Model (3) is an AutoRegressive model of order 3, AR(3), incorporating up to y_{t-3}.

In the fourth model, y_t is regressed on a constant, its first two lags, and the first lag of x_t. The regression equation is:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \delta_1 x_{t-1} + u_t$$

This is an Autoregressive Distributed Lag model, denoted as ADL(p, q), where $p = 2$ represents the number of lags of the dependent variable y_t, and $q = 1$ represents the number of lags of the independent variable x_t. Thus, this model is classified as ADL(2,1).

Similarly, Model (5) follows an ADL(2,2) structure, including two lags of y_t and two lags of x_t.

The last model, Model (6), differs significantly from the previous ones as it does not include lagged values of y_t. Instead, it is a regression of y_t on x_t and its lags:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \beta_3 x_{t-2} + \beta_4 x_{t-3} + u_t$$

This model follows a purely Distributed Lag (DL) structure, denoted as DL(3), since it contains up to three lags of x_t. Unlike the autoregressive models, it does not incorporate past values of y_t.
This is a dynamic regression model where the coefficients $\beta_1, \beta_2, \beta_3$, and β_4 represent the dynamic multipliers.

2. The choice of the appropriate model depends on the underlying theoretical framework and statistical criteria such as Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

   We can compute AIC and BIC using on the Residual Standard Error (SER) provided in the six R output.

   The SER is:

$$SER = \sqrt{\frac{\sum u_i^2}{T-k}} = \sqrt{\frac{RSS}{T-k}}$$

   So, the Residual Sum of Squares (RSS) can be computed from the SER using the formula:

$$RSS = SER^2 \cdot (T-k)$$

where T is the number of observations and k is the number of estimated parameters.

The total number of observations from 1970 to 2019 is 50 ($T = 50$). However, due to the inclusion of lagged values of y_t or x_t, some observations are lost.

To identify the value of k for the models of interest, we need to count the number of estimated parameters. For example, in the ADL(2,2) model: $k = 1 + 2 + 2 = 5$, where 1 represents the intercept, 2 are the lags of y_t, and 2 are the lags of x_t.

It follows that:

- The first model is an AR(1), using $T = 49$ observations and $k = 2$.
- The second model is an AR(2), using $T = 48$ observations and $k = 3$.
- The third model is an AR(3), using $T = 47$ observations and $k = 4$.
- The fourth model is an ADL(2,1), using $T = 48$ observations and $k = 4$.
- The fifth model is an ADL(2,2), using $T = 48$ observations and $k = 5$.
- The sixth model is a DL(3), using $T = 47$ observations and $k = 5$.

Using the given SER values and corresponding T values, the Residual Sum of Squares (RSS) for each model is computed as follows:

$$
\begin{aligned}
RSS_1 &= 0.7547^2 \cdot 47 = 26.770 \\
RSS_2 &= 0.7116^2 \cdot 45 = 22.787 \\
RSS_3 &= 0.6803^2 \cdot 43 = 19.901 \\
RSS_4 &= 0.7183^2 \cdot 44 = 22.702 \\
RSS_5 &= 0.7146^2 \cdot 43 = 21.958 \\
RSS_6 &= 1.57^2 \cdot 42 = 103.526
\end{aligned}
$$

The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are defined as:

$$
\begin{aligned}
AIC &= \ln\left(\frac{RSS}{T}\right) + \frac{2k}{T} \\
BIC &= \ln\left(\frac{RSS}{T}\right) + \frac{k\ln(T)}{T}
\end{aligned}
$$

We now compute AIC and BIC for each model using their respective RSS, T, and k values:

- Model 1 (AR(1), $k = 2$, $T = 49$)

$$AIC_1 = \ln\left(\frac{26.770}{49}\right) + \frac{2 \times 2}{49} = -0.5228$$
$$BIC_1 = \ln(0.5463) + \frac{2\ln(49)}{49} = -0.4455$$

- Model 2 (AR(2), $k = 3$, $T = 48$)

$$AIC_2 = \ln\left(\frac{22.787}{48}\right) + \frac{2 \times 3}{48} = -0.6194$$
$$BIC_2 = \ln(0.4747) + \frac{3\ln(48)}{48} = -0.5024$$

- Model 3 (AR(3), $k = 4$, $T = 47$)

$$AIC_3 = \ln\left(\frac{19.901}{47}\right) + \frac{2 \times 4}{47} = -0.6904$$
$$BIC_3 = \ln(0.4234) + \frac{4\ln(47)}{47} = -0.5329$$

- Model 4 (ADL(2,1), $k = 4$, $T = 48$)

$$AIC_4 = \ln\left(\frac{22.702}{48}\right) + \frac{2 \times 4}{48} = -0.5829$$
$$BIC_4 = \ln(0.4729) + \frac{4\ln(48)}{48} = -0.4270$$

- Model 5 (ADL(2,2), $k = 5$, $T = 48$)

$$AIC_5 = \ln\left(\frac{21.958}{48}\right) + \frac{2 \times 5}{48} = -0.5735$$
$$BIC_5 = \ln(0.4574) + \frac{5\ln(48)}{48} = -0.3786$$

- Model 6 (DL(3), $k = 5$, $T = 47$)

$$AIC_6 = \ln\left(\frac{103.526}{47}\right) + \frac{2 \times 5}{47} = 1.0023$$
$$BIC_6 = \ln(2.2027) + \frac{5\ln(47)}{47} = 1.1991$$

Model	RSS	AIC	BIC
1	26.770	-0.5228	-0.4455
2	22.787	-0.6194	-0.5024
3	19.901	-0.6904	-0.5329
4	22.702	-0.5829	-0.4270
5	21.958	-0.5735	-0.3786
6	103.526	1.0023	1.1991

Lower AIC and BIC values indicate a better model, as they balance goodness of fit and model complexity. Model (3) has the lowest AIC (-0.6904) and BIC (-0.5329), suggesting it is the best among the given models in terms of both prediction accuracy and parsimony. Model (2) is the second-best option, with slightly higher AIC and BIC values compared to Model (3). Model (6) has by far the worst AIC (1.0023) and BIC (1.1991), indicating poor model fit and/or overfitting. Models (4) and (5) (the model in which we added the auxiliary variable) do not improve upon Model (3) and have higher AIC/BIC values, making them less preferable.

3. A way to estimate the $RM\hat{S}FE$ is with the Regression Standard Error (SER):

$$RM\hat{S}FE_{SER} = \sqrt{\frac{RSS}{T-k}}$$

For the Model (1) we will have:

$$RM\hat{S}FE_{SER,1} = \sqrt{\frac{26.770}{49-2}} = \sqrt{\frac{26.770}{47}} = \sqrt{0.5696} = 0.7547$$

Similarly, for the other models (but we have the SER in the R output):

$$RM\hat{S}FE_{SER,2} = \sqrt{\frac{22.787}{48-k_2}} = \sqrt{\frac{22.787}{45}} = 0.7116$$

$$RM\hat{S}FE_{SER,3} = \sqrt{\frac{19.901}{47-k_3}} = \sqrt{\frac{19.901}{43}} = 0.6803$$

$$RM\hat{S}FE_{SER,4} = \sqrt{\frac{22.702}{48-k_4}} = \sqrt{\frac{22.702}{44}} = 0.7183$$

$$RM\hat{S}FE_{SER,5} = \sqrt{\frac{21.958}{48-k_5}} = \sqrt{\frac{21.958}{43}} = 0.7146$$

$$RM\hat{S}FE_{SER,6} = \sqrt{\frac{103.526}{47-k_6}} = \sqrt{\frac{103.526}{42}} = 1.57$$

4. The actual values of unemployment (x_t) and inflation (y_t) for the years 2019 to 2023 are as follows:

Year (t)	2019	2020	2021	2022	2023
Unemployment (x_t)	9.95	9.16	9.50	8.07	7.63
Inflation (y_t)	0.61	-0.14	1.87	8.20	5.62

The Model (1) is:

$$\hat{y}_t = 1.178 + 0.879 y_{t-1}$$

It is possible to use this model to get Out-Of-Sample (OOS) forecasts one step ahead, as:

$$\hat{y}_{t+1|t} = 1.178 + 0.879 y_t$$

For example, the expected value of inflation for $t + 1 = 2020$, based on the information available in $t = 2019$ would be:

$$\hat{y}_{t+1|t} = 1.178 + 0.879 \times 2.13 = 1.71$$

The out-of-sample (OOS) forecasts $\hat{y}_{t+1|t}$ from 2020 to 2023 computed using the actual values of y_t from the previous year are:

$$\begin{aligned}
\hat{y}_{2020|2019} &= 1.178 + 0.879 \cdot y_{2019} = 1.178 + 0.879 \cdot 0.61 = 1.71 \\
\hat{y}_{2021|2020} &= 1.178 + 0.879 \cdot y_{2020} = 1.178 + 0.879 \cdot (-0.14) = 1.05 \\
\hat{y}_{2022|2021} &= 1.178 + 0.879 \cdot y_{2021} = 1.178 + 0.879 \cdot 1.87 = 2.82 \\
\hat{y}_{2023|2022} &= 1.178 + 0.879 \cdot y_{2022} = 1.178 + 0.879 \cdot 8.20 = 8.39
\end{aligned}$$

Thus, the OOS forecasts are:

Year (t)	2019	2020	2021	2022	2023
Inflation (y_t)	0.61	-0.14	1.87	8.20	5.62
OOS Forecast ($\hat{y}_{t+1|t}$)		1.71	1.05	2.82	8.39

For example, the the actual value of inflation for 2020 (y_{2020}) will deviate from the OOS forecast ($\hat{y}_{2020|2019}$) of:

$$y_{2020} - \hat{y}_{2020|2019} = -0.14 - 1.71 = -1.85$$

This value is the forecast error, $\tilde{u}_{2020}$.

To estimate $R\hat{MSFE}_{OOP}$ the formula is:

$$R M\hat{S}FE_{OOP} = \sqrt{\frac{1}{P}\sum_{t=1}^{P}(y_t - \hat{y}_t)^2} = \sqrt{\frac{1}{P}\sum_{i=1}^{P}\tilde{u}_i^2}$$

where $P = 4$ is the number of out-of-sample observations (forecasts), y_t are the actual values, $\hat{y}_t$ are the forecasts, and $\tilde{u}_t$ the forecast errors.

Year (t)	y_t	$\hat{y}_{t+1\|t}$	$\tilde{u}_t = y_t - \hat{y}_t$	$\tilde{u}_t^{\,2}$
2020	-0.14	1.71	-1.85	3.42
2021	1.87	1.05	0.82	0.67
2022	8.20	2.82	5.38	28.94
2023	5.62	8.39	-2.77	7.67

The mean squared error is:

$$\frac{1}{4}\sum_{t=1}^{4}\tilde{u}_t^{\,2} = \frac{3.42 + 0.67 + 28.80 + 7.67}{4} = 10.64$$

The $RMS\hat{F}E_{OOS}$ is:

$$RMS\hat{F}E_{OOS} = \sqrt{10.64} = 3.19$$

The $RM\hat{S}FE_{SER}$ is 0.7547, whereas the $RM\hat{S}FE_{OOS} = 3.19$.

The significant discrepancy between these values suggests that the model performed well in-sample but failed to generalize due to the unique and unpredictable nature of the COVID-19 period.

5. The one step ahead forecast for 2022 is:

$$\hat{y}_{2023|2022} = 1.178 + 0.879 \cdot y_{2022} = 1.178 + 0.879 \cdot 8.20 = 8.39$$

The 95% prediction interval for this prediction, using the $RM\hat{S}FE_{OOS}$, is:

$$[\hat{y}_{t+1|t} \pm z_{1-\alpha/2} \times RM\hat{S}FE_{OOP}] = [8.39 \pm 1.96 \times 3.19] = [2.14; 14.64]$$

With a 95% probability, inflation for 2022 will be between 2.14% and 14.64%.

If we have used the $RM\hat{S}FE_{SER}$ the 95% prediction interval, will be:

$$[\hat{y}_{t+1|t} \pm z_{1-\alpha/2} \times RM\hat{S}FE_{SER}] = [8.39 \pm 1.96 \times 0.7547] = [6.912, 9.868]$$

A way smaller interval than the previous one.

6. The Model (3) is an AR(3):

$$\hat{y}_t = 1.614 + 1.099y_{t-1} + 0.048y_{t-2} - 0.317y_{t-3}$$

It is suspected that the process y_t may exhibit a unit root.

To test this hypothesis, is computed the first difference of the process to apply an ADF test.

The equation of the first difference of the process is:

$$\begin{array}{c} \Delta\hat{y}_t = 0.019 + 0.00005t - 0.901y_{t-1} + 0.048\Delta y_{t-1} - 0.317\Delta y_{t-2} \\ (0.012) \quad (0.0002) \quad (0.2403) \quad (0.0382) \qquad\quad (0.042) \end{array}$$

We have to compare the estimated test statistic with the critical values of the ADF test to determine whether a stochastic trend is present.

We have to test if γ, the coefficient of the first lag, y_{t-1} is:

$$H_0 : \gamma = 0 \quad \text{(the series has a unit root, meaning it is non-stationary)}$$

vs.

$$H_1 : \gamma < 0 \quad \text{(the series is stationary)}$$

The test statistic is:

$$t = \frac{\hat{\gamma}}{\text{SE}(\hat{\gamma})}$$

where $\hat{\gamma}$ is the estimated coefficient and $\text{SE}(\hat{\gamma})$ is its standard error.

Given the estimated coefficient $\hat{\gamma} = -0.901$ and its standard error (0.2403), we have the corresponding t-statistic:

$$t = -3.75$$

Comparing the t-statistic with the critical values from the Dickey-Fuller table if it is smaller than the critical value, we **reject** H_0 and conclude that the series is **stationary**, if it is greater than the critical value, we **fail to reject** H_0, meaning the series may be **non-stationary**.

Model	**1%**	**5%**	**10%**
Intercept only	-3.43	-2.86	-2.57
Intercept and trend	-3.96	-3.41	-3.12

Tabella 1: Critical values for the ADF test

In our model we have a time trend term ($\beta \cdot t = 0.00005 \cdot t$).

Comparing the test statistic -3.75 with the 5% critical value with 'Intercept and trend' of -3.41, since $-3.75 < -2.98$, we reject the null hypothesis of a unit root, indicating stationarity (not for 1%).

7. To test if Model (3) may undergo a structural break at observation $t^* = 24$, we perform the Chow test:

$$H_0 : \text{There is } \textbf{no structural break}$$
$$H_1 : \text{There is a } \textbf{structural break}$$

The test statistic is given by $F = 1.39$.

We compare the test statistic with the critical value of the F-distribution at significance level $\alpha = 5\%$, with degrees of freedom $k = 4$ (the number of parameters in the model AR(3)) and $(n - 2k) = 39$ (where $n = 47$ is the number of observations).

The critical value for $F_{4,39}$ at the 5% significance level is 2.61.

Since $F = 1.39 < 2.61$, we **do not reject** the null hypothesis of no structural break.

Thus, with a 5% significance level, we can conclude that there is **no structural break at $t^* = 24$.**

8. To further examine model stability, perform a QLR test (with 15% trimming) for potential structural breaks in the series.

The test statistic is given by $QLR = 5.21$.

Restrictions (q)	Significance Level		
	1%	5%	10%
1	12.16	8.67	7.12
2	7.78	5.86	5.00
3	6.02	4.71	4.09
4	5.12	4.09	3.59
5	4.53	3.66	3.26
6	4.12	3.37	3.02
7	3.82	3.15	2.84

Tabella 2: QLR critical values with 15% trimming

Comparing with the 1% critical value with Restrictions $q = 4$ (the number of parameters in the model AR(3)) of 5.12, since $QLR = 5.21 > 5.12$, we reject the null hypothesis, indicating a structural break within the given interval.

For the AR(3):

- The ADF test indicates stationarity, rejecting the presence of a stochastic trend.

- The Chow test for a break at $t^* = 24$ does not show evidence of a structural change.
- The QLR test suggests a break in the time series, indicating possible instability in the model.

9. The Model (6) is a dynamic regression model DL(3) represented as:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \beta_3 x_{t-2} + \beta_4 x_{t-3} + u_t$$

The coefficients of the model β_1, β_2, β_3 and β_4 are the dynamic multipliers that represent the effect of a one-unit change in x_t on y_t in the current and lagged periods.

$$\begin{aligned} \hat{y}_t = {} & \underset{(0.3305)}{10.1910} - \underset{(0.1191)}{0.2266}x_t - \underset{(0.1490)}{0.0777}x_{t-1} + \underset{(0.1535)}{0.0293}x_{t-2} + \underset{(0.1052)}{0.1355}x_{t-3} \end{aligned}$$

The impact factor is the coefficient that captures the immediate (or direct) effect of x_t on y_t without accounting for any lagged effects $(x_{t-1}, x_{t-2}, x_{t-3})$. In our model β_1 is the impact factor:

$$\beta_1 = -0.2266$$

A one-unit increase in x_t results in an immediate decrease of y_t by 0.2266 units.

The significance of the impact factor is tested using a t-test:

$$H_0 : \beta_1 = 0 \quad \text{vs.} \quad H_1 : \beta_1 \neq 0$$

A significant t-statistic implies that x_t has a statistically significant immediate effect on y_t.

The t-statistic for β_1 is calculated as:

$$t = \frac{\hat{\beta}_1}{\text{SE}(\hat{\beta}_1)} = \frac{-0.2266}{0.1191} = -1.903$$

The p-value associated with this t-statistic is $p = 0.0636$, which is greater than the standard significance level of 0.05. Thus, we fail to reject the null hypothesis, meaning the impact factor (x_t) is not statistically significant at the 5% level. However, it is significant at the 10% level.

10. The cumulative multipliers represent the total effect of a one-unit change in x_t over multiple time periods:

$$\delta_i = \sum_{j=1}^{i} \beta_j$$

For each i:
$\delta_0 = \beta_0 = 10.191$
$\delta_1 = \beta_1 = -0.2266$
$\delta_2 = \beta_1 + \beta_2 = -0.2266 - 0.0777 = -0.3043$
$\delta_3 = \beta_1 + \beta_2 + \beta_3 = -0.2266 - 0.0777 + 0.0293 = -0.2750$
$\delta_4 = \beta_1 + \beta_2 + \beta_3 + \beta_4 = -0.2266 - 0.0777 + 0.0293 + 0.1355 = -0.1395$

	Dynamic Multiplier (β_i)	Cumulative Multiplier (γ_i)
Intercept	10.191	10.191
*Lag*0	-0.2266	-0.2266
*Lag*1	-0.0777	-0.3043
*Lag*2	0.0293	-0.2750
*Lag*3	0.1355	-0.1395

The cumulative multiplier at $t + 3$ indicates a total effect of -0.1395, which suggests a small overall negative effect.