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Game Theory

Duopoly Cournot-Nash

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ECONOMIC MODEL

(DUOPOLY)

WITH CONTINUOUS CHOICES

Continuous strategies: there is a continuous space of strategies also excluding the use of mixed strategies, we have a balance in pure strategies

Reflect on discrete-chosen strategies: High-Low; Enter-Do not Enter, etc.

COURNOT – DUPOLIO (1838)

1) **Players = 2 firms**

2) **Strategies = quantity; production**

- **The different quantities that can be produced (set of non-negative real numbers) MARKET:**

$$Q = q_1 + q_2$$

$$P(Q) = a - Q$$

- **Since $q_1 + q_2 = Q \geq a \rightarrow P(Q) = 0$ then neither of the two firms will produce $q_i > a$**
- **a = reserve price (price that consumers are willing to pay for the first unit produced)**

Production with constant returns, and same technology (cost function):

$$C_i = c\, q_i$$

4) payoff of each firm is the profit $\pi_i(q_i, q_j)$

Cournot

q_1, q_2 ; $P(Q) = a - Q$; $Q = q_1 + q_2$

- Constant marginal costs c with $c<a$
- Firms choose simultaneously q_i e q_j

$$\pi_i(q_i, q_j) = q_i\,[(P(Q)-c] = q_i\,[a-(q_i+q_j)-c]$$

→ **The quantity pair is an Equil. of Nash (q_i^*, q_j^*) if for each firm, q^* is the solution of the problem**

$$\max_{q_i} \pi_i(q_i, q_j) = q_i\,[a-(q_i+q_j^*)-c]$$

For each firm:

$$\text{MAX } \pi_i = q_i\,(p-c) = [a-(q_i+q_j)-c]$$

Firm I

$$\underset{q_1}{\text{MAX}}\ \pi_1 = q_1\,[a-q_1-q_2^*-c]$$

$$\frac{\partial(\pi_1)}{\partial q_1} = a-2q_1-q_2^*-c \rightarrow \underbrace{q_1 = \tfrac{1}{2}\,(a-q_2^*-c)}$$

Firm II

$$\underset{q_2}{\text{MAX}}\ \pi_2 = q_2\,[a-q_1^*-q_2-c]$$

$$\frac{\partial(\pi_2)}{\partial q_2} = a-q_1^*-2q_2-c \rightarrow \underbrace{q_2 = \tfrac{1}{2}\,(a-q_1^*-c)}$$

$$\begin{cases} q_1 = \frac{1}{2}(a - q_2^* - c) \\ q_2 = \frac{1}{2}(a - q_1^* - c) \end{cases}$$

The solution is $q_1^* = \frac{a-c}{3}$; $q_2^* = \frac{a-c}{3}$

Note: it is an optimal mutual reaction: Nash equilibrium

We can ask ourselves if this equilibrium is stable. In other words, if there is an incentive for firms to move from this solution in terms of quantity of production. Obviously, given the market price, every firm wants to produce $Q > q_i$

Profit for each firm is

$$\pi = \frac{(a-c)^2}{9}$$

how do you calculate it ? (here the steps of elementary mathematics)

$$\pi_i = (a - q_i - q_j - c)q_i \quad \rightarrow \quad q_i = q_j = q$$

$$= (a - 2q - c)q \quad = \quad (a-c)q - 2q^2$$

$$con \quad q^* = \frac{(a-c)}{3}$$

$$= (a-c)\frac{(a-c)}{3} - 2\frac{(a-c)^2}{9} \quad =$$

$$\frac{3(a-c)^2 - 2(a-c)^2}{9}$$

What happens with a more efficient firm?

that is, with a firm with lower marginal costs?

$$C_i = cq_i \quad ora \quad C_i = c_i q_i$$

$$\pi_i = (a - q_i - q_j - c_i)q_i$$

$$\frac{\partial \pi_1}{\partial q_1} = a - 2q_1 - q_2 - c_1 = 0$$

$$\frac{\partial \pi_2}{\partial q_2} = a - 2q_2 - q_1 - c_2 = 0$$

I find for example q1 from the latter and insert it in the first and vice versa. The new Cournot quantities are:

$$q_1^* = \frac{a - 2c_1 + c_2}{3}$$

$$q_2^* = \frac{a - 2c_2 + c_1}{3}$$

$$q_1^* = \frac{a - 2c_1 + c_2}{3}$$

$$q_2^* = \frac{a - 2c_2 + c_1}{3}$$

$$ora:$$

$$\frac{\partial q_i^*}{\partial c_i} = -\frac{2}{3} \qquad \frac{\partial q_i^*}{\partial c_j} = \frac{1}{3}$$

The quantity produced is DECREASING at ITS marginal cost; INCREASING at the marginal cost of the opposing firm

In Cournot Duopoly the most efficient firm produces more with higher profits in equilibrium: when the marginal costs of my opponent increase, I produce more and therefore, in equilibrium, I have higher profits.

- **Every firm wants to be a monopoly and choose qi that maximizes $\pi_i(q_i,0)$**

$$Max\,\pi_i(q_i,0) = (a - q_i - c)q_i$$

$$\frac{\partial \pi_i}{\partial q_i} = a - 2q_i - c = 0$$

$$q_M = \frac{(a-c)}{2} \quad \Rightarrow \quad q_M > q_C$$

Quindi

$$\pi_i(q_i,0) = (a - q_i - c)q_i \quad = \frac{(a-c)^2}{4} \quad \Rightarrow \quad \pi_M > \pi_C$$

Is there a possibility of agreement?

$\longrightarrow$ **Given 2 firms, the aggregate profits of the duopoly can be maximized by producing $Q=q_1+q_2=q_m$ of monopoly and therefore:**

$$q_1 = \frac{q_M}{2}; \qquad q_2 = \frac{q_M}{2}$$

$$dove$$

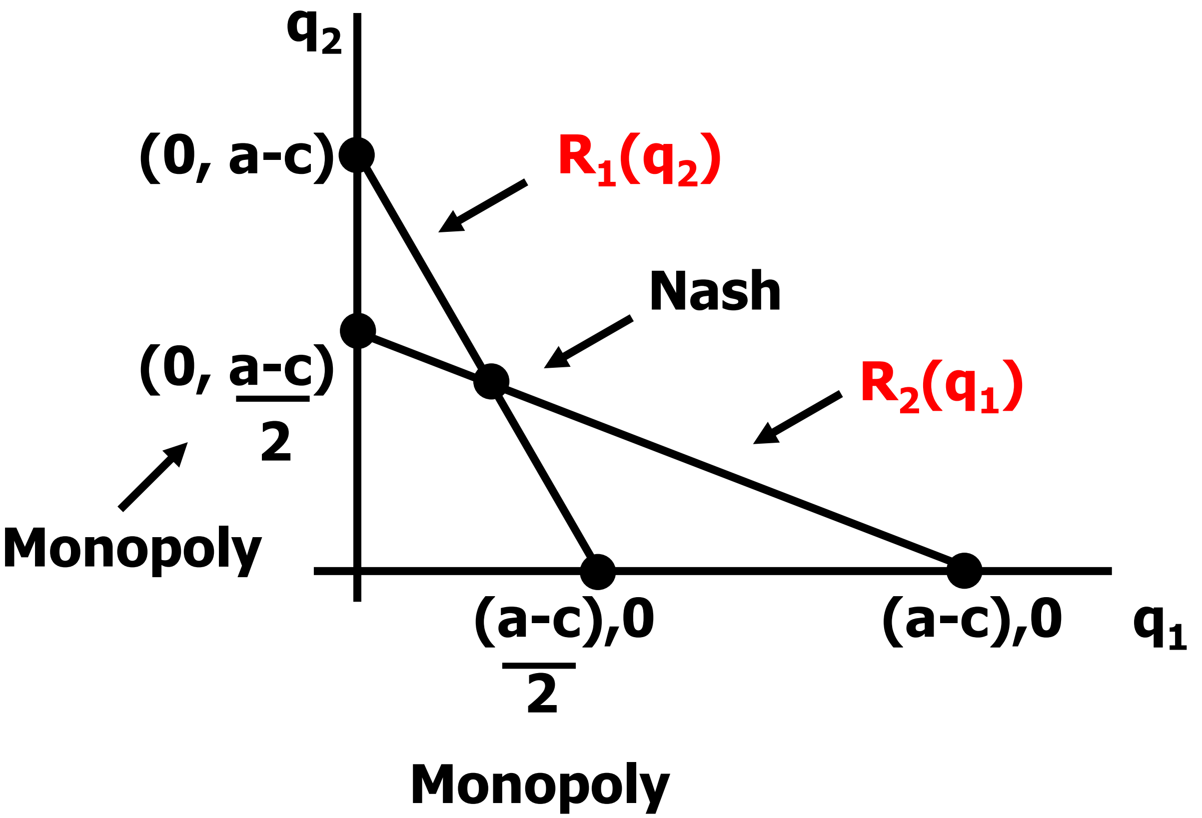
$$q_i = \frac{q_M}{2} < q_C$$

The amount of monopoly is lower than that of monopoly competition. The amount of lower monopoly reduces production costs and increases prices, thus increasing profits.

there is an incentive to deviate! and do not respect the agreement:

- **The quantity of monopoly qm has a lower production and therefore a higher price. At this high price every firm has the incentive to increase its quantity, although this will decrease the market price.**

- **In Cournot equilibrium, the aggregate quantity is greater and $P(q_1,q_2)$ is lower, with a lower temptation to deviate.**



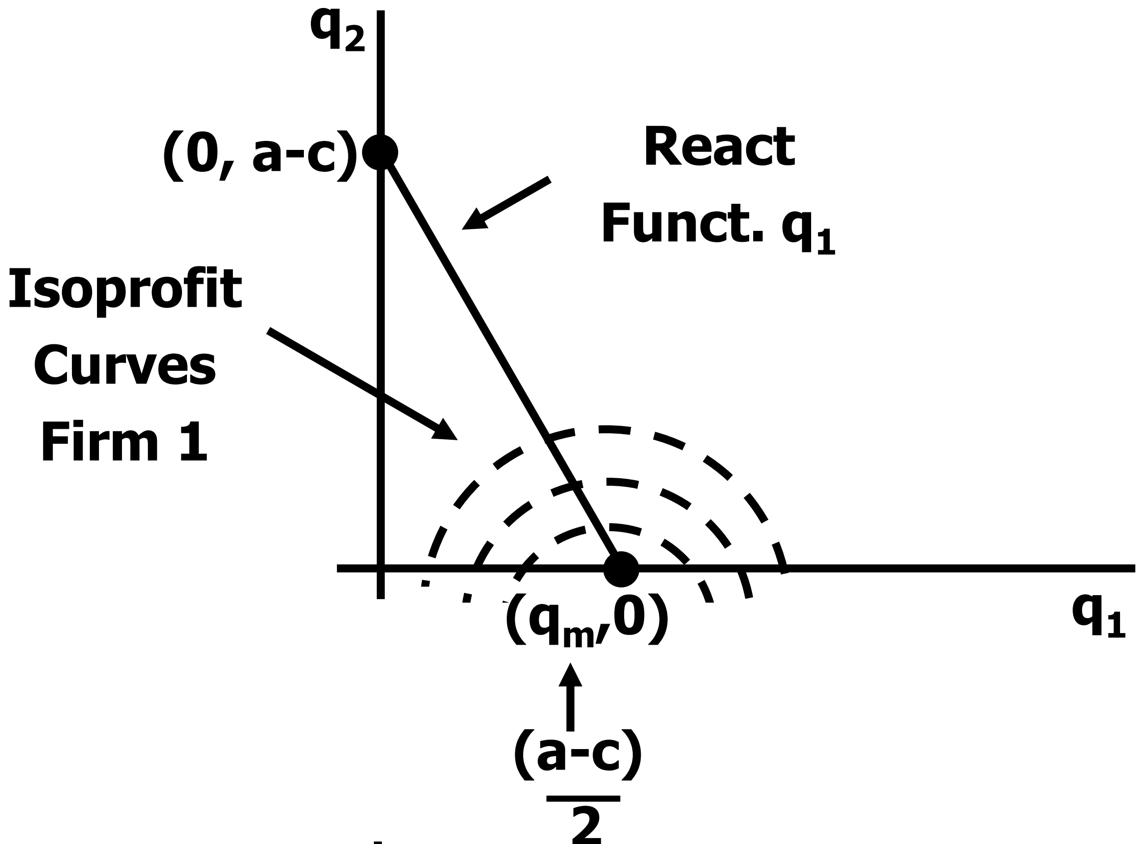
$R_2(q_1)$	$q_2 = ½ (a-q_1-c)$	Player I
$R_1(q_2)$	$q_1 = ½ (a-q_2-c)$	Player II

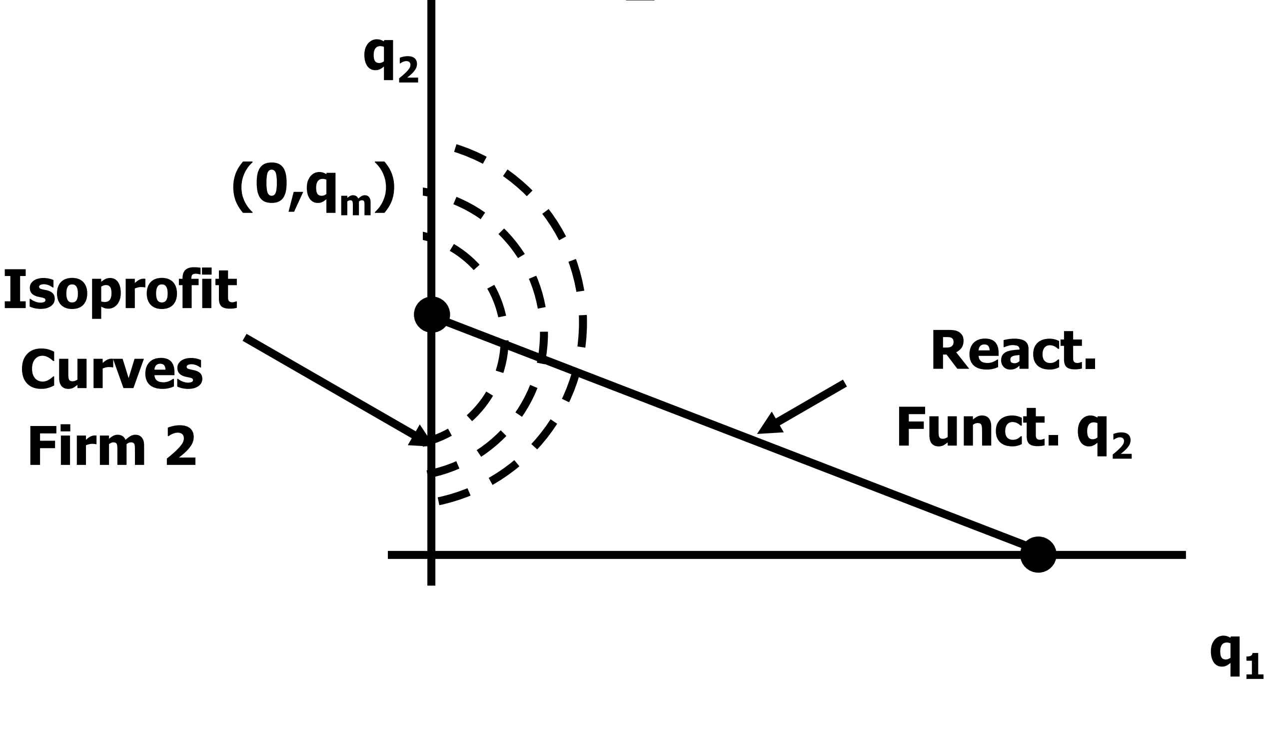
Example. La $R_2(q_1)$ è $q_2= ½ (a-q_1-c)$:

With $q_1=0$, Solution (0, Monopoly): First Intercept
Con $q_2=0$, Soluzione ((a-c), 0): Second Intercept

$$\begin{cases} q_1 = \frac{1}{2}(a - q_2 - c) \\ q_2 = \frac{1}{2}(a - q_1 - c) \end{cases}$$

Reaction functions





keep in mind!

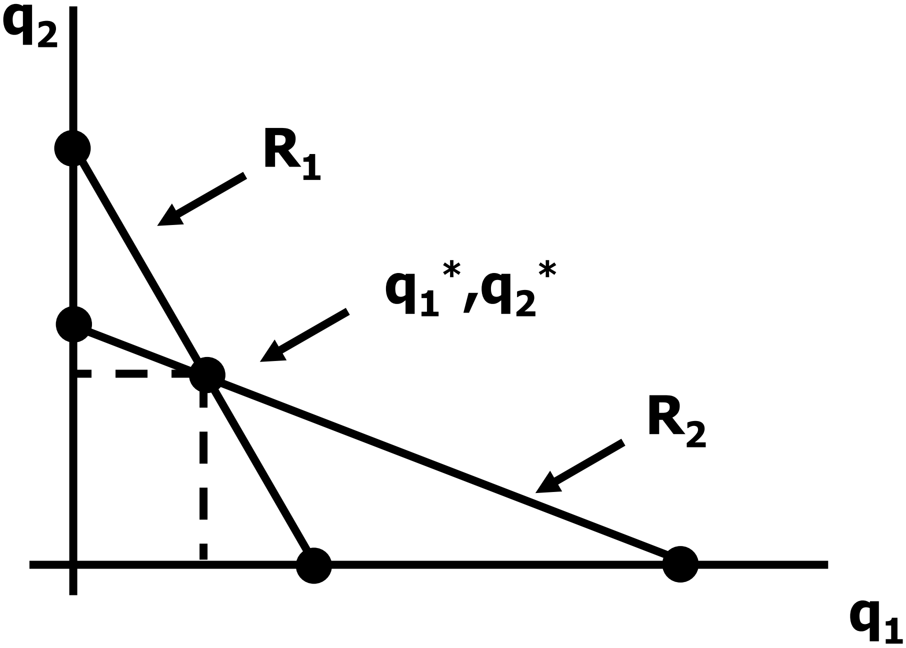
$\longrightarrow$ **Isoprofit Curves**

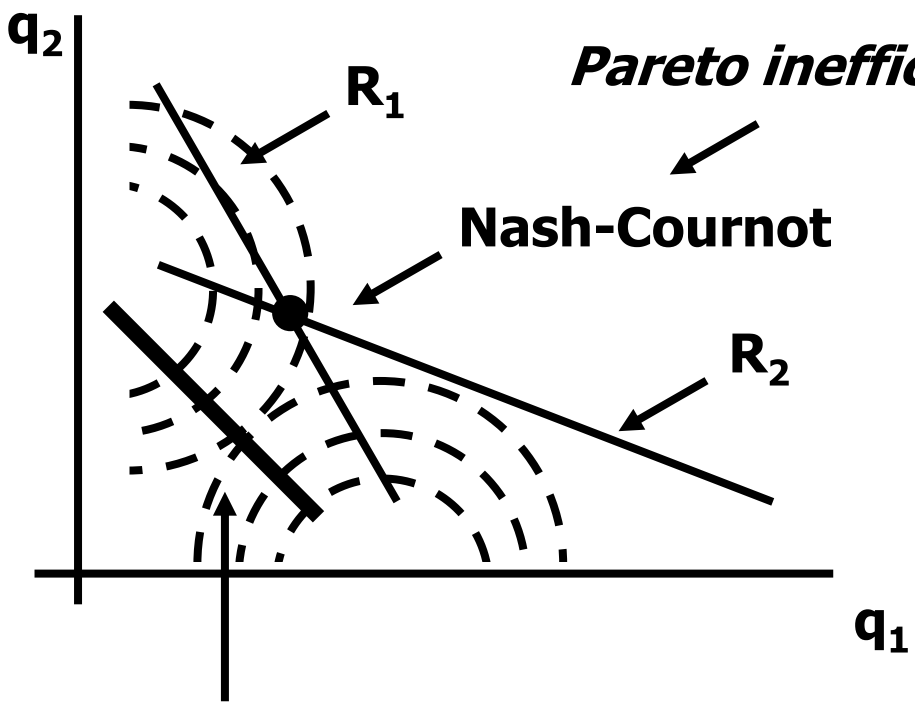
$$\pi_i\,(q_1\;q_2) = [a-(q_1+q_2)-c]q_1 = aq_1-q_1{}^2-q_1q_2-q_1c$$

The curves represent the combinations of q1 and q2 that give the company (in this case, the firm I) a constant profit level:

$$aq_1-q_1{}^2-q_1q_2-q_1c = \pi_1$$

$$\pi_i = q_1\,[\underbrace{a-(q_1+q_2)}_{P}-c] = \underbrace{q_1a-q_1^2-q_1q_2-q_1c}_{\text{Isoprofit}}$$





Contract curve: it is not possible to know where the Pareto efficient equilibrium is on this curve: the exact allocation will depend on the relative contractual strength

The CURVE OF CONTRACTS is the set of all combinations of q1 and q2 that maximize the sum of the joint profits of the two firms.
Along the curve of contracts, the profit of INDUSTRY is everywhere equal to monopoly profit.

Bertrand:

Price-fixing is a crucial component of a firm's decision-making process.
• In oligopoly, the demand that is addressed to a firm (and therefore its profits) also depends on the prices charged by the competitors.

Assume that firms have to choose their prices simultaneously. Clearly, the situation described is that of a strategy game, in which the decisions of each company depend from conjecture on the price of the rival.

The payoff functions are given from the value of profits:

- $\pi_i(p_i, p_j) = p_i D_i(p_i, p_j) - C_i(D_i(p_i, p_j))$
  - $= (p_i - c)D_i(p_i, p_j)$

Where D(p) is the market demand

Bertrand

Duopoly, homogeneous product, constant returns to scale, identical technologies.

$$CP \quad \Rightarrow \quad p = c$$

$$Giocatore 1 \qquad p_1 = c + \lambda$$

$$con \quad \lambda = mark - up \quad \lambda > 0$$

$$Giocatore 2\ ?:$$

$$p_2 = p_1 - \eta \quad = \quad c + \lambda - \mu$$

$$dato: \quad \mu < \lambda, \qquad p_2 \geq c$$

The optimal response of the firm 2 (player2) is to set a price between c and p1. Consumption will be directed exclusively towards player-firm 2. Firm 2 is the monopolist because the product is homogeneous: this can not be an equilibrium

Given the behavior of firm 2, for firm 1 it will be optimal to set the price:

$$Giocatore 1 \quad p_1 = p_2 - \sigma$$
$$con \quad \sigma \quad che\ assicura \quad p_1 \geq c$$

Now it is the firm 1 which holds the market monopoly. This will bring firm 2 to further reduce the price and so on.

An undercutting mechanism is generated which will stop when:

$$p_1 = p_2 = c$$

At this level, no firm can lower the price (produces losses) and no firm can raise it without the other firm following it: The only Nash equilibrium in Bertrand's model is to set prices:

$$p_1^B = p_2^B = c$$

With an homogeneous good, and constant scale returns, Bertrand reproduces the equilibrium of Perfect Competition

Bertrand paradox: the equilibrium of PC emerges from a price war, where each firm seeks to acquire monopoly power via undercutting

If firms have different technologies, then the most efficient firm becomes a monopolist. Example:

$$Se \quad c_2 > c_1 \quad \Rightarrow$$

$$qualsiasi \quad p_1 \in (c_1, c_2)$$

…and Firm 1 becomes a monopoly

Notice:

Even with just two firms the equilibrium price is (as in perfect competition) equal to the marginal cost, and therefore profits are zero (and would be negative if there were fixed costs)!
This result is contradicted by empirical evidence, which suggests:
1) normally the duopolists make high profits;
2) the increase in the number of competitors gradually decreases the market price.
An explanation refers to the possibility that a small price reduction is not enough to get all the demand, or that it is not convenient to lower the price for the competitors.

This happens if:
- 1) the products are differentiated (a small price reduction is not enough to obtain the movement of all consumers)

- 2) interaction is "dynamic" (in "repeated games" it is necessary to examine the effect connected to possible future retaliation to behavior opportunistic)