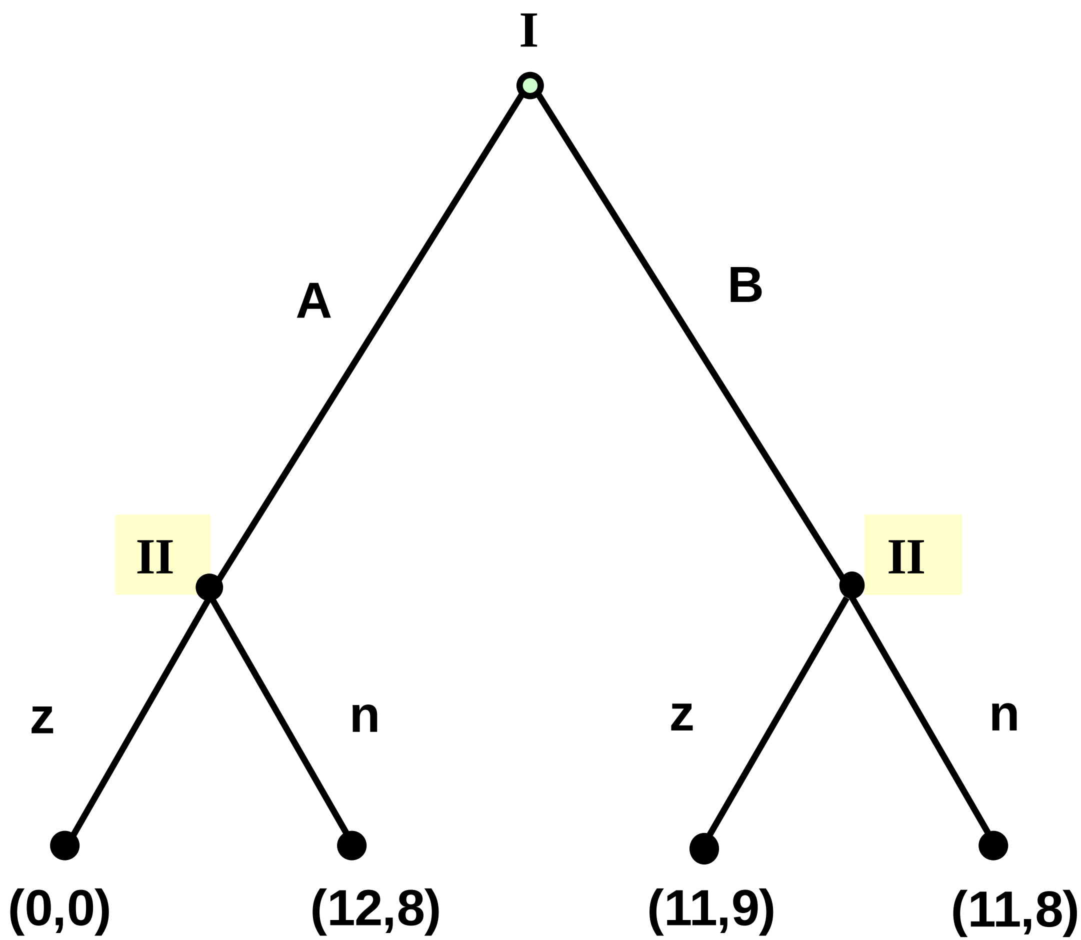
EXAMPLES

Tor Vergata Game Theory course


I
A
B
II
II
z
n
z
n
(0,0)
(12,8)
(11,9)
(11,8)

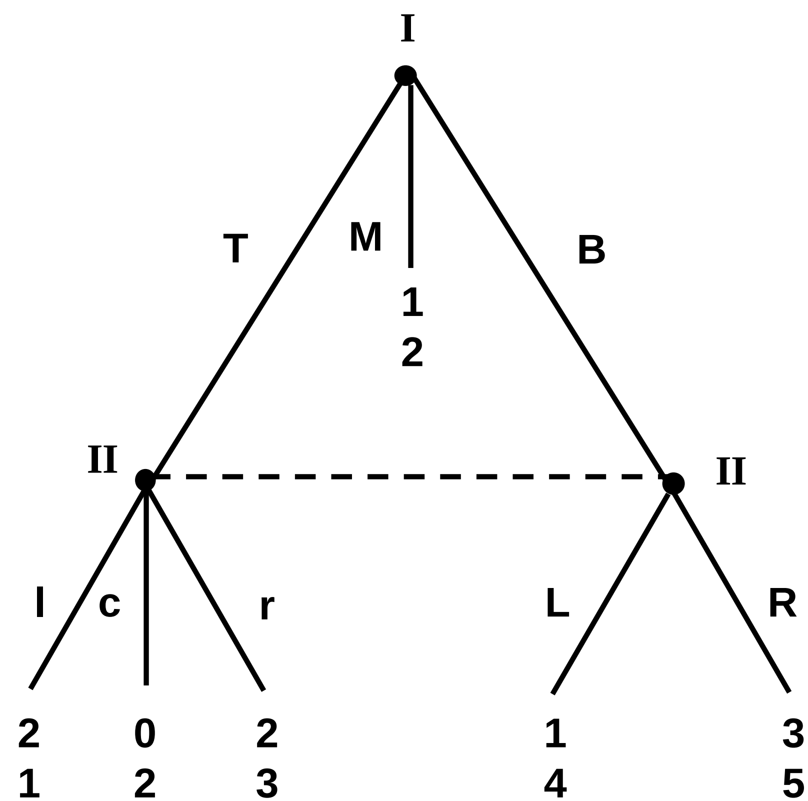
	zz	zn	nz	nn
A	0,0	0,0	12,8	12,8
B	11,9	11,8	11,9	11,8

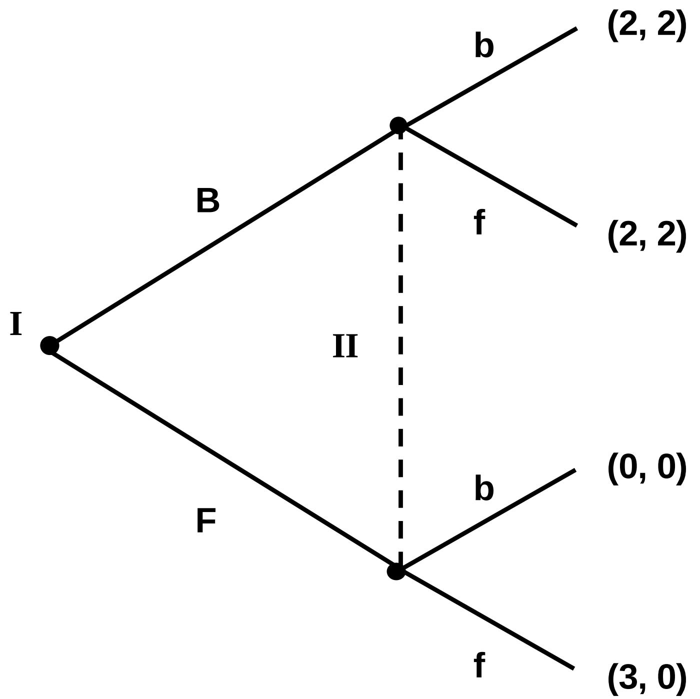
	f
B	12,8
F	11,9

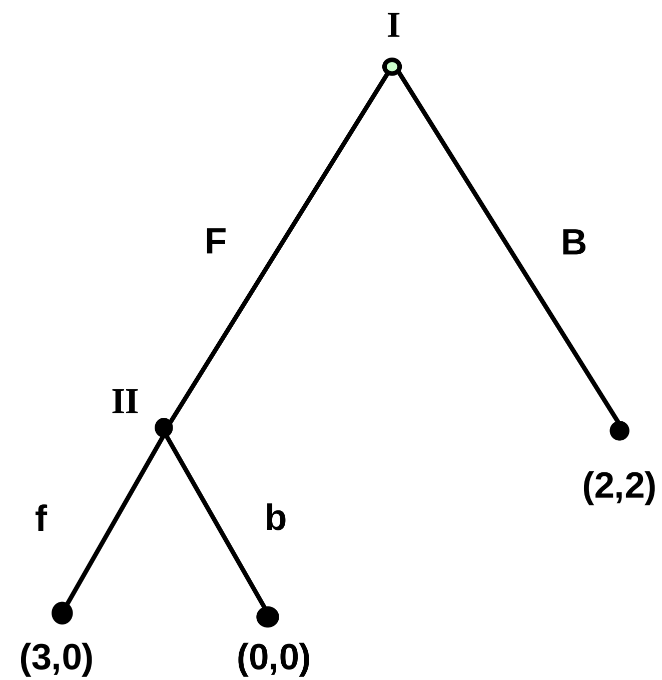
Unique Nash equilibrium: Bf

Three Nash (A,nz); (A,nn);(B,zz)

what's wrong with this game?







Find the strategic form

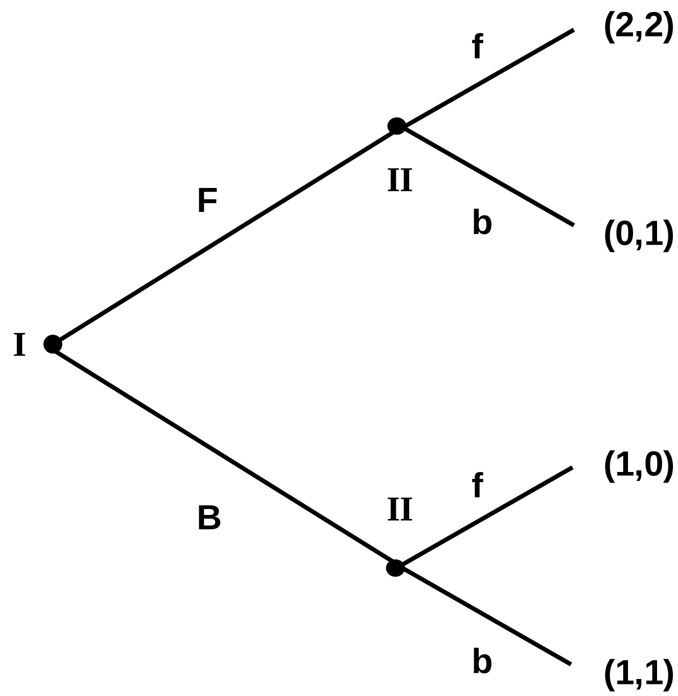
	b	f
B	2, 2	2, 2
F	0, 0	3, 0

different extensive forms can provide the same strategic form

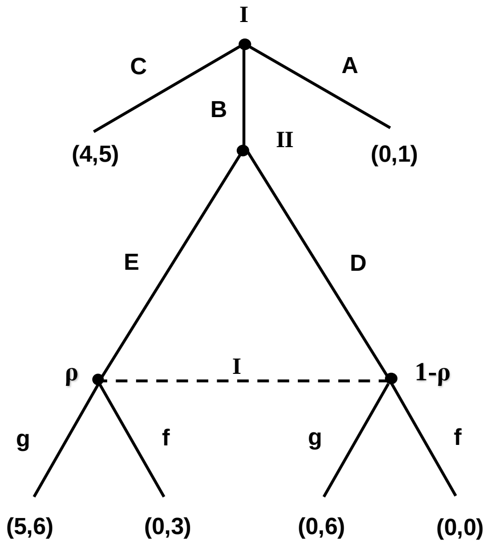
notice two quite different games, one with imperfect information and the other with perfect information

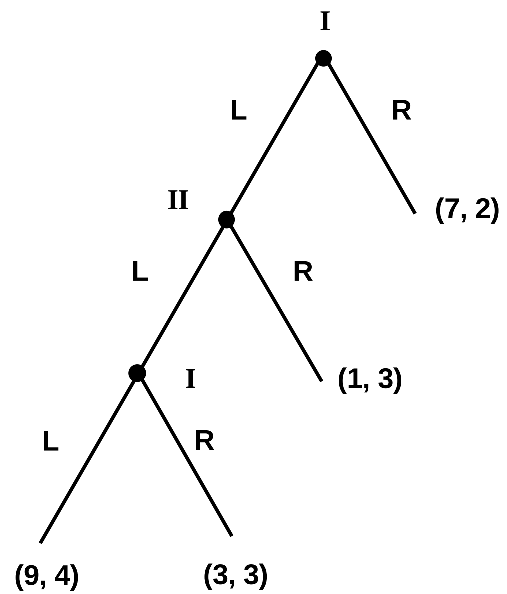
	b	f
B	1, 1	1, 0
F	0, 1	2, 2

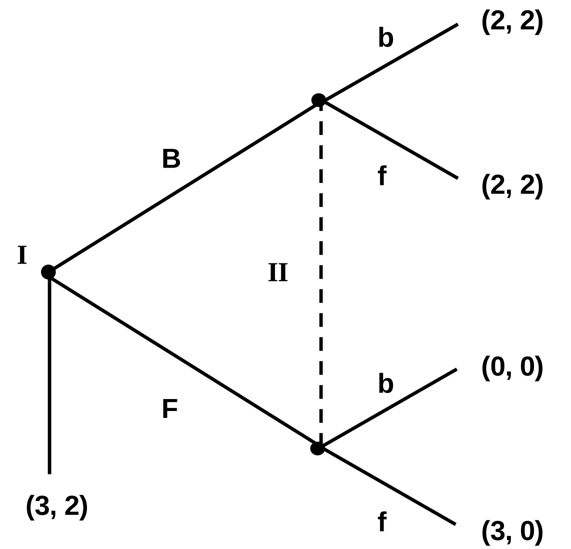
Could this strategic form be represented by a graph with complete information such as the following?

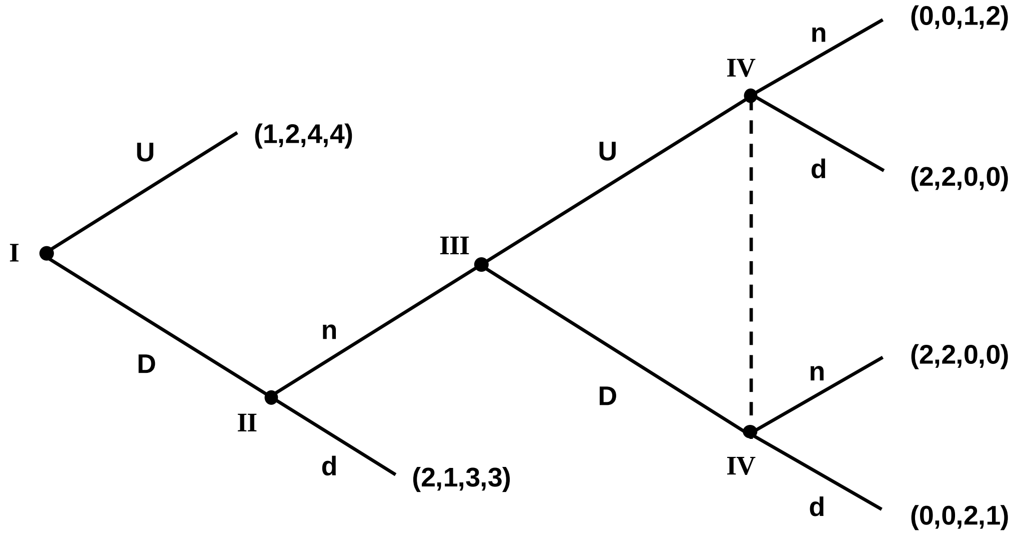


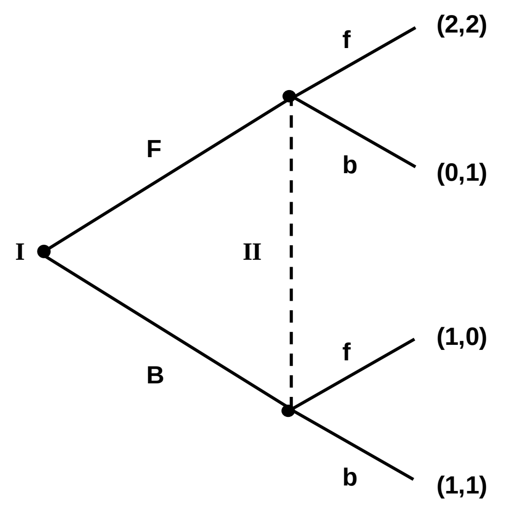
Subgames


I
C
A
B
II
(4,5)
(0,1)
E
D
ρ
I
1-ρ
g
f
g
f
(5,6)
(0,3)
(0,6)
(0,0)


I
L
R
II
(7, 2)
L
R
I
(1, 3)
L
R
(9, 4)
(3, 3)


b
(2, 2)
B
f
(2, 2)
I
II
b
(0, 0)
F
(3, 2)
f
(3, 0)


n
(0,0,1,2)
IV
U
(1,2,4,4)
U
d
(2,2,0,0)
I
III
n
D
n
(2,2,0,0)
D
II
IV
d
(2,1,3,3)
d
(0,0,2,1)

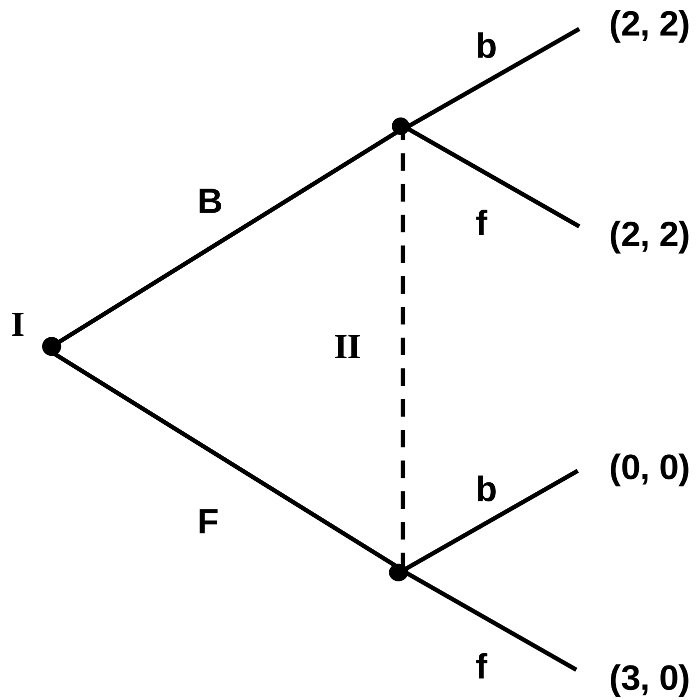

f
(2,2)
F
b
(0,1)
I
II
f
(1,0)
B
b
(1,1)

	c	nc
C	180, 164	100, 172
NC	204, 84	124, 92

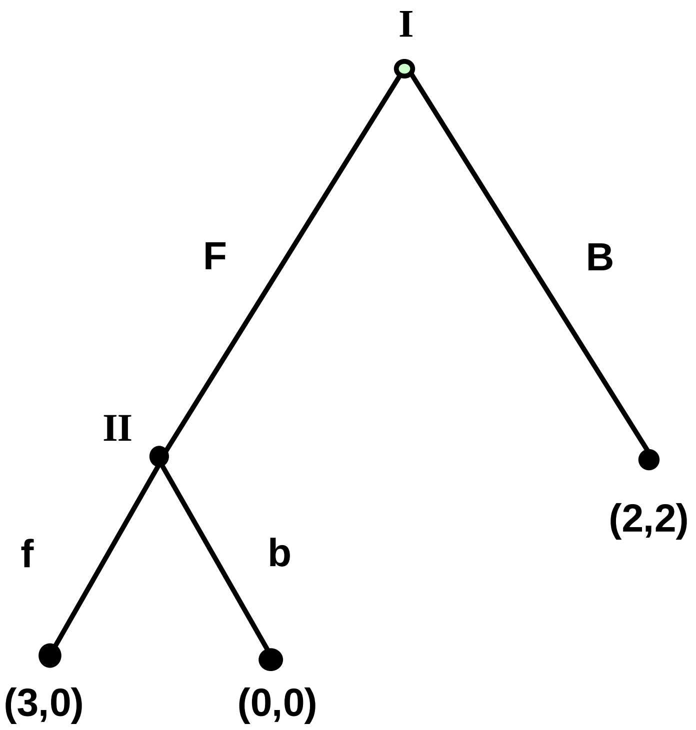
	c	nc
C	9, 9	0, 12
NC	12, 0	7, 7

	s	g
a	8, 8	0, 9
b	9, 0	2, 2

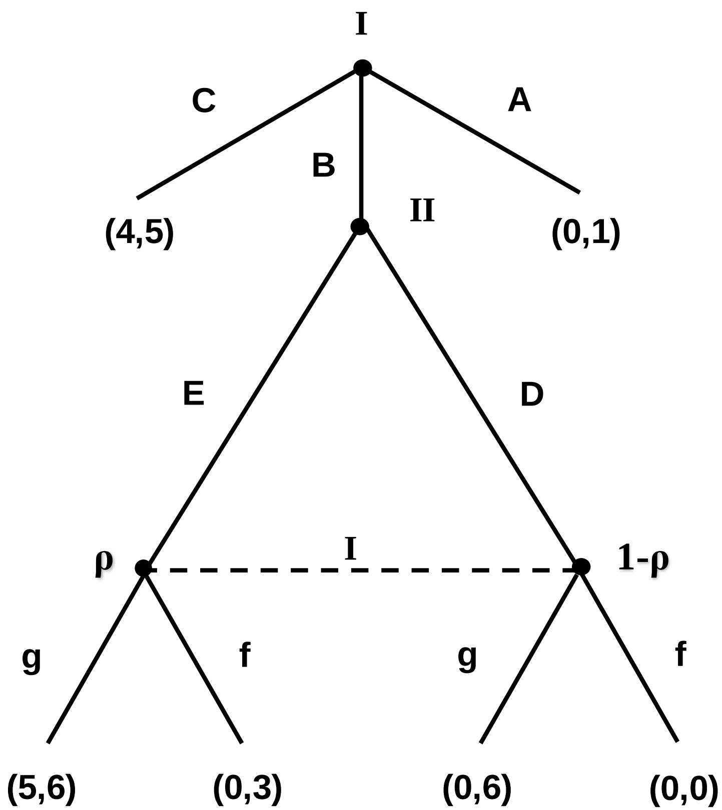
	a	b
A	5, 5	-3, 8
B	8, -3	0, 0

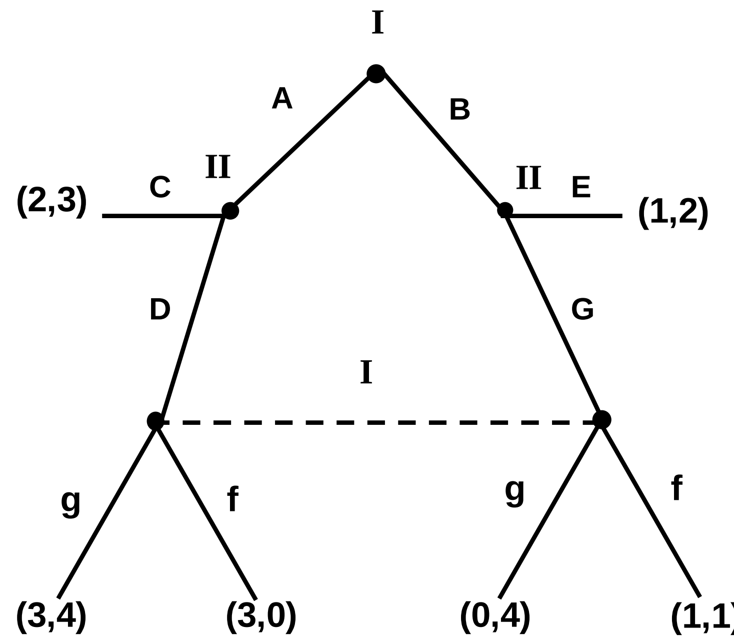


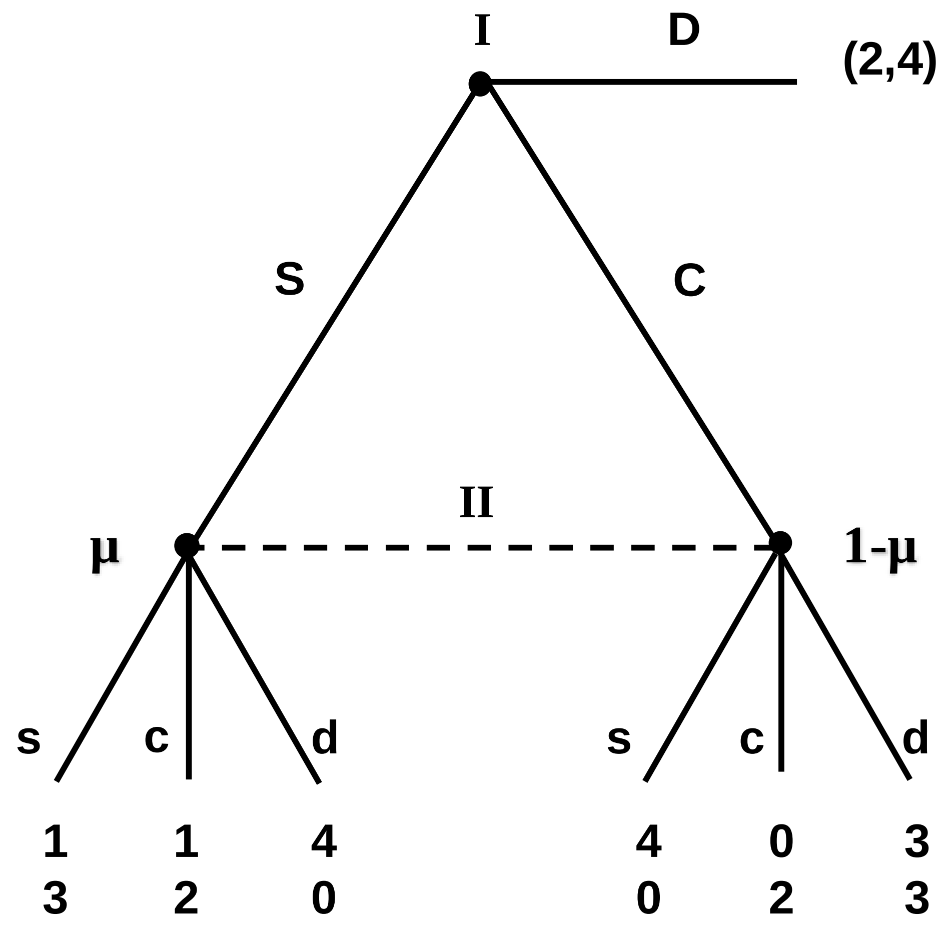
Subgames?



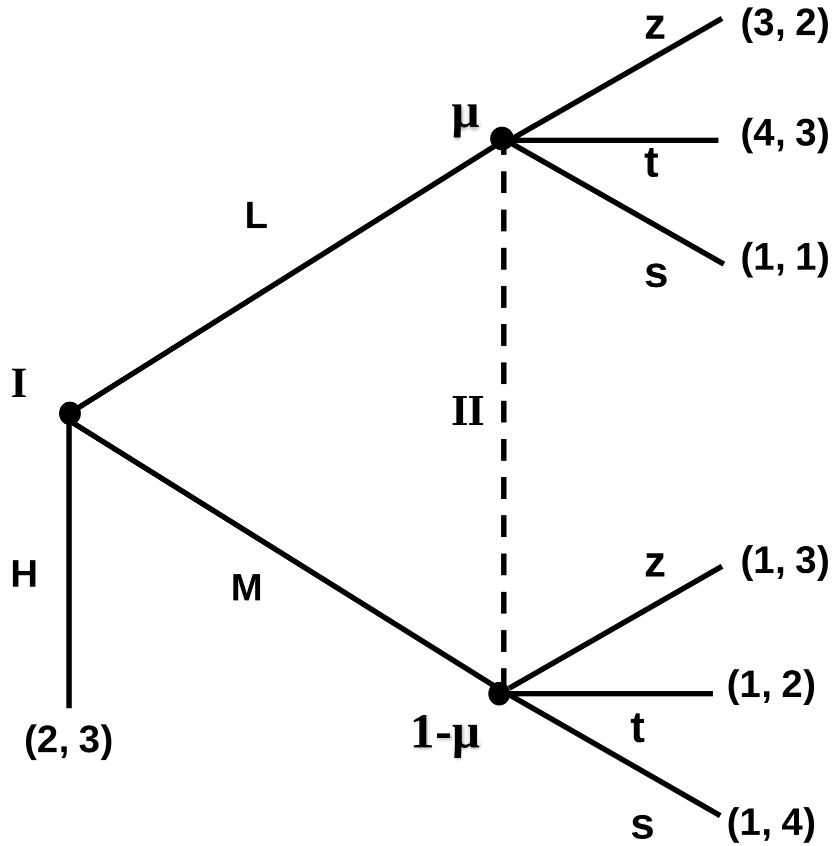
	b	f
B	2, 2	2, 2
F	0, 0	3, 0


I
C
A
B
II
(4,5)
(0,1)
E
D
ρ
I
1-ρ
g
f
g
f
(5,6)
(0,3)
(0,6)
(0,0)


I
A
B
II
II
C
E
(2,3)
(1,2)
D
G
I
g
f
g
f
(3,4)
(3,0)
(0,4)
(1,1)

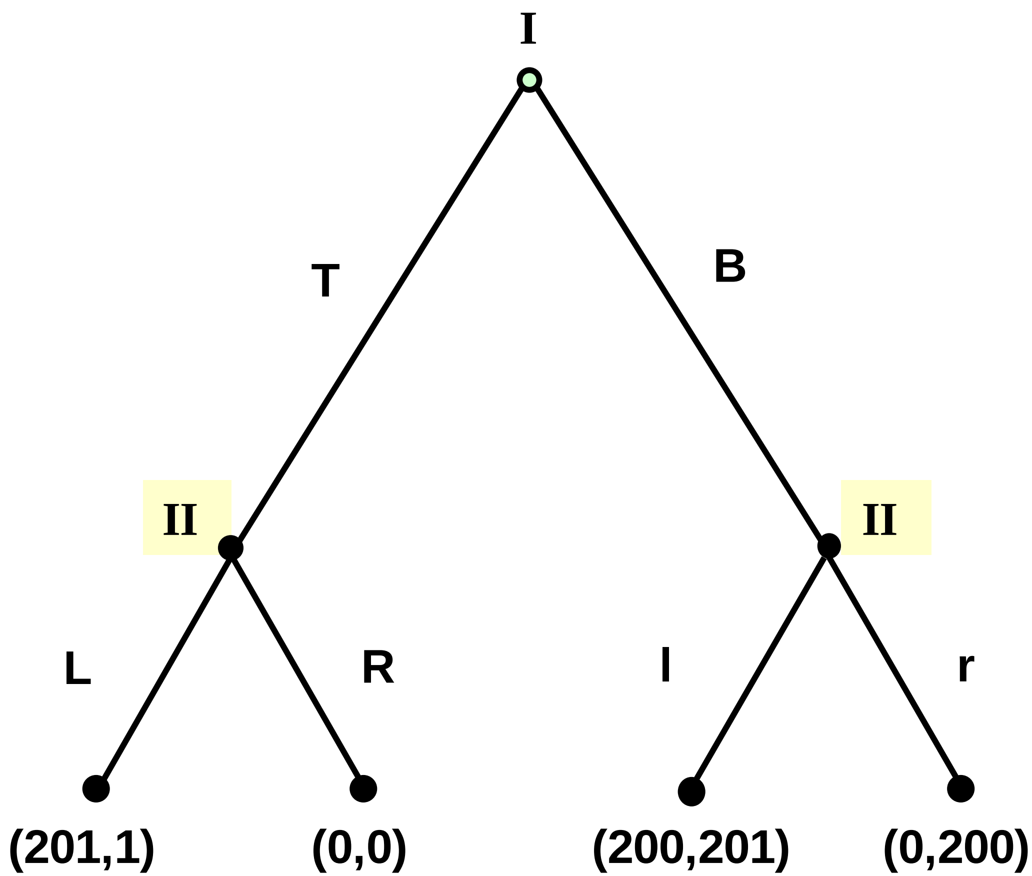

I
D
(2,4)
S
C
II
μ
1-μ
s
c
d
s
c
d
1 1 4 4 0 3
3 2 0 0 2 3

	s	c	d
S	1,3	1,2	4,0
C	4,0	0,2	3,3
D	2,4	$\underline{2}$,$\underline{4}$	2,4

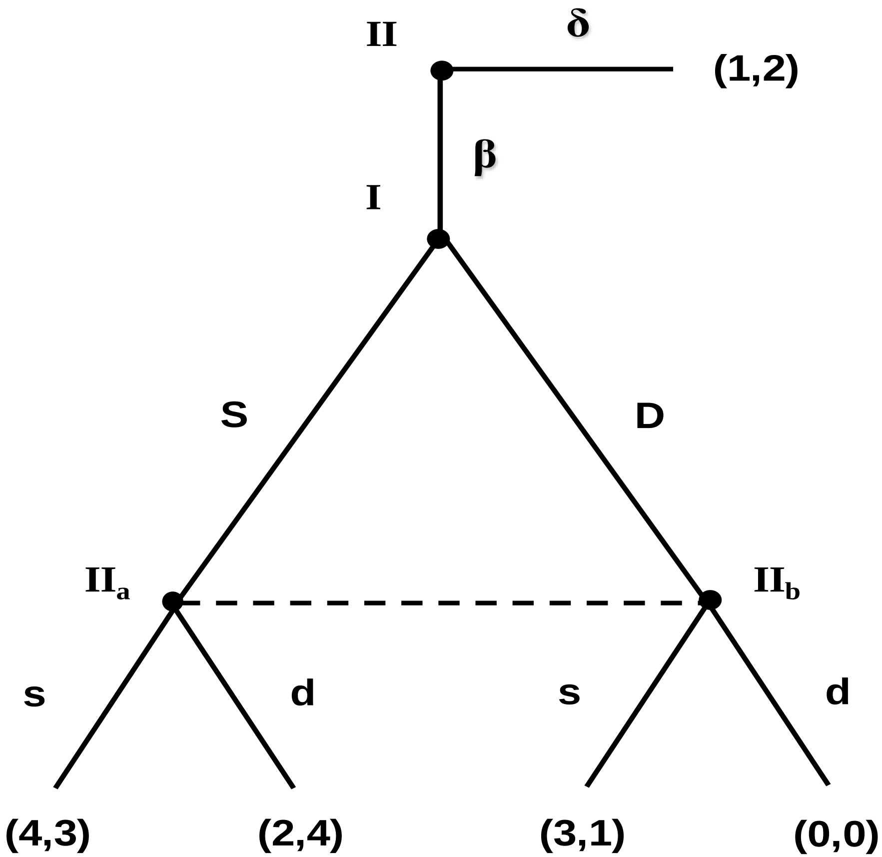


z
(3, 2)
μ
(4, 3)
t
L
(1, 1)
s
I
II
z
(1, 3)
H
M
(1, 2)
t
(2, 3)
1-μ
s
(1, 4)

	s	t	z
H	2, 3	2, 3	2, 3
M	1, 4	1, 2	1, 3
L	1, 1	4, 3	3, 2

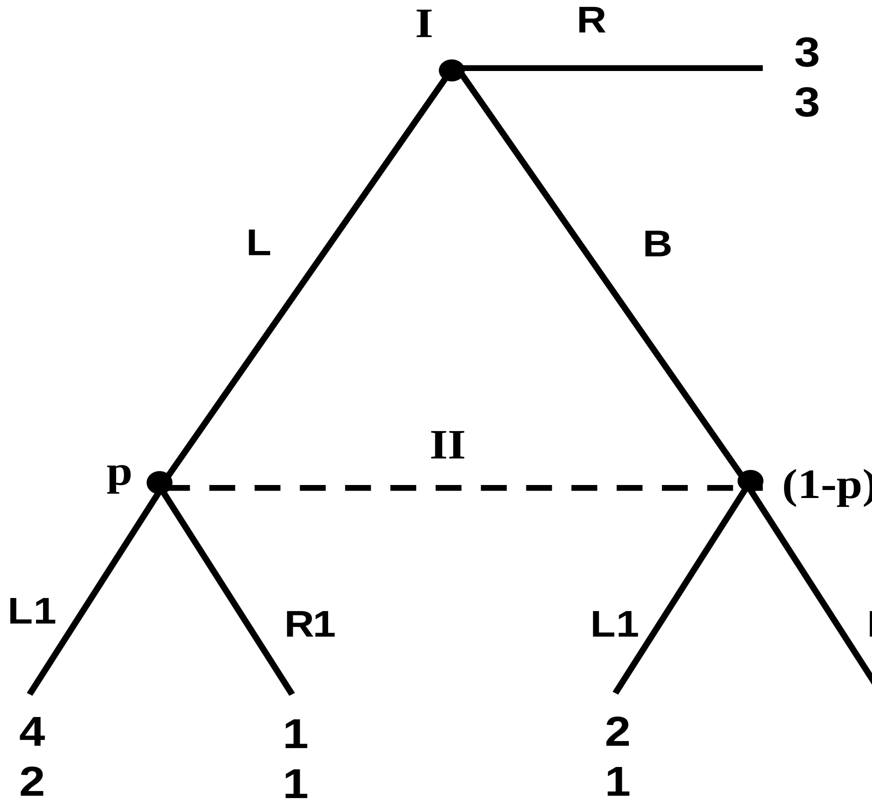

I
T
B
II
II
L
R
l
r
(201,1)
(0,0)
(200,201)
(0,200)

	Ll	Lr	Rl	Rr
T	<u>201</u>,<u>1</u>	<u>201</u>,<u>1</u>	0,0	0,0
B	200,201	0,200	<u>200</u>,<u>201</u>	0,200


II
δ
(1,2)
β
I
S
D
IIa
IIb
s
d
s
d
(4,3)
(2,4)
(3,1)
(0,0)

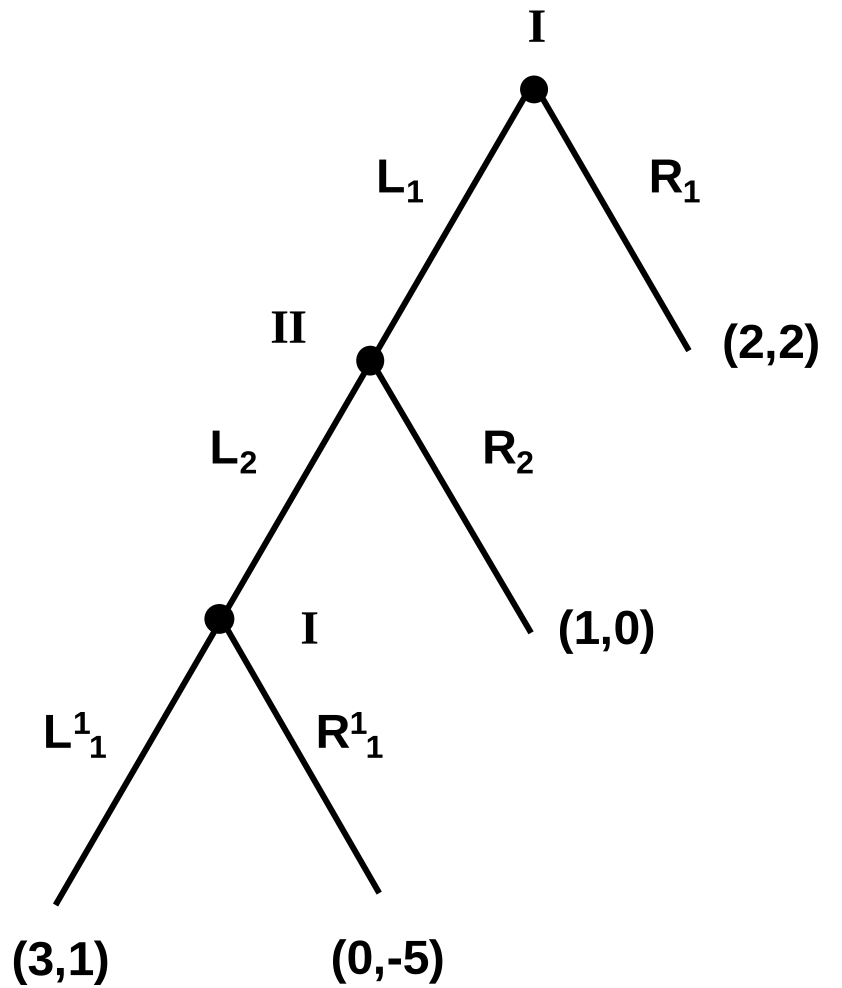
3 NE: $(S, \beta d)$, $(D, \delta s)$, $(D, \delta d)$.

UNIQUE SPNE IS $(S, \beta d)$.


I
R
3
3
L
B
II
p
(1-p)
L1
R1
L1
R1
4
2
1
1
2
1
1
2

	L1	R1
R	3, 3	3, 3
L	4, 2	1, 1
B	2, 1	1, 2

TWO SPNE (R,R1) e (L,L1).


I
L_1
R_1
II
(2,2)
L_2
R_2
I
(1,0)
L^1_1
R^1_1
(3,1)
(0,-5)

	L_2	R_2
R_1, L_1^1	$2,\underline{2}$	$\underline{2},\underline{2}$
R_1, R_1^1	$2,\underline{2}$	$\underline{2},\underline{2}$
L_1, L_1^1	$\underline{3},\underline{1}$	$\underline{1},0$
L_1, R_1^1	$0,-5$	$1,\underline{0}$

	q(1)	(1-q)(2)
p(1)	A, C	a, D
(1-p)(2)	B, c	b, d

$$q = \frac{b-a}{b-a+A-B}$$

$$p = \frac{d-c}{d-c+C-D}$$

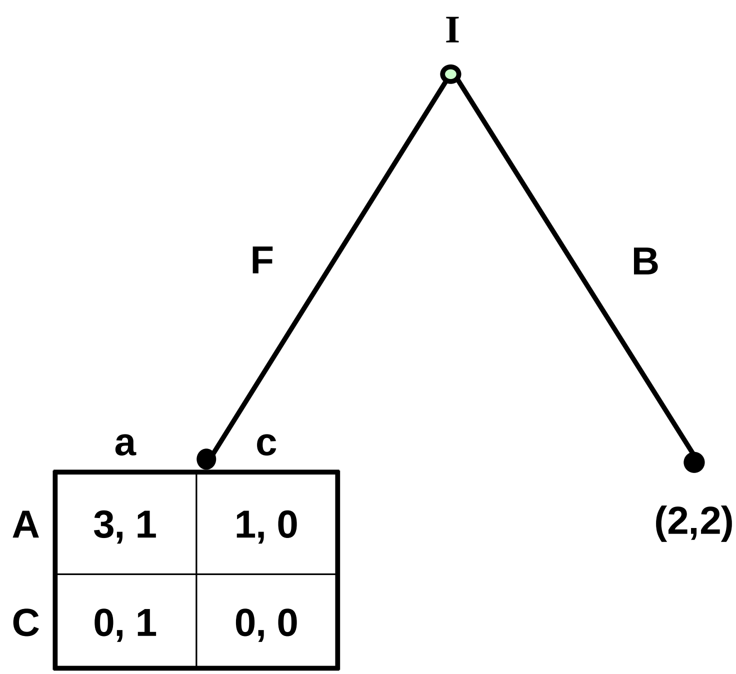
	b	f
B	1,6	0,4
F	1,1	0,4

$$b: \quad 6*0.5+1*0.5=3.5$$

$$f: \quad 4*0.5+4*0.5=4$$

(B,b) is a Nash, althouth it is dominant with respect to the payoffs it could not be played. Notice the expected utilities of player II

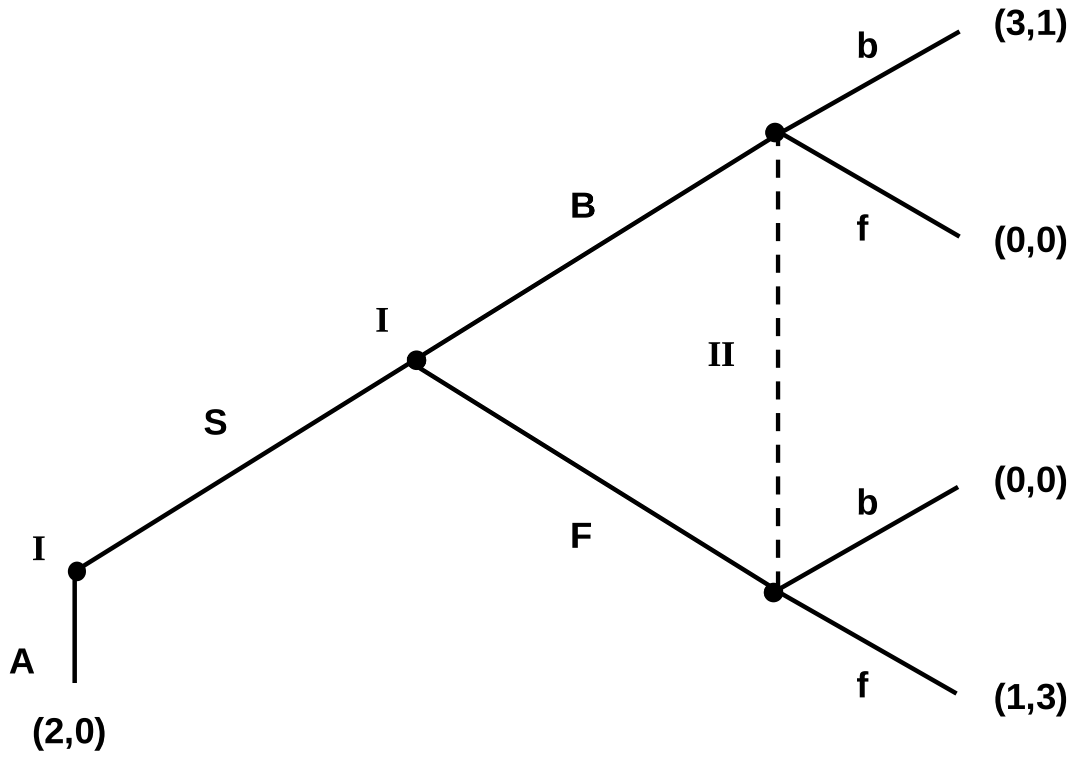
a) Write in strategic form; b) find Nash equilibria c) find SPNE



Notice: the game 2x2 is a subgame of the whole game and it has a unique NE, (A,a). The unique SPNE is, therefore, (FA,a).

	a	c
B	2, 2	2, 2
FA	3, 1	1, 0
FC	0, 1	0, 0

why doesn't player I take 2 and end the game?: **Forward Induction**

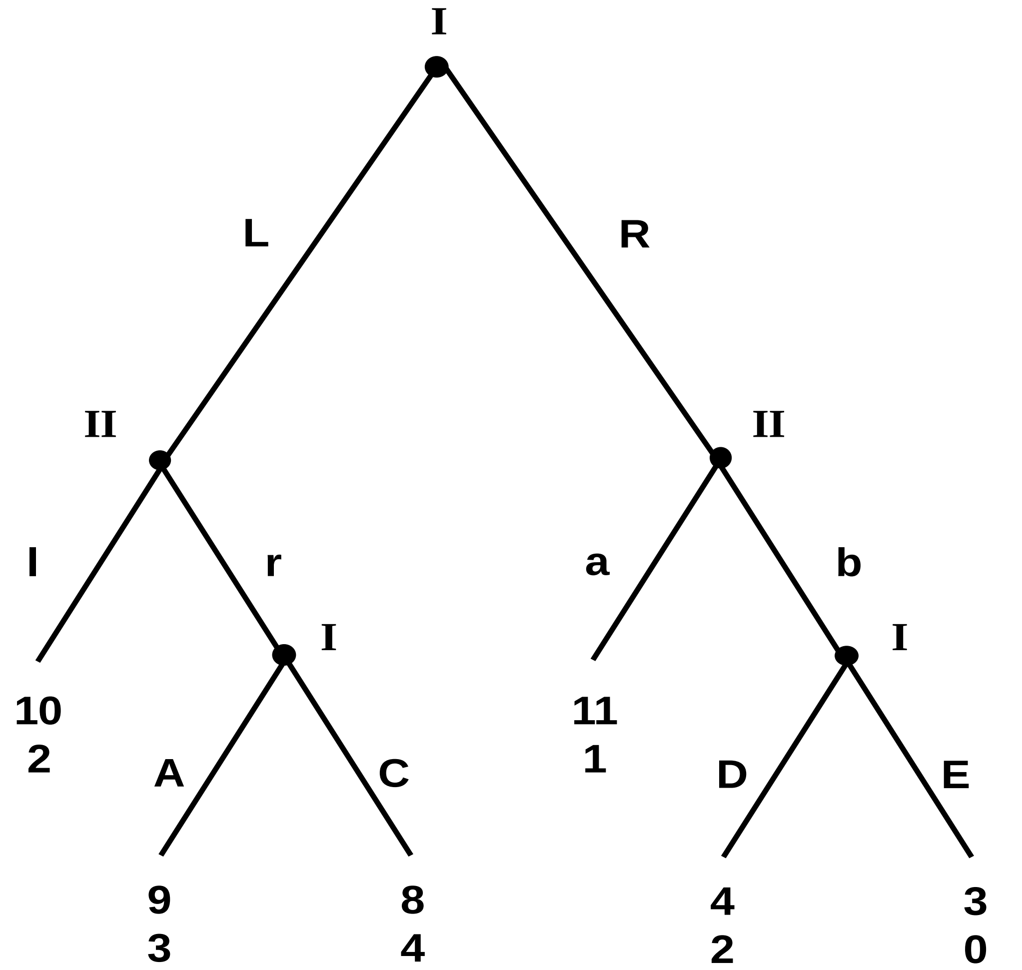


	b	f
AB	2,0	$\underline{2},\underline{0}$
AF	2,0	$\underline{2},\underline{0}$
SB	$\underline{3},\underline{1}$	0,0
SF	0,0	1,3

	b	f
B	$\underline{3},\underline{1}$	0,0
F	0,0	$\underline{1},\underline{3}$

3 Nash: (AB,f); (AF,f); (SB,b). The subgame (BOS) provides 2 Nash: (B,b) e (F,f). This implies 2 SPNE: (AF,f) e (SB,b).

1) BI ?   2) how many Subgames ? 3) assume an information set for player I in the last stage of the game: comment the game and the equilibria



	la	lb	rb	ra
LAD	10, 2	10, 2	9, 3	9, 3
LAE	10, 2	10, 2	9, 3	9, 3
LCE	10, 2	10, 2	8, 4	8, 4
LCD	10, 2	10, 2	8, 4	8, 4
RAD	11, 1	4, 2	4, 2	11, 1
RAE	11, 1	3, 0	3, 0	11, 1
RCE	11, 1	3, 0	3, 0	11, 1
RCD	11, 1	4, 2	4, 2	11, 1

	l	r
A	10, 2	9, 3
C	10, 2	8, 4

NE: (A,r)

	a	b
D	11, 1	4, 2
E	11, 1	3, 0

NE: (D,b) and (E,b)

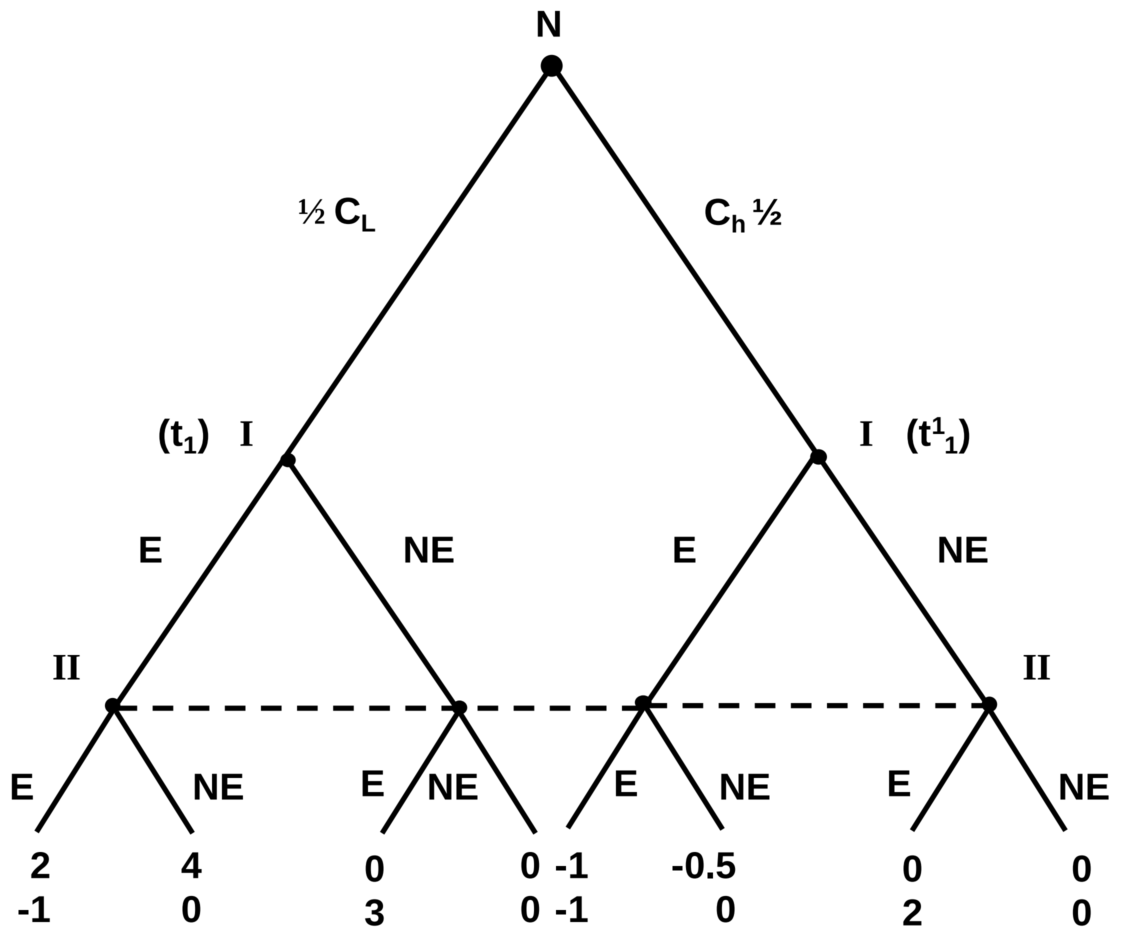
	r
A	9, 3
C	8, 4

EN A

	b
D	4, 2
E	3, 0

EN D

SPNE: (LAD, rb).


N
½ C_L
C_h ½
(t1) I
I (t11)
E
NE
E
NE
II
II
E
NE
E
NE
E
NE
E
NE
2 -1
4 0
0 3
0 0
-1 -1
-0.5 0
0 2
0 0

	E	NE
E	2,-1	4,0
NE	0,3	0,0

LOW COST

	E	NE
E	-1,-1	-0.5, 0
NE	0,3	0,0

HIGH COST

PLAYER II EXPECTED PROFITS

$$E: \quad 0.5(-1)+0.5(3)$$

$$NE: 0.5(0)+0.5(0)$$

$$[(E,NE),E]; \quad \left\{ \begin{pmatrix} E & t_1 \\ NE & t_1^1 \end{pmatrix}; E \quad II \right\}$$

BAYESAN NASH EQUILIBRIUM

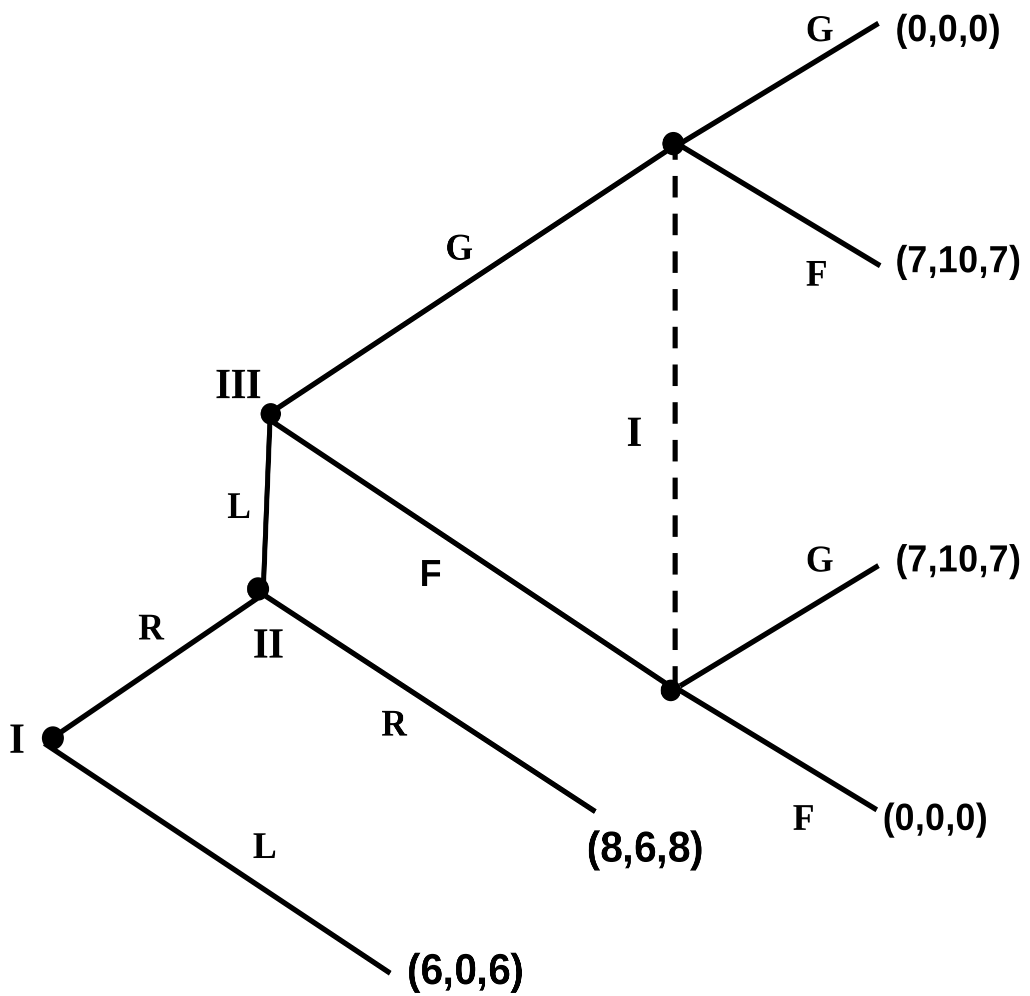
BI: (L,A,D; r,b); 4 subgames ; Imperfect recall

Consider the following prisoner dilemma (10, 10), (2, 14), (14, 2), (5, 5). If both players play the **dominated strategy** forever, what is their payoff considering that the discount factor is 0.95? explain

WRITE the strategic form and note that the dominated strategy leads to the "cooperative" equilibrium whose playoffs are (10, 10).

$$(1+0.95+0.95^2+........)10=\frac{1}{1-0.95}10=\frac{1}{0.05}10=20\cdot 10=200$$

The last stage has three NE two in pure strategies and one in mixed strategies. Make your considerations on any equilibrium.



The solution is related to the coordination game between player III and player I: two Nash equilibria in pure strategies and a balance in mixed strategies: (G, F), (F, G) and (1/2, 1/2).
(GF) and (FG): PAYOFF (7,7). WITH (1/2,1/2) EXPECTED UTILITIES, (3.5, 3.5).

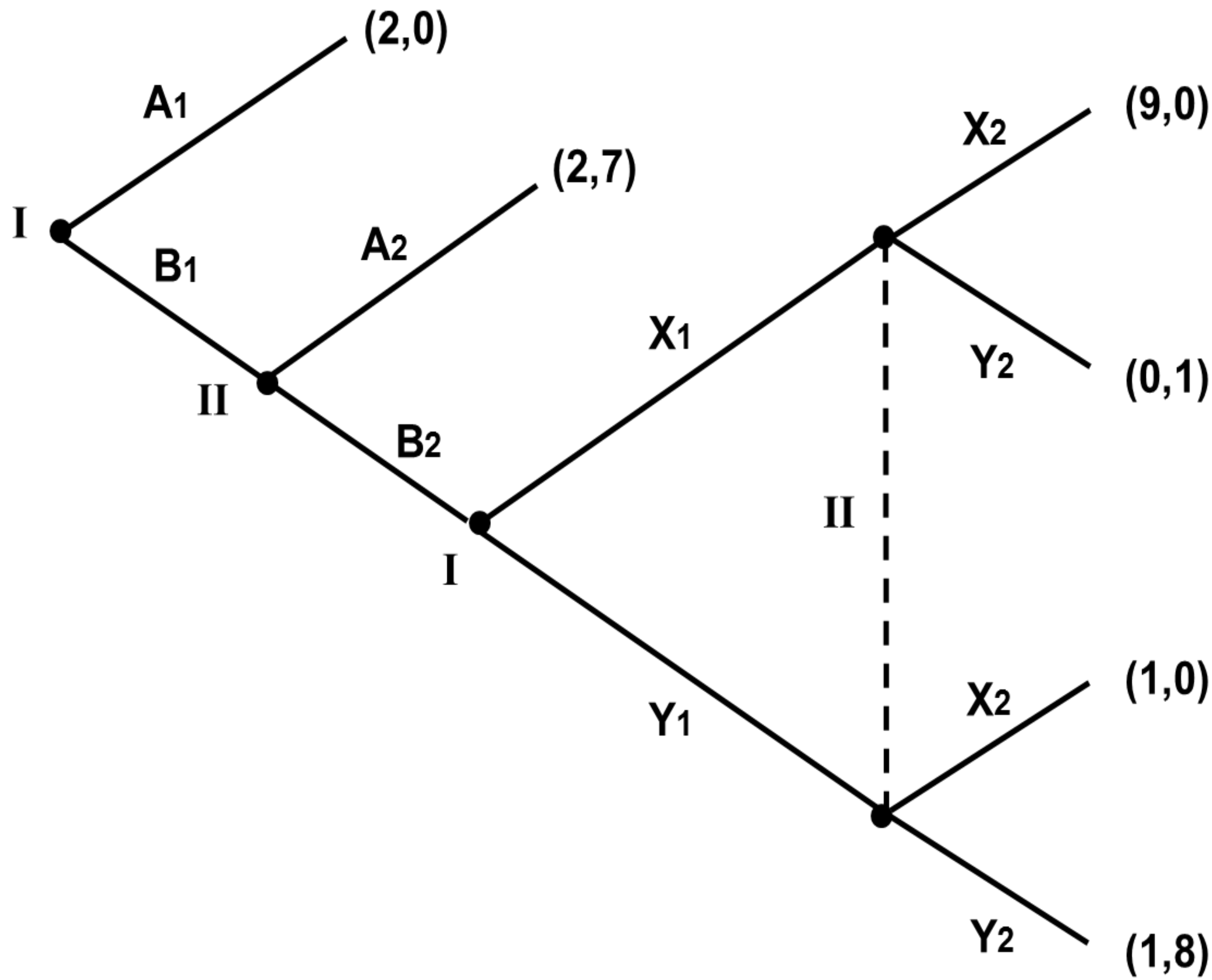
If the two players manage to coordinate then player II will opt for L (take 10) and, consequently, player I for R (take 7).

If the two players (I and III) in the third stage of the game will NOT be able to coordinate (i.e. they do not trust to play (G, F or F, G) but will randomize (play mixed strategies), then player II will choose R and, again, player I will opt for R. So in every ENPS player I is called to choose R.

However, it may be reasonable for I to choose to play L: if I thinks it will be difficult to coordinate with III in the last stage of the game, his payoff expected from the balance of mixed strategies in this stage is 3.5 and for fear that II, instead, will give for granted coordination and games L, player I could end the game with the immediate choice of L.

The point to underline (as a problem) is that the equilibrium of subgames requires not only that Nash is played in all subgames, but also that all players expect the same equilibrium.

II takes for granted a coordination that may not take place !!!!


(2,0)
A1
I
B1
(2,7)
A2
II
B2
I
X1
Y1
(9,0)
X2
Y2
(0,1)
II
(1,0)
X2
Y2
(1,8)

	v	d
L	170, 154	90, 162
R	194, 74	114, 82

TELL ME SOMETHING OF THIS GAME

$$\delta \geq \frac{\pi^{Dev} - \pi^{coop}}{\pi^{Dev} - \pi^{Nash}} = \frac{T-R}{T-P} = \frac{162-154}{162-82} = 8/80 = 0.1 \quad II$$

$$\frac{T-R}{T-P} = \frac{194-170}{194-114} = 24/30 = 0.3 \quad I$$

TELL ME SOMETHING OF THESE DISCONT FACTORS FOR GRIM STRATEGIES

	C	F
C	5,5	-3,8
F	8,-3	0,0

It is clear that cooperation is more likely to:

i) lower values of T;

ii) for more severe punishments, with lower values of P;

can you try it (with a grim strategy) ?

$$\delta \geq \frac{\pi^{Dev} - \pi^{coop}}{\pi^{Dev} - \pi^{Nash}} = \frac{T - R}{T - P} = \frac{8-5}{8-0} = \frac{3}{8} = 0.375$$

$$es. \quad T = 10 \quad \frac{10-5}{10-0} = \frac{5}{10} = 0.5$$

$$es. \quad T = 6 \quad \frac{6-5}{6-0} = \frac{1}{6} = 0{,}166$$

$$es. \quad P = -2 \quad \frac{8-5}{8+2} = \frac{3}{10} = 0.3$$

	R	C
E	-1,-2	1,0
NE	1,0	2,-1

	R	C
E	2,-2	4,0
NE	1,0	2,-1

Suppose 2 types firm I: high productivity and low productivity. Firm II raises prices or keeps them constant

1) find BE and
2) the likelihood that Firm II will choose a price increase

$$R: \quad \frac{1}{2}(-2) + \frac{1}{2}(0) = -1$$

$$C: \quad \frac{1}{2}(0) + \frac{1}{2}(-1) = -\frac{1}{2}$$

So.. It is advantageous to keep prices constant

BE: (E,NE; C)
NE for the type low probability
E for the type high probability
C for firm II

$$R: \quad p(-2) + (1-p)(0) = -2p$$

$$C: \quad p(0) + (1-p)(-1) = -1 + p$$

$$R > C \qquad raice > cons\tan t$$

$$-2p > -1 + p$$

$$p < \frac{1}{3} \quad increase$$

$$p > \frac{1}{3} \quad keep\ cons\tan t$$

Of course this Probability is exogenous and given by NATURE

	E	NE
E	1,2	-1,0
NE	0,3	1,2

	E	NE
E	1,2	-1,3
NE	0,3	1,5

Firms decide to go into the market

Firm 2: two dominant strategies
Distribution Probabilities on Firm 2 types is given by Nature and are CK

1) Find, however, the probability that it is convenient for firm I to play E and the probability that it is convenient to play NE
2) find the respective Bayesian Equilibria

$$E: \quad p(1)+(1-p)(-1)=2p-1$$

$$NE: \quad p(0)+(1-p)(1)=1-p$$

$$p=\frac{2}{3} \quad I \quad indifferent\ to\ E,\ NE$$

$$P>\frac{2}{3} \quad I \quad Enter$$

$$BE:$$

$$(E;(E,NE)) \quad with \quad p>\frac{2}{3};$$

$$(NE;(E,NE)) \quad with \quad p<\frac{2}{3}$$

These are the BEs for the different extractions of Natura