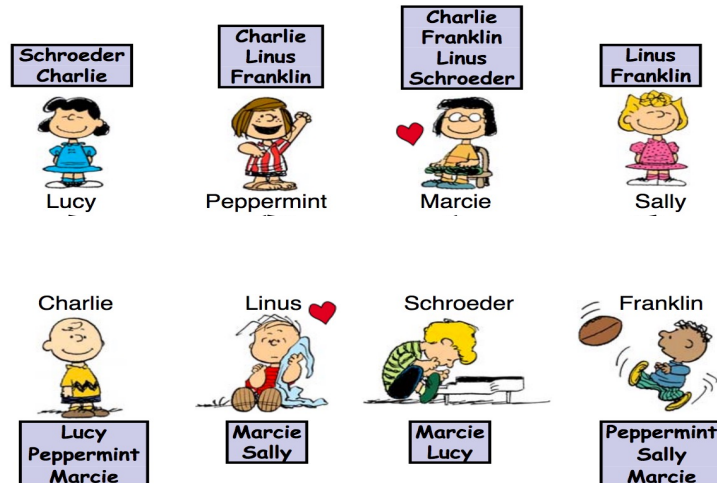


## Exercise

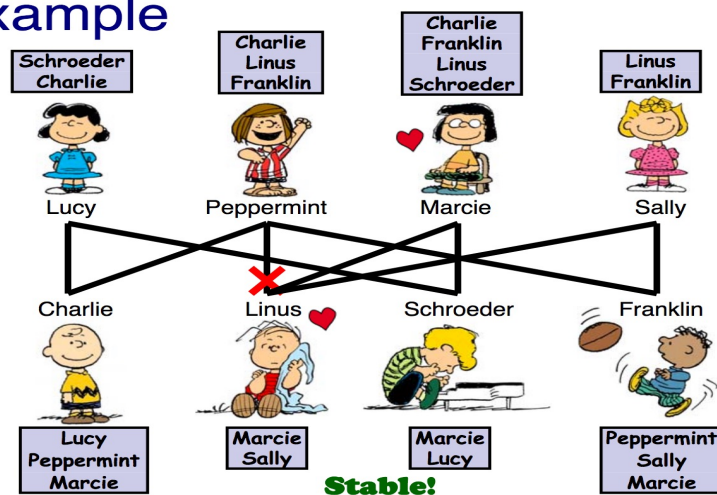
- Find a stable marriage for the Peanuts



## Exercise

- Find a stable marriage for the Peanuts

### Example



# Gale-Shapley algorithm

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- Fix some ordering on the women
- Repeat until everyone is matched
  - Let X be the first unmatched woman in the ordering
  - Find man Y such that Y is the most desirable man in X's list such that Y is unmatched, or Y is currently matched to Z and X is more preferable to Y than Z
  - Match X and Y; possible this turns Z to be unmatched

## Women-optimal solution

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- Let's find a **women optimal stable marriage**
  - Women preference lists
    - Lucy: Schroeder, Charlie
    - Peppermint: Charlie, Linus, Franklin
    - Marcie: Charlie, Franklin, Linus, Schroeder
    - Sally: Linus, Franklin
  - Men preference lists
    - Charlie: Lucy, Peppermint, Marcie
    - Linus: Marcie, Sally
    - Schroeder: Marcie, Lucy
    - Franklin: Peppermint, Sally, Marcie
- Ordering on the women:  
Lucy Peppermint Marcie Sally
- Lucy: Schroeder
  - Peppermint: Charlie
  - Marcie: Franklin
  - Sally: Linus
- Marcie prefers Charlie to Franklin, but Charlie prefers Peppermint to Marcie

## Men-optimal solution

---

- Let's find a **men optimal stable marriage**

Ordering on the men:

Charlie Linus Schroeder Franklin

- Women preference lists

- Lucy: Schroeder, Charlie
- Peppermint: Charlie, Linus, Franklin
- Marcie: Charlie, Franklin, Linus, Schroeder
- Sally: Linus, Franklin

- Charlie: Lucy
- Linus: Marcie
- Schroeder: ~~Marcie~~ **Lucy**
- Franklin:

- Men preference lists

- Charlie: Lucy, Peppermint, Marcie
- Linus: Marcie, Sally
- Schroeder: Marcie, Lucy
- Franklin: Peppermint, Sally, Marcie

- Charlie:
- Linus: Marcie
- Schroeder: Lucy
- Franklin:

## Men-optimal solution

---

- Let's find a **men optimal stable marriage**

- Women preference lists

- Lucy: Schroeder, Charlie
- Peppermint: Charlie, Linus, Franklin
- Marcie: Charlie, Franklin, Linus, Schroeder
- Sally: Linus, Franklin

- Men preference lists

- Charlie: Lucy, Peppermint, Marcie
- Linus: Marcie, Sally
- Schroeder: Marcie, Lucy
- Franklin: Peppermint, Sally, Marcie

- Charlie: Peppermint
- Linus: Marcie
- Schroeder: Lucy
- Franklin:

- Charlie: Peppermint
- Linus: Marcie
- Schroeder: Lucy
- Franklin: ~~Peppermint~~ Sally

Franklin prefers Peppermint to Sally, but Peppermint prefers Charlie to Franklin

## Men-optimal solution

---

- What happens if at this point we match Franklin before Charlie?
  - Women preference lists
    - Lucy: Schroeder, Charlie
    - Peppermint: Charlie, Linus, Franklin
    - Marcie: Charlie, Franklin, Linus, Schroeder
    - Sally: Linus, Franklin
  - Men preference lists
    - Charlie: Lucy, Peppermint, Marcie
    - Linus: Marcie, Sally
    - Schroeder: Marcie, Lucy
    - Franklin: Peppermint, Sally, Marcie
- Charlie:
  - Linus: Marcie
  - Schroeder: Lucy
  - Franklin:
- Charlie:
  - Linus: Marcie
  - Schroeder: Lucy
  - Franklin: Peppermint

## Men-optimal solution

---

- What happens if we match Franklin before Charlie?
  - Women preference lists
    - Lucy: Schroeder, Charlie
    - Peppermint: Charlie, Linus, Franklin
    - Marcie: Charlie, Franklin, Linus, Schroeder
    - Sally: Linus, Franklin
  - Men preference lists
    - Charlie: Lucy, Peppermint, Marcie
    - Linus: Marcie, Sally
    - Schroeder: Marcie, Lucy
    - Franklin: Peppermint, Sally, Marcie
- Charlie: Peppermint
  - Linus: Marcie
  - Schroeder: Lucy
  - Franklin: Peppermint
- Charlie: Peppermint
  - Linus: Marcie
  - Schroeder: Lucy
  - Franklin:
- Charlie: Peppermint
  - Linus: Marcie
  - Schroeder: Lucy
  - Franklin: Sally

## Wrapping up

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| Women-optimal stable marriage | Men-optimal stable marriage |
|-------------------------------|-----------------------------|
| (Lucy, Schroeder)             | (Charlie, Peppermint)       |
| (Peppermint, Charlie)         | (Linus, Marcie)             |
| (Marcie, Franklin)            | (Schroeder, Lucy)           |
| (Sally, Linus)                | (Franklin, Sally)           |

**Women-optimal:** Marcie prefers Franklin to Linus, Sally prefers Linus to Franklin: such matching is best for all women among all stable matchings, and worst for all men

**Men-optimal:** Linus prefers Marcie to Sally, Franklin prefers Sally to Marcie: such matching is best for all men among all stable matchings, and worst for all women