

Exercise on hashing (Hashing.pdf, slide 32)

Let's consider a hash table of size 7 and open addressing with quadratic probe using the hash function $h(k) = k \bmod 7$. Insert the keys 18, 14, 21, 1 and 35 (in this order) and write down the entries of the hash table after each insertion.

Recall that with quadratic hashing the probe sequence is

$$h_i(k) = (h(k) + i^2) \bmod \text{TableSize}$$

where $h(k)$ is the original hash function (in our case $h(k) = k \bmod 7$). That means:

$$0^{\text{th}} \text{ probe: } h(k)$$

$$1^{\text{st}} \text{ probe: } h_1(k) = (h(k) + 1^2) \bmod \text{TableSize} = (h(k) + 1) \bmod \text{TableSize}$$

$$2^{\text{nd}} \text{ probe: } h_2(k) = (h(k) + 2^2) \bmod \text{TableSize} = (h(k) + 4) \bmod \text{TableSize}$$

$$3^{\text{rd}} \text{ probe: } h_3(k) = (h(k) + 3^2) \bmod \text{TableSize} = (h(k) + 9) \bmod \text{TableSize}$$

$$4^{\text{th}} \text{ probe: } h_4(k) = (h(k) + 4^2) \bmod \text{TableSize} = (h(k) + 16) \bmod \text{TableSize}$$

$$5^{\text{th}} \text{ probe: } h_5(k) = (h(k) + 5^2) \bmod \text{TableSize} = (h(k) + 25) \bmod \text{TableSize}$$

...

We consider a new probe until an empty (i.e., *open*) slot is found.

Let's **insert key 18**.

$$h(18) = 18 \bmod 7 = 4$$

and slot 4 is empty, so we insert key 18 into slot 4.

0	
1	
2	
3	
4	18
5	
6	

Insert 18

Let's **insert key 14**.

$$\text{Since } h(14) = 14 \bmod 7 = 0$$

and slot 0 is empty, we insert key 14 into slot 0.

0	14
1	
2	
3	
4	18
5	
6	

Insert 14

Let's **insert key 21**.

$$\text{Since } h(21) = 21 \bmod 7 = 0$$

and slot 0 is full, there is a collision.

Let's consider the quadratic probe sequence, starting from $i=1$

$$h_1(21) = ((21 \bmod 7) + 1^2) \bmod 7 = (0 + 1) \bmod 7 = 1$$

Slot 1 is empty, so we insert key 21 into slot 1.

0	14	↻ $i=1$
1	21	
2		
3		
4	18	
5		
6		

Insert 21

Let's **insert key 1**.

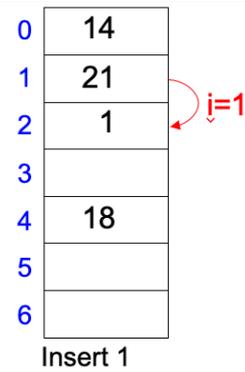
Since $h(1) = 1 \bmod 7 = 1$

and slot 1 is full, there is a collision.

Let's consider the quadratic probe sequence, starting from $i=1$

$h_1(1) = ((1 \bmod 7) + 1^2) \bmod 7 = (1 + 1) \bmod 7 = 2 \bmod 7 = 2$

Slot 2 is empty, so we insert key 1 into slot 2.



Let's **insert key 35**.

Since $h(35) = 35 \bmod 7 = 0$

and slot 0 is full, there is a collision.

Let's consider the quadratic probe sequence, starting from $i=1$

$h_1(35) = ((35 \bmod 7) + 1^2) \bmod 7 = (0 + 1) \bmod 7 = 1$

Slot 1 is full, again a collision.

Let's try with the 2nd step in the probe sequence ($i=2$):

$h_2(35) = ((35 \bmod 7) + 2^2) \bmod 7 = (0 + 4) \bmod 7 = 4$

Slot 4 is full, again a collision.

Let's try with the 3rd step in the probe sequence ($i=3$):

$h_3(35) = ((35 \bmod 7) + 3^2) \bmod 7 = (0 + 9) \bmod 7 = 2$

Slot 2 is full, again a collision.

Let's try with the 4th step in the probe sequence ($i=4$):

$h_4(35) = ((35 \bmod 7) + 4^2) \bmod 7 = (0 + 16) \bmod 7 = 2$

Slot 2 is full, again a collision.

Let's try with the 5th step in the probe sequence ($i=5$):

$h_5(35) = ((35 \bmod 7) + 5^2) \bmod 7 = (0 + 25) \bmod 7 = 4$

Slot 4 is full, again a collision.

Let's try with the 6th step in the probe sequence ($i=6$):

$h_6(35) = ((35 \bmod 7) + 6^2) \bmod 7 = (0 + 36) \bmod 7 = 1$

Slot 1 is full, again a collision.

Let's try with the 7th step in the probe sequence ($i=7$):

$h_7(35) = ((35 \bmod 7) + 7^2) \bmod 7 = (0 + 49) \bmod 7 = 0$

Slot 0 is full, again a collision.

Let's try with the 8th step in the probe sequence ($i=8$):

$h_8(35) = ((35 \bmod 7) + 8^2) \bmod 7 = (0 + 64) \bmod 7 = 1$

Slot 1 is full, again a collision.

Let's try with the 9th step in the probe sequence ($i=9$):

$h_9(35) = ((35 \bmod 7) + 9^2) \bmod 7 = (0 + 81) \bmod 7 = 4$

Slot 4 is full, again a collision.

Let's try with the 10th step in the probe sequence ($i=10$):

$h_{10}(35) = ((35 \bmod 7) + 10^2) \bmod 7 = (0 + 100) \bmod 7 = 6$

Slot 6 is empty! We insert key 35 into slot 6.

