

### Exercise on hashing (Hashing.pdf, slide 45)

Let's consider a hash table of size 7 using open addressing with quadratic probing. The hash function is  $h(k) = k \bmod 7$

Insert the keys 18, 14, 21, 1, and 35 (in this order) and write down the table entries after each insertion.

Quadratic probing uses a quadratic function to calculate the next slot to check. Therefore, the probe sequence is:

$$h_i(k) = (h(k) + i^2) \bmod \text{TableSize}$$

where  $i=0, 1, 2, \dots$  and  $h(k)$  is the original hash function (in our case  $h(k) = k \bmod 7$ ). The  $\bmod \text{TableSize}$  ensures that the new index wraps around to stay within the bounds of the table.

That means:

$$0^{\text{th}} \text{ probe: } h_0(k) = h(k)$$

$$1^{\text{st}} \text{ probe: } h_1(k) = (h(k) + 1^2) \bmod \text{TableSize} = (h(k) + 1) \bmod \text{TableSize}$$

$$2^{\text{nd}} \text{ probe: } h_2(k) = (h(k) + 2^2) \bmod \text{TableSize} = (h(k) + 4) \bmod \text{TableSize}$$

$$3^{\text{rd}} \text{ probe: } h_3(k) = (h(k) + 3^2) \bmod \text{TableSize} = (h(k) + 9) \bmod \text{TableSize}$$

$$4^{\text{th}} \text{ probe: } h_4(k) = (h(k) + 4^2) \bmod \text{TableSize} = (h(k) + 16) \bmod \text{TableSize}$$

$$5^{\text{th}} \text{ probe: } h_5(k) = (h(k) + 5^2) \bmod \text{TableSize} = (h(k) + 25) \bmod \text{TableSize}$$

...

We continue probing until an empty (i.e., *open*) slot is found.

Informally, starting from the initial slot the key would be inserted (the  $0^{\text{th}}$  probe), we move ahead by 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 ... slots until we found an empty slot.

Let's **insert key 18**.

$$h(18) = 18 \bmod 7 = 4$$

Slot 4 is empty, so we insert key 18 into slot 4.

|   |    |
|---|----|
| 0 |    |
| 1 |    |
| 2 |    |
| 3 |    |
| 4 | 18 |
| 5 |    |
| 6 |    |

Insert 18

Let's **insert key 14**.

$$\text{Since } h(14) = 14 \bmod 7 = 0$$

and slot 0 is empty, so we insert key 14 into slot 0.

|   |    |
|---|----|
| 0 | 14 |
| 1 |    |
| 2 |    |
| 3 |    |
| 4 | 18 |
| 5 |    |
| 6 |    |

Insert 14

Let's **insert key 21**.

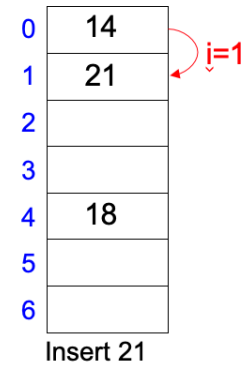
Since  $h(21) = 21 \bmod 7 = 0$

and slot 0 is occupied, there is a collision.

Let's consider the quadratic probe sequence, starting from  $i=1$

$h_1(21) = ((21 \bmod 7) + 1^2) \bmod 7 = (0 + 1) \bmod 7 = 1$

Slot 1 is empty, so we insert key 21 into slot 1.



Let's **insert key 1**.

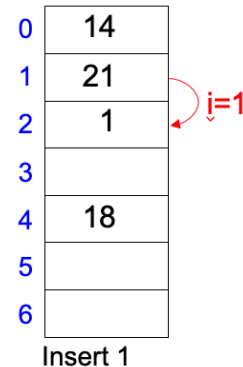
Since  $h(1) = 1 \bmod 7 = 1$

and slot 1 is full, there is a collision.

Let's consider the quadratic probe sequence, starting from  $i=1$

$h_1(1) = ((1 \bmod 7) + 1^2) \bmod 7 = (1 + 1) \bmod 7 = 2 \bmod 7 = 2$

Slot 2 is empty, so we insert key 1 into slot 2.



Let's **insert key 35**.

Since  $h(35) = 35 \bmod 7 = 0$

and slot 0 is full, there is a collision.

Let's consider the quadratic probe sequence, starting from  $i=1$

$h_1(35) = ((35 \bmod 7) + 1^2) \bmod 7 = (0 + 1) \bmod 7 = 1$

Slot 1 is full, again a collision.

Let's try with the 2<sup>nd</sup> step in the probe sequence ( $i=2$ ):

$h_2(35) = ((35 \bmod 7) + 2^2) \bmod 7 = (0 + 4) \bmod 7 = 4$

Slot 4 is full, again a collision.

Let's try with the 3<sup>rd</sup> step in the probe sequence ( $i=3$ ):

$h_3(35) = ((35 \bmod 7) + 3^2) \bmod 7 = (0 + 9) \bmod 7 = 2$

Slot 2 is full, again a collision.

Let's try with the 4<sup>th</sup> step in the probe sequence ( $i=4$ ):

$h_4(35) = ((35 \bmod 7) + 4^2) \bmod 7 = (0 + 16) \bmod 7 = 2$

Slot 2 is full, again a collision.

Let's try with the 5<sup>th</sup> step in the probe sequence ( $i=5$ ):

$h_5(35) = ((35 \bmod 7) + 5^2) \bmod 7 = (0 + 25) \bmod 7 = 4$

Slot 4 is full, again a collision.

Let's try with the 6<sup>th</sup> step in the probe sequence ( $i=6$ ):

$h_6(35) = ((35 \bmod 7) + 6^2) \bmod 7 = (0 + 36) \bmod 7 = 1$

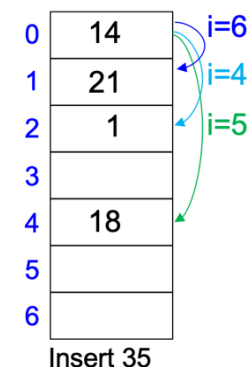
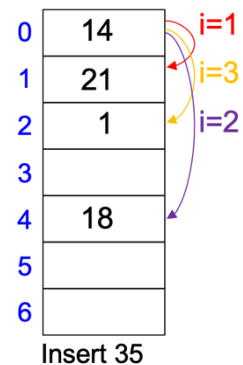
Slot 1 is full, again a collision.

Let's try with the 7<sup>th</sup> step in the probe sequence ( $i=7$ ):

$h_7(35) = ((35 \bmod 7) + 7^2) \bmod 7 = (0 + 49) \bmod 7 = 0$

Slot 0 is full, again a collision.

Let's try with the 8<sup>th</sup> step in the probe sequence ( $i=8$ ):



$h_8(35) = ((35 \bmod 7) + 8^2) \bmod 7 = (0 + 64) \bmod 7 = 1$   
Slot 1 is full, again a collision.

Let's try with the 9<sup>th</sup> step in the probe sequence (i=9):  
 $h_9(35) = ((35 \bmod 7) + 9^2) \bmod 7 = (0 + 81) \bmod 7 = 4$   
Slot 4 is full, again a collision.

Let's try with the 10<sup>th</sup> step in the probe sequence (i=10):  
 $h_{10}(35) = ((35 \bmod 7) + 10^2) \bmod 7 = (0 + 100) \bmod 7 = 2$   
Slot 2 is full, again a collision.

Let's try with the 11<sup>th</sup> step in the probe sequence (i=11):  
 $h_{11}(35) = ((35 \bmod 7) + 11^2) \bmod 7 = (0 + 121) \bmod 7 = 2$   
Slot 2 is full, again a collision.

Let's try with the 12<sup>th</sup> step in the probe sequence (i=12):  
 $h_{12}(35) = ((35 \bmod 7) + 12^2) \bmod 7 = (0 + 144) \bmod 7 = 4$   
Slot 4 is full, again a collision.

Let's try with the 13<sup>th</sup> step in the probe sequence (i=13):  
 $h_{13}(35) = ((35 \bmod 7) + 13^2) \bmod 7 = (0 + 169) \bmod 7 = 1$   
Slot 1 is full, again a collision.

Let's try with the 14<sup>th</sup> step in the probe sequence (i=14):  
 $h_{14}(35) = ((35 \bmod 7) + 14^2) \bmod 7 = (0 + 196) \bmod 7 = 0$   
Slot 0 is full, again a collision.

This is a very unlucky situation: the probe sequence is 0, 1, 4, 2, 2, 4, 1 and then repeats endlessly. As a result, we are unable to find a free slot for the key. The underlying reason is that the quadratic probing sequence does not cover all slots in the table, so some positions remain unreachable.